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CONCRETE ENGINEERS' HANDBOOK

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S. F.

The Concrete Engineer's Handbook

A CONVENIENT REFERENCE BOOK
For All Persons Interested In

Cement, Plain and Reinforced Concrete,
Building Construction, Architecture,
Concrete Blocks, Mill Building,
Office Building, Fireproof
Houses, Etc.

BY

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PREFACE

The publishers have not attempted in this work to produce a condensed cyclopedia covering the broad field of concrete engineering, but they have aimed to present to the public a handy reference book convenient to carry in the pocket—a pocketbook in reality—and containing rules, formulas, tables, and diagrams that are often used and needed by architects, concrete engineers, inspectors, superintendents, foremen, carpenters, contractors, draftsmen, designers, house owners and prospective house owners; in fact, every one engaged in any profession or trade connected with building of concrete, or in any way interested therein.

The aim of the publishers has been to select from the vast amount of material on hand only that portion which is most likely to be used in connection with daily work, or which will be most frequently consulted. Although the treatment of some subjects is of necessity brief, it has been the aim to so distribute the space available that it would cover the more important subjects as fully as possible. It has been found impracticable to give tables for the strength of concrete beams and slabs, because the stresses employed are subject to city regulations. Until the large cities adopt uniform requirements, such tables cannot come into general use. The tables selected throughout this work are those most in demand, and the applications of the rules and

formulas are shown in many cases by practical examples and solutions, together with explanations.

The material is conveniently arranged for ready reference. Particular attention has been given to the weights of various materials. Most tables on weights are incomplete and it is often necessary to search some time before the weight of any particular material can be ascertained. It is hoped that this volume will obviate tedious work of this kind.

The section on tests of cement will be found to contain useful information that is usually found only in special treatises on the subject.

INTERNATIONAL CORRESPONDENCE SCHOOLS,
July, 1911. Scranton, Pa.

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The Concrete Engineer's Handbook

USEFUL TABLES

WEIGHTS AND MEASURES

LINEAR MEASURE

12	inches (in.)	=1 foot	ft.
3	feet	=1 yard	yd.
5.5	yards	=1 rod	rd.
40	rods	=1 furlong	fur.
8	furlongs	=1 mile	mi.

in.	*ft.*	*yd.*	*rd.*	*fur.*	*mi.*
36=	3=	1			
198=	16.5=	5.5=	1		
7,920=	660=	220=	40=	1	
63,360=	5,280=	1,760=	320=	8=	1

SURVEYOR'S MEASURE

7.92 inches	= 1 link	li.
25 links	= 1 rod	rd.
4 rods 100 links 66 feet	=1 chain	ch.
80 chains	=1 mile	mi.

1 mi. = 80 ch. = 320 rd. = 8,000 li. = 63,360 in.

SQUARE MEASURE

144 square inches (sq. in.)	= 1 square foot	sq. ft.
9 square feet	= 1 square yard	sq. yd.
$30\frac{1}{4}$ square yards	= 1 square rod	sq. rd.
160 square rods	= 1 acre	A.
640 acres	= 1 square mile	sq. mi.

sq. mi.	*A*	*sq. rd.*	*sq. yd.*	*sq. ft.*	*sq. in.*
1	= 640	= 102,400	= 3,097,600	= 27,878,400	= 4,014,489,600

SURVEYOR'S SQUARE MEASURE

625 square links (sq. li.)	= 1 square rod	sq. rd.
16 square rods	= 1 square chain	sq. ch.
10 square chains	= 1 acre	A.
640 acres	= 1 square mile	sq. mi.
36 square miles (6 mi. square)	= 1 township	Tp.

1 sq. mi. = 640 A. = 6,400 sq. ch. = 102,400 sq. rd. = 64,000,000 sq. li.

The acre contains 4,840 sq. yd., or 43,560 sq. ft., and is equal to the area of a square measuring 208.71 ft. on a side.

CUBIC MEASURE

1,728 cubic inches (cu. in.)	= 1 cubic foot	cu. ft.
27 cubic feet	= 1 cubic yard	cu. yd.
128 cubic feet	= 1 cord	cd.
$24\frac{3}{4}$ cubic feet	= 1 perch	P.

1 cu. yd. = 27 cu. ft. = 46,656 cu. in.

MEASURE OF ANGLES OR ARCS

60 seconds (″)	= 1 minute	′
60 minutes	= 1 degree	°
90 degrees	= 1 rt. angle or quadrant	□
360 degrees	= 1 circle	cir.

1 cir. = 360° = 21,600′ = 1,296,000″

AVOIRDUPOIS WEIGHT

437.5 grains (gr.)	= 1 ounce	oz.
16 ounces	= 1 pound	lb.
100 pounds	= 1 hundredweight	cwt.
20 cwt., or 2,000 lb.	= 1 ton	T.

1 T. = 20 cwt. = 2,000 lb. = 32,000 oz. = 14,000,000 gr.

The avoirdupois pound contains 7,000 gr.

LONG-TON TABLE

16 ounces	= 1 pound	lb.
112 pounds	= 1 hundredweight	cwt.
20 cwt., or 2,240 lb.	= 1 ton	T.

TROY WEIGHT

24 grains (gr.)	= 1 pennyweight	pwt.
20 pennyweights	= 1 ounce	oz.
12 ounces	= 1 pound	lb.

1 lb. = 12 oz. = 240 pwt. = 5,760 gr.

DRY MEASURE

2 pints (pt.)	= 1 quart	qt.
8 quarts	= 1 peck	pk.
4 pecks	= 1 bushel	bu.

1 bu. = 4 pk. = 32 qt. = 64 pt.

The U. S. struck bushel contains 2,150.42 cu. in. = 1.2444 cu. ft. By law, its dimensions are those of a cylinder 18½ in. in diameter and 8 in. deep. The heaped bushel is equal to 1¼ struck bushels, the cone being 6 in. high. The dry gallon contains 268.8 cu. in., being ⅛ struck bushel.

For approximations, the bushel may be taken at 1¼ cu. ft.; or 1 cu. ft. may be considered ⅘ bushel.

The British bushel contains 2,218.19 cu. in. = 1.2837 cu. ft. = 1.032 U. S. bushels.

LIQUID MEASURE

4 gills (gi.)	= 1 pint	pt.
2 pints	= 1 quart	qt.
4 quarts	= 1 gallon	gal.
31½ gallons	= 1 barrel	bbl.
2 barrels, or 63 gallons	= 1 hogshead	hhd.

1 hhd. = 2 bbl. = 63 gal. = 252 qt. = 504 pt. = 2,016 gi.

The U. S. gallon contains 231 cu. in. = .134 cu. ft., nearly; or 1 cu. ft. contains 7.481 gal. The following cylinders contain the given measures very closely:

	Diam.	*Height*		*Diam.*	*Height*
Gill........	$1\frac{3}{4}$ in.	3 in.	Gallon....	7 in.	6 in.
Pint......	$3\frac{1}{2}$ in.	3 in.	8 gallons	14 in.	12 in.
Quart.....	$3\frac{1}{2}$ in.	6 in.	10 gallons	14 in.	15 in.

When water is at its maximum density, 1 cu. ft. weighs 62.425 lb. and 1 gal. weighs 8.345 lb.

For approximations, 1 cu. ft. of water is considered equal to $7\frac{1}{2}$ gal., and 1 gal. as weighing $8\frac{1}{3}$ lb.

The British imperial gallon, both liquid and dry, contains 277.274 cu. in. = .16046 cu. ft., and is equivalent to the volume of 10 lb. of pure water at 62° F. To reduce British to U. S. liquid gallons, multiply by 1.2. Conversely, to convert U. S. into British liquid gallons, divide by 1.2; or, increase the number of gallons one-fifth.

THE METRIC SYSTEM

The metric system is based on the meter, which, according to the United States Coast and Geodetic Survey Report of 1884, is equal to 39.370432 in. The value commonly used is 39.37 in., and is authorized by the United States government. The meter was originally intended to be one ten-millionth of the distance from the pole to the equator, measured on a meridian passing near Paris. This distance was carefully calculated, and a standard meter bar made and deposited among the archives of France, at Paris. It has since been discovered that the original calculations were at fault and the standard meter is somewhat short of one ten-millionth of the earth's quadrant. Nevertheless, the error is so small that it was not considered necessary to change the standard to make it correct, and the original meter length is still preserved.

There are three principal units—the *meter*, the *liter* (pronounced "lee-ter"), and the *gram*, the units of length, capacity, and weight, respectively. Multiples of these units

are obtained by prefixing to the names of the principal units the Greek words *deca* (10), *hecto* (100), and *kilo* (1,000); the submultiples, or divisions, are obtained by prefixing the Latin words *deci* ($\frac{1}{10}$), *centi* ($\frac{1}{100}$), and *milli* ($\frac{1}{1000}$). These prefixes form the key to the entire system. The abbreviations of the principal units of these submultiples begin with a small letter, and those of the multiples begin with a capital letter.

MEASURES OF LENGTH

10 millimeters (mm.)	= 1 centimeter	cm.
10 centimeters	= 1 decimeter	dm.
10 decimeters	= 1 meter	m.
10 meters	= 1 decameter	Dm.
10 decameters	= 1 hectometer	Hm.
10 hectometers	= 1 kilometer	Km.

MEASURES OF SURFACE (NOT LAND)

100 square millimeters (sq. mm.)	= 1 square centimeter	sq. cm.
100 square centimeters	= 1 square decimeter	sq. dm.
100 square decimeters	= 1 square meter	sq. m.

MEASURES OF VOLUME

1,000 cubic millimeters (cu. mm.)	= 1 cubic centimeter	cu. cm.
1,000 cubic centimeters	= 1 cubic decimeter	cu. dm.
1,000 cubic decimeters	= 1 cubic meter	cu. m.

MEASURES OF CAPACITY

10 milliliters (ml.)	= 1 centiliter	cl.
10 centiliters	= 1 deciliter	dl.
10 deciliters	= 1 liter	l.
10 liters	= 1 decaliter	Dl.
10 decaliters	= 1 hectoliter	Hl.
10 hectoliters	= 1 kiloliter	Kl.

The liter is equal to the volume occupied by 1 cu. dm.

MEASURES OF WEIGHT

10 milligrams (mg.)	= 1 centigram	cg.
10 centigrams	= 1 decigram	dg.
10 decigrams	= 1 gram	g.
10 grams	= 1 decagram	Dg.
10 decagrams	= 1 hectogram	Hg.
10 hectograms	= 1 kilogram	Kg. or kilo
1,000 kilograms	= 1 ton	T.

The gram is the weight of 1 cu. cm. of pure distilled water at a temperature of 39.2° F.; the kilogram is the weight of 1 liter of water; the ton is the weight of 1 cu. m. of water.

CONVERSION TABLES

By means of the tables on pages 8 and 9, metric measures can be converted into English, and vice versa, by simple addition. All the figures of the values given are not required, four or five digits being all that are commonly used; it is only in very exact calculations that all the digits are necessary. Using table, proceed as follows: Change 6,471.8 ft. into meters. Any number, as 6,471.8, may be regarded as $6{,}000+400+70+1+.8$; also, $6{,}000=1{,}000\times 6$; $400=100\times 4$; etc. Hence, looking in the left-hand column of the upper part of the table, page 8, for figure 6 (the first figure of the given number), we find opposite it in the third column, which is headed Feet to Meters, the number 1.8287838. Now, using but five digits and increasing the fifth digit by 1 (since the next is greater than 5), we get 1.8288. In other words, 6 ft. = 1.8288 m.; hence, 6,000 ft. $=1{,}000\times 1.8288=1{,}828.8$, simply moving the decimal point three places to the right. Likewise, 400 ft. = 121.92 m.; 70 ft. = 21.336 m.; 1 ft. = .3048 m.; and .8 ft. = .2438 m. Adding as shown above, we get 1,972.6046 m.

1,828.8
121.92
21.336
.3048
.2438
1,972.6046

Again, convert 19.635 kilos into pounds. The work should be perfectly clear from the explanation given above. The result is 43.2875 lb.

22.046
19.8416
1.3228
.0661
.0110
43.2875

The only difficulty in applying these tables lies in locating the decimal point. It may always be found thus: If the figure considered lies to the left of the decimal point, count each figure in

order, beginning with units (but calling unit's place zero) until the desired figure is reached; then move the decimal point to the *right* as many places as the figure being considered is to the left of the unit figure. Thus, in the first case above, 6 lies three places to the left of 1, which is in unit's place; hence, the decimal point is moved three places to the *right*. By exchanging the words "right" and "left," the statement will also apply to decimals. Thus, in the second case above, the 5 lies three places to the *right* of unit's place; hence, the decimal point in the number taken from the table is moved three places to the *left*.

CONVERSION TABLE

ENGLISH MEASURES INTO METRIC

English	Metric Inches to Meters	Metric Feet to Meters	Metric Pounds to Kilos	Metric Gallons to Liters
1	.0253998	.3047973	.4535925	3.7853122
2	.0507996	.6095946	.9071850	7.5706244
3	.0761993	.9143919	1.3607775	11.3559366
4	.1015991	1.2191892	1.8143700	15.1412488
5	.1269989	1.5239865	2.2679625	18.9265610
6	.1523987	1.8287838	2.7215550	22.7118732
7	.1777984	2.1335811	3.1751475	26.4971854
8	.2031982	2.4383784	3.6287400	30.2824976
9	.2285980	2.7431757	4.0823325	34.0678098
10	.2539978	3.0479730	4.5359250	37.8531220

English	Metric Square Inches to Square Meters	Metric Square Feet to Square Meters	Metric Cubic Feet to Cubic Meters	Metric Pounds per Square Inch to Kilo per Square Meter
1	.000645150	.092901394	.028316094	703.08241
2	.001290300	.185802788	.056632188	1,406.16482
3	.001935450	.278704182	.084948282	2,109.24723
4	.002580600	.371605576	.113264376	2,812.32964
5	.003225750	.464506970	.141580470	3,515.41205
6	.003870900	.557408364	.169896564	4,218.49446
7	.004516050	.650309758	.198212658	4,921.57687
8	.005161200	.743211152	.226528752	5,624.65928
9	.005806350	.836112546	.254844846	6,327.74169
10	.006451500	.929013940	.283160940	7,030.82410

CONVERSION TABLE

Metric Measures Into English

Metric	English: Meters to Inches	English: Meters to Feet	English: Kilos to Pounds	English: Liters to Gallons
1	39.370432	3.2808693	2.2046223	.2641790
2	78.740864	6.5617386	4.4092447	.5283580
3	118.111296	9.8426079	6.6138670	.7925371
4	157.481728	13.1234772	8.8184894	1.0567161
5	196.852160	16.4043465	11.0231117	1.3208951
6	236.222592	19.6852158	13.2277340	1.5850741
7	275.593024	22.9660851	15.4323564	1.8492531
8	314.963456	26.2469544	17.6369787	2.1134322
9	354.333888	29.5278237	19.8416011	2.3776112
10	393.704320	32.8086930	22.0462234	2.6417902

Metric	English: Square Meters to Square Inches	English: Square Meters to Square Feet	English: Cubic Meters to Cubic Feet	English: Kilos per Square Meter to Pounds per Square Inch
1	1,550.03092	10.7641034	35.3156163	.001422310
2	3,100.06184	21.5282068	70.6312326	.002844620
3	4,650.09276	32.2923102	105.9468489	.004266930
4	6,200.12368	43.0564136	141.2624652	.005689240
5	7,750.15460	53.8205170	176.5780815	.007111550
6	9,300.18552	64.5846204	211.8936978	.008533860
7	10,850.21644	75.3487238	247.2093141	.009956170
8	12,400.24736	86.1128272	282.5249304	.011378480
9	13,950.27828	96.8769306	317.8405467	.012800790
10	15,500.30920	107.6410340	353.1561630	.014223100

MATHEMATICAL TABLES

CIRCUMFERENCES AND AREAS OF CIRCLES FROM 1-64 TO 100

Diam.	Circum.	Area	Diam.	Circum.	Area
1/64	.0491	.0002	4	12.5664	12.5664
1/32	.0982	.0008	4 1/8	12.9591	13.3641
1/16	.1963	.0031	4 1/4	13.3518	14.1863
1/8	.3927	.0123	4 3/8	13.7445	15.0330
3/16	.5890	.0276	4 1/2	14.1372	15.9043
1/4	.7854	.0491	4 5/8	14.5299	16.8002
5/16	.9817	.0767	4 3/4	14.9226	17.7206
3/8	1.1781	.1104	4 7/8	15.3153	18.6555
7/16	1.3744	.1503	5	15.7080	19.6350
1/2	1.5708	.1963	5 1/8	16.1007	20.6290
9/16	1.7671	.2485	5 1/4	16.4934	21.6476
5/8	1.9635	.3068	5 3/8	16.8861	22.6907
11/16	2.1598	.3712	5 1/2	17.2788	23.7583
3/4	2.3562	.4418	5 5/8	17.6715	24.8505
13/16	2.5525	.5185	5 3/4	18.0642	25.9673
7/8	2.7489	.6013	5 7/8	18.4569	27.1086
15/16	2.9452	.6903	6	18.8496	28.2744
1	3.1416	.7854	6 1/8	19.2423	29.4648
1 1/8	3.5343	.9940	6 1/4	19.6350	30.6797
1 1/4	3.9270	1.2272	6 3/8	20.0277	31.9191
1 3/8	4.3197	1.4849	6 1/2	20.4204	33.1831
1 1/2	4.7124	1.7671	6 5/8	20.8131	34.4717
1 5/8	5.1051	2.0739	6 3/4	21.2058	35.7848
1 3/4	5.4978	2.4053	6 7/8	21.5985	37.1224
1 7/8	5.8905	2.7612	7	21.9912	38.4846
2	6.2832	3.1416	7 1/8	22.3839	39.8713
2 1/8	6.6759	3.5466	7 1/4	22.7766	41.2826
2 1/4	7.0686	3.9761	7 3/8	23.1693	42.7184
2 3/8	7.4613	4.4301	7 1/2	23.5620	44.1787
2 1/2	7.8540	4.9087	7 5/8	23.9547	45.6636
2 5/8	8.2467	5.4119	7 3/4	24.3474	47.1731
2 3/4	8.6394	5.9396	7 7/8	24.7401	48.7071
2 7/8	9.0321	6.4918	8	25.1328	50.2656
3	9.4248	7.0686	8 1/8	25.5255	51.8487
3 1/8	9.8175	7.6699	8 1/4	25.9182	53.4563
3 1/4	10.2102	8.2958	8 3/8	26.3109	55.0884
3 3/8	10.6029	8.9462	8 1/2	26.7036	56.7451
3 1/2	10.9956	9.6211	8 5/8	27.0963	58.4264
3 5/8	11.3883	10.3206	8 3/4	27.4890	60.1322
3 3/4	11.7810	11.0447	8 7/8	27.8817	61.8625
3 7/8	12.1737	11.7933	9	28.2744	63.6174

TABLE—(*Continued*)

Diam.	Circum.	Area	Diam.	Circum.	Area
9⅛	28.6671	65.3968	19½	61.2612	298.648
9¼	29.0598	67.2008	19¾	62.0466	306.355
9⅜	29.4525	69.0293	20	62.8320	314.160
9½	29.8452	70.8823	20¼	63.6174	322.063
9⅝	30.2379	72.7599	20½	64.4028	330.064
9¾	30.6306	74.6621	20¾	65.1882	338.164
9⅞	31.0233	76.589	21	65.9736	346.361
10	31.4160	78.540	21¼	66.7590	354.657
10¼	32.2014	82.516	21½	67.5444	363.051
10½	32.9868	86.590	21¾	68.3298	371.543
10¾	33.7722	90.763	22	69.1152	380.134
11	34.5576	95.033	22¼	69.9006	388.822
11¼	35.3430	99.402	22½	70.6860	397.609
11½	36.1284	103.869	22¾	71.4714	406.494
11¾	36.9138	108.434	23	72.2568	415.477
12	37.6992	113.098	23¼	73.0422	424.558
12¼	38.4846	117.859	23½	73.8276	433.737
12½	39.2700	122.719	23¾	74.6130	443.015
12¾	40.0554	127.677	24	75.3984	452.390
13	40.8408	132.733	24¼	76.1838	461.864
13¼	41.6262	137.887	24½	76.9692	471.436
13½	42.4116	143.139	24¾	77.7546	481.107
13¾	43.1970	148.490	25	78.5400	490.875
14	43.9824	153.938	25¼	79.3254	500.742
14¼	44.7678	159.485	25½	80.1108	510.706
14½	45.5532	165.130	25¾	80.8962	520.769
14¾	46.3386	170.874	26	81.6816	530.930
15	47.1240	176.715	26¼	82.4670	541.190
15¼	47.9094	182.655	26½	83.2524	551.547
15½	48.6948	188.692	26¾	84.0378	562.003
15¾	49.4802	194.828	27	84.8232	572.557
16	50.2656	201.062	27¼	85.6086	583.209
16¼	51.0510	207.395	27½	86.3940	593.959
16½	51.8364	213.825	27¾	87.1794	604.807
16¾	52.6218	220.354	28	87.9648	615.754
17	53.4072	226.981	28¼	88.7502	626.798
17¼	54.1926	233.706	28½	89.5356	637.941
17½	54.9780	240.529	28¾	90.3210	649.182
17¾	55.7634	247.450	29	91.1064	660.521
18	56.5488	254.470	29¼	91.8918	671.959
18¼	57.3342	261.587	29½	92.6772	683.494
18½	58.1196	268.803	29¾	93.4626	695.128
18¾	58.9050	276.117	30	94.2480	706.860
19	59.6904	283.529	30¼	95.0334	718.690
19¼	60.4758	291.040	30½	95.8188	730.618

TABLE—(*Continued*)

Diam.	Circum.	Area	Diam.	Circum.	Area
30¾	96.6042	742.645	42	131.947	1,385.450
31	97.3896	754.769	42¼	132.733	1,401.990
31¼	98.1750	766.992	42½	133.518	1,418.630
31½	98.9604	779.313	42¾	134.303	1,435.370
31¾	99.7458	791.732	43	135.089	1,452.200
32	100.5312	804.250	43¼	135.874	1,469.140
32¼	101.3166	816.865	43½	136.660	1,486.170
32½	102.1020	829.579	43¾	137.445	1,503.300
32¾	102.8874	842.391	44	138.230	1,520.530
33	103.673	855.301	44¼	139.016	1,537.860
33¼	104.458	868.309	44½	139.801	1,555.29
33½	105.244	881.415	44¾	140.587	1,572.81
33¾	106.029	894.620	45	141.372	1,590.43
34	106.814	907.922	45¼	142.157	1,608.16
34¼	107.600	921.323	45½	142.943	1,625.97
34½	108.385	934.822	45¾	143.728	1,643.89
34¾	109.171	948.420	46	144.514	1,661.91
35	109.956	962.115	46¼	145.299	1,680.02
35¼	110.741	975.909	46½	146.084	1,698.23
35½	111.527	989.800	46¾	146.870	1,716.54
35¾	112.312	1,003.790	47	147.655	1,734.95
36	113.098	1,017.878	47¼	148.441	1,753.45
36¼	113.883	1,032.065	47½	149.226	1,772.06
36½	114.668	1,046.349	47¾	150.011	1,790.76
36¾	115.454	1,060.732	48	150.797	1,809.56
37	116.239	1,075.213	48¼	151.582	1,828.46
37¼	117.025	1,089.792	48½	152.368	1,847.46
37½	117.810	1,104.469	48¾	153.153	1,866.55
37¾	118.595	1,119.244	49	153.938	1,885.75
38	119.381	1,134.118	49¼	154.724	1,905.04
38¼	120.166	1,149.089	49½	155.509	1,924.43
38½	120.952	1,164.159	49¾	156.295	1,943.91
38¾	121.737	1,179.327	50	157.080	1,963.50
39	122.522	1,194.593	50½	158.651	2,002.97
39¼	123.308	1,209.958	51	160.222	2,042.83
39½	124.093	1,225.420	51½	161.792	2,083.08
39¾	124.879	1,240.981	52	163.363	2,123.72
40	125.664	1,256.640	52½	164.934	2,164.76
40¼	126.449	1,272.400	53	166.505	2,206.19
40½	127.235	1,288.250	53½	168.076	2,248.01
40¾	128.020	1,304.210	54	169.646	2,290.23
41	128.806	1,320.260	54½	171.217	2,332.83
41¼	129.591	1,336.410	55	172.788	2,375.83
41½	130.376	1,352.660	55½	174.359	2,419.23
41¾	131.162	1,369.000	56	175.930	2,463.01

TABLE—(*Continued*)

Diam.	Circum.	Area	Diam.	Circum.	Area
56½	177.500	2,507.19	78½	246.616	4,839.83
57	179.071	2,551.76	79	248.186	4,901.68
57½	180.642	2,596.73	79½	249.757	4,963.92
58	182.213	2,642.09	80	251.328	5,026.56
58½	183.784	2,687.84	80½	252.899	5,089.59
59	185.354	2,733.98	81	254.470	5,153.01
59½	186.925	2,780.51	81½	256.040	5,216.82
60	188.496	2,827.44	82	257.611	5,281.03
60½	190.067	2,874.76	82½	259.182	5,345.63
61	191.638	2,922.47	83	260.753	5,410.62
61½	193.208	2,970.58	83½	262.324	5,476.01
62	194.779	3,019.08	84	263.894	5,541.78
62½	196.350	3,067.97	84½	265.465	5,607.95
63	197.921	3,117.25	85	267.036	5,674.51
63½	199.492	3,166.93	85½	268.607	5,741.47
64	201.062	3,217.00	86	270.178	5,808.82
64½	202.633	3,267.46	86½	271.748	5,876.56
65	204.204	3,318.31	87	273.319	5,944.69
65½	205.775	3,369.56	87½	274.890	6,013.22
66	207.346	3,421.20	88	276.461	6,082.14
66½	208.916	3,473.24	88½	278.032	6,151.45
67	210.487	3,525.66	89	279.602	6,221.15
67½	212.058	3,578.48	89½	281.173	6,291.25
68	213.629	3,631.69	90	282.744	6,361.74
68½	215.200	3,685.29	90½	284.315	6,432.62
69	216.770	3,739.29	91	285.886	6,503.90
69½	218.341	3,793.68	91½	287.456	6,575.56
70	219.912	3,848.46	92	289.027	6,647.63
70½	221.483	3,903.63	92½	290.598	6,720.08
71	223.054	3,959.20	93	292.169	6,792.92
71½	224.624	4,015.16	93½	293.740	6,866.16
72	226.195	4,071.51	94	295.310	6,939.79
72½	227.766	4,128.26	94½	296.881	7,013.82
73	229.337	4,185.40	95	298.452	7,088.24
73½	230.908	4,242.93	95½	300.023	7,163.04
74	232.478	4,300.85	96	301.594	7,238.25
74½	234.049	4,359.17	96½	303.164	7,313.84
75	235.620	4,417.87	97	304.735	7,389.83
75½	237.191	4,476.98	97½	306.306	7,466.21
76	238.762	4,536.47	98	307.877	7,542.98
76½	240.332	4,596.36	98½	309.448	7,620.15
77	241.903	4,656.64	99	311.018	7,697.71
77½	243.474	4,717.31	99½	312.589	7,775.66
78	245.045	4,778.37	100	314.160	7,854.00

DECIMAL EQUIVALENTS

1-64	.015625	17-64	.265625	33-64	.515625	49-64	.765625
1-32	.031250	9-32	.281250	17-32	.531250	25-32	.781250
3-64	.046875	19-64	.296875	35-64	.546875	51-64	.796875
1-16	.062500	5-16	.312500	9-16	.562500	13-16	.812500
5-64	.078125	21-64	.328125	37-64	.578125	53-64	.828125
3-32	.093750	11-32	.343750	19-32	.593750	27-32	.843750
7-64	.109375	23-64	.359375	39-64	.609375	55-64	.859375
1-8	.125000	3-8	.375000	5-8	.625000	7-8	.875000
9-64	.140625	25-64	.390625	41-64	.640625	57-64	.890625
5-32	.156250	13-32	.406250	21-32	.656250	29-32	.906250
11-64	.171875	27-64	.421875	43-64	.671875	59-64	.921875
3-16	.187500	7-16	.437500	11-16	.687500	15-16	.937500
13-64	.203125	29-64	.453125	45-64	.703125	61-64	.953125
7-32	.218750	15-32	.468750	23-32	.718750	31-32	.968750
15-64	.234375	31-64	.484375	47-64	.734375	63-64	.984375
1-4	.250000	1-2	.500000	3-4	.750000	1	1

DECIMALS OF A FOOT FOR EACH 1-32 INCH

Inch	0″	1″	2″	3″	4″	5″
0	0	.0833	.1667	.2500	.3333	.4167
$\frac{1}{32}$	.0026	.0859	.1693	.2526	.3359	.4193
$\frac{1}{16}$	.0052	.0885	.1719	.2552	.3385	.4219
$\frac{3}{32}$	.0078	.0911	.1745	.2578	.3411	.4245
$\frac{1}{8}$	.0104	.0937	.1771	.2604	.3437	.4271
$\frac{5}{32}$	.0130	.0964	.1797	.2630	.3464	.4297
$\frac{3}{16}$	.0156	.0990	.1823	.2656	.3490	.4323
$\frac{7}{32}$	.0182	.1016	.1849	.2682	.3516	.4349
$\frac{1}{4}$	.0208	.1042	.1875	.2708	.3542	.4375
$\frac{9}{32}$	.0234	.1068	.1901	.2734	.3568	.4401
$\frac{5}{16}$	.0260	.1094	.1927	.2760	.3594	.4427
$\frac{11}{32}$	.0286	.1120	.1953	.2786	.3620	.4453
$\frac{3}{8}$	.0312	.1146	.1979	.2812	.3646	.4479
$\frac{13}{32}$	.0339	.1172	.2005	.2839	.3672	.4505
$\frac{7}{16}$	.0365	.1198	.2031	.2865	.3698	.4531
$\frac{15}{32}$	.0391	.1224	.2057	.2891	.3724	.4557
$\frac{1}{2}$	.0417	.1250	.2083	.2917	.3750	.4583
$\frac{17}{32}$	.0443	.1276	2109	.2943	.3776	.4609
$\frac{9}{16}$	.0469	.1302	.2135	.2969	.3802	.4635
$\frac{19}{32}$	.0495	.1328	.2161	.2995	.3828	.4661
$\frac{5}{8}$	.0521	.1354	.2188	.3021	.3854	.4688
$\frac{21}{32}$	.0547	.1380	.2214	.3047	.3880	.4714
$\frac{11}{16}$	.0573	.1406	.2240	.3073	.3906	.4740
$\frac{23}{32}$	.0599	.1432	.2266	.3099	.3932	.4766

DECIMALS OF A FOOT FOR EACH 1-32 INCH—(*Continued*)

Inch	0″	1″	2″	3″	4″	5″
3/4	.0625	.1458	.2292	.3125	.3958	.4792
25/32	.0651	.1484	.2318	.3151	.3984	.4818
13/16	.0677	.1510	.2344	.3177	.4010	.4844
27/32	.0703	.1536	.2370	.3203	.4036	.4870
7/8	.0729	.1562	.2396	.3229	.4062	.4896
29/32	.0755	.1589	.2422	.3255	.4089	.4922
15/16	.0781	.1615	.2448	.3281	.4115	.4948
31/32	.0807	.1641	.2474	.3307	.4141	.4974

DECIMALS OF A FOOT FOR EACH 1-32 INCH

Inch	6″	7″	8″	9″	10″	11″
0	.5000	.5833	.6667	.7500	.8333	.9167
1/32	.5026	.5859	.6693	.7526	.8359	.9193
1/16	.5052	.5885	.6719	.7552	.8385	.9219
3/32	.5078	.5911	.6745	.7578	.8411	.9245
1/8	.5104	.5937	.6771	.7604	.8437	.9271
5/32	.5130	.5964	.6797	.7630	.8464	.9297
3/16	.5156	.5990	.6823	.7656	.8490	.9323
7/32	.5182	.6016	.6849	.7682	.8516	.9349
1/4	.5208	.6042	.6875	.7708	.8542	.9375
9/32	.5234	.6068	.6901	.7734	.8568	.9401
5/16	.5260	.6094	.6927	.7760	.8594	.9427
11/32	.5286	.6120	.6953	.7786	.8620	.9453
3/8	.5312	.6146	.6979	.7812	.8646	.9479
13/32	.5339	.6172	.7005	.7839	.8672	.9505
7/16	.5365	.6198	.7031	.7865	.8698	.9531
15/32	.5391	.6224	.7057	.7891	.8724	.9557
1/2	.5417	.6250	.7083	.7917	.8750	.9583
17/32	.5443	.6276	.7109	.7943	.8776	.9609
9/16	.5469	.6302	.7135	.7669	.8802	.9635
19/32	.5495	.6328	.7161	.7995	.8828	.9661
5/8	.5521	.6354	.7188	.8021	.8854	.9688
21/32	.5547	.6380	.7214	.8047	.8880	.9714
11/16	.5573	.6406	.7240	.8073	.8906	.9740
23/32	.5599	.6432	.7266	.8099	.8932	.9766
3/4	.5625	.6458	.7292	.8125	.8958	.9792
25/32	.5651	.6484	.7318	.8151	.8984	.9818
13/16	.5677	.6510	.7344	.8177	.9010	.9844
27/32	.5703	.6536	.7370	.8203	.9036	.9870
7/8	.5729	.6562	.7396	.8229	.9062	.9896
29/32	.5755	.6589	.7422	.8255	.9089	.9922
15/16	.5781	.6615	.7448	.8281	.9115	.9948
31/32	.5807	.6641	.7474	.8307	.9141	.9974

MATHEMATICS

FORMULAS

$$=\{+[-:(\sqrt{\times / \div}):-]\}=$$

The term *formula*, as used in mathematics and in technical books, may be defined as *a rule in which symbols are used instead of words;* in fact, a formula may be regarded as a shorthand method of expressing a rule.

Most persons having no knowledge of algebra regard formulas with distrust; they think that a person must be a good algebraic scholar in order to be able to use formulas. This idea, however, is erroneous. As a rule, no knowledge of any branch of mathematics except arithmetic is required to enable one to use a formula. Any formula can be expressed in words, and when so expressed it becomes a rule.

Formulas are much more convenient than rules. They show at a glance all the operations that are to be performed; they do not have to be read three or four times, as is the case with most rules, to enable one to understand their meaning; they take up less space, both in the printed book and in one's notebook, than rules; in short, whenever a rule can be expressed as a formula, the formula is to be preferred. It is the intention in the following pages to show how to use such formulas as are likely to be encountered in "handbooks," or other works of like nature.

The signs used in formulas are the ordinary signs indicative of operations and the signs of aggregation. All these signs are used in arithmetic, but, to refresh the reader's memory, their nature and uses will be explained before proceeding further.

The signs indicative of operations are six in number, viz.: $+$, $-$, $\times$, $\div$, $|$, and $\sqrt{}$.

The sign $(+)$ indicates addition, and is called *plus;* when placed between two quantities, it indicates that the two quantities are to be added. Thus, in the expression $25+17$, the sign $(+)$ shows that 17 is to be added to 25.

The sign ($-$) indicates subtraction, and is called *minus;* when placed between two quantities, it indicates that the quantity on the right is to be subtracted from that on the left. Thus, in the expression $25-17$, the sign ($-$) shows that 17 is to be subtracted from 25.

The sign ($\times$) indicates multiplication, and is read *times*, or *multiplied by;* when placed between two quantities, it indicates that the quantity on the left is to be multiplied by that on the right. Thus, in the expression 25×17, the sign ($\times$) shows that 25 is to be multiplied by 17.

The sign ($\div$) indicates division, and is read *divided by;* when placed between two quantities, it indicates that the quantity on the left is to be divided by that on the right. Thus, in the expression $25\div 17$, the sign ($\div$) shows that 25 is to be divided by 17.

Division is also indicated by placing a straight line between the two quantities. Thus, $25 \mid 17$, $25/17$, and $\frac{25}{17}$ all indicate that 25 is to be divided by 17. If both quantities are placed on the same horizontal line, the straight line indicates that the quantity on the left is to be divided by that on the right. When one quantity is below the other, the straight line between indicates that the quantity above the line is to be divided by the one below it.

The sign ($\sqrt{}$) indicates that some root of the quantity to the right is to be taken; it is called the *radical sign.* To indicate what root is to be taken, a small figure, called the *index*, is placed within the sign, this being always omitted when the square root is to be indicated. Thus $\sqrt{25}$ indicates that the square root of 25 is to be taken; $\sqrt[3]{25}$ indicates that the cube root of 25 is to be taken; etc.

NOTE.—As the term "quantity" is a very convenient one to use, it will be defined. In mathematics the word *quantity* is applied to anything that is to be subjected to the ordinary operations of addition, subtraction, multiplication, etc., when it is desired not to be more explicit and not to state exactly what the thing is. Thus, the terms "two or more numbers," or "two or more quantities" may be used. However, the word quantity is more general in its meaning than the word number.

The signs of aggregation are four in number, viz.: $\overline{\quad}$, ( ), [ ], and $\{\ \}$, respectively called the *vinculum*, the *parenthesis*, the *brackets*, and the *brace*. They are used when it is

desired to indicate that all the quantities included by them are to be subjected to the same operation. Thus, suppose that the sum of 5 and 8 is to be multiplied by 7, and that the addition is to precede the multiplication. Any one of the four signs of aggregation may be employed to indicate the operation. Thus, $\overline{5+8}\times 7$, $(5+8)\times 7$, $[5+8]\times 7$, $\{5+8\}\times 7$ The vinculum is placed above the quantities that are to be treated as one quantity and subjected to the same operations.

While any one of the four signs may be used as shown above, custom has restricted their use somewhat. The vinculum is rarely used except in connection with the radical sign. Thus, instead of writing $\sqrt[3]{}(5+8)$, $\sqrt[3]{}[5+8]$, or $\sqrt[3]{}\{5+8\}$ for the cube root of 5 plus 8, all of which would be correct, the vinculum is nearly always used, $\sqrt[3]{5+8}$.

In cases where only one sign of aggregation is needed (except, of course, when a root is to be indicated), the parenthesis is always used. Hence, $(5+8)\times 7$ would be the usual way of expressing the product of 5 plus 8 by 7.

If two signs of aggregation are needed, the brackets and parenthesis are used, so as to avoid having a parenthesis within a parenthesis, the brackets being placed outside. For example, $[(20-5)\div 3]\times 9$ means that the difference between 20 and 5 is to be divided by 3, and this result multiplied by 9.

If three signs of aggregation are required, the brace, brackets, and parenthesis are used, the brace being placed outside, the brackets next, and the parenthesis inside. For example, $\{[(20-5)\div 3]\times 9-21\}\div 8$ means that the quotient obtained by dividing the difference between 20 and 5 by 3 is to be multiplied by 9, and that 21 is to be subtracted from the product thus obtained, and the result divided by 8.

Should it be necessary to use all four signs of aggregation, the brace would be put outside, the brackets next, the parenthesis next, and the vinculum inside. For example, $\{[(\overline{20-5}\div 3)\times 9-21]\div 8\}\times 12$. The reason for using the brace in this last instance will be explained, as it is not generally understood.

When several quantities are connected by the various signs indicating addition, subtraction, multiplication, and division, the operation indicated by the sign of multiplication must always be performed first. Thus, $2+3\times 4$ equals 14, 3 being multiplied by 4 before adding to 2. Similarly, $10\div 2\times 5$ equals 1, since 2×5 equals 10, and $10\div 10$ equals 1. Hence, in the preceding case, if the brace were omitted, the result would be $\frac{1}{4}$; whereas, by inserting the brace, the result is 36.

Following the sign of multiplication comes the sign of division in its order of importance. For example, $5-9\div 3$ equals 2, 9 being divided by 3 before subtracting from 5. The signs of addition and subtraction are of equal value; that is, if several quantities are connected by plus and minus signs, the indicated operations may be performed in the order in which the quantities are placed.

There is one other sign used, which is neither a sign of aggregation nor a sign indicative of an operation to be performed; it is $(=)$, and is called the sign of *equality;* it means that all on one side of it is exactly equal to all on the other side. For example, $2=2$, $5-3=2$, $5\times(14-9)=25$.

Having described the signs used in formulas, the formulas themselves will now be explained. First, consider the well-known rule for finding the safe load that a rectangular white-oak post will carry, which may be stated as follows:

From unity subtract 1 one-hundredth of the dividend obtained by dividing the length of the post in inches by the least dimension of its cross-section in inches. Multiply the remainder by 1,000 times the area of the post section in square inches. The result is the safe load the post will carry.

This rule is rather complicated, and it can be greatly simplified by putting it in the form of a formula.

An examination of the rule will show that three quantities (viz., the area of the section, the length, and the least dimension of the cross-section) are involved. Hence, the rule might be expressed as follows:

$$\text{Safe load} = 1{,}000 \times \text{area of section in square inches} \times \left(1 - \frac{\text{length in inches}}{100 \times \text{least dimension of cross-section in inches}}\right)$$

This expression could be shortened by representing each quantity by a single letter. Thus, representing the safe load by W, the area of the cross-section of the post in square inches by A, the length of the post in inches by L, the least dimension of its cross-section in inches by D, and substituting these letters for the quantities that they represent, the preceding expression would reduce to

$$W=1{,}000\times A\times\left(1-\frac{L}{100\times D}\right),$$

a much simpler and shorter expression. This last expression is called a *formula*.

As a further example, consider the rule as explained on page 300 for finding the safe resisting moment of a reinforced-concrete beam, which is as follows:

Multiply the area of the steel reinforcement in square inches by the distance from the center of the steel to the top of the beam. Multiply the product by 13,760. The result is the safe resisting moment of the beam in inch-pounds.

In this case, let M represent the safe resisting moment in inch-pounds; a, the area of the steel, in square inches; and d, the distance, in inches, from the center of the reinforcement to the top of the beam. Then, the preceding rule may be expressed by the following formula:

$$M=13{,}760\times d\times a$$

The formula just given shows, as was stated in the beginning, that a formula is really a shorthand method of expressing a rule. It is customary, however, to omit the sign of multiplication between two or more quantities when they are to be multiplied together, or between a number and a letter representing a quantity, it being always understood that when two letters are adjacent with no sign between them, the quantities represented by these letters are to be multiplied. Bearing this fact in mind, the formula just given can be further simplified to

$$M=13{,}760da$$

The sign of multiplication, evidently, cannot be omitted between two or more numbers, as it would then be impossible to distinguish the numbers. A near approach to this, however, may be attained by placing a dot between the numbers

that are to be multiplied together, and this is frequently done in works on mathematics when it is desired to economize space. In such cases it is customary to put the dot higher than the position occupied by the decimal point. Thus, 2·3 means the same as 2×3; 542·749·1,006 indicates that the numbers 542, 749, and 1,006 are to be multiplied together.

It is also customary to omit the sign of multiplication in expressions similar to the following: $a\times\sqrt{b+c}$, $3\times(b+c)$, $(b+c)\times a$, etc., writing them $a\sqrt{b+c}$, $3(b+c)$, $(b+c)a$, etc. The sign is not omitted when several quantities are included by a vinculum, and it is desired to indicate that the quantities so included are to be multiplied by another quantity. For example, $3\times\overline{b+c}$, $\overline{b+c}\times a$, $\sqrt{b+c}\times a$, etc., are always written as here printed.

Before proceeding further, it may be well to explain one other device used by formula makers, and which is apt to puzzle one who encounters it for the first time. It is the use of what mathematicians call *primes* and *subs.*, and what printers call *superior* and *inferior* characters. As a rule, formula makers designate quantities by the initial letters of the names of the quantities. For example, they represent moment by M, stress by s, length by l, etc. This practice is to be commended, as the letter itself serves in many cases to identify the quantity that it represents. Some authors carry the practice a little further and represent all quantities of the same nature by the same letter throughout the book, always having the same letter represent the same thing. Now, this practice necessitates the use of the primes and subs. above mentioned when two quantities have the same name, but represent different things. Thus, consider the word *moment* as applied to beams. The safe moment equals the ultimate moment divided by the factor of safety. If it is decided to designate all moments by M, it will be necessary to make a distinction between the M that refers to safe moment and the M that refers to ultimate moment. This may be effected by designating the safe moment as M_s, and the ultimate moment as M_u. The formula may then be written $M_s=\frac{M_u}{F}$, in which F equals the factor of safety.

The main thing to be remembered is that *when a formula is given in which the same letters occur several times, all like letters having the same primes or subs. represent the same quantities, while those which differ in any respect represent different quantities.* Thus, in the formula

$$R=\sqrt{\frac{I_1+I_2+I_3+I_4}{A}},$$

R represents the radius of gyration of a section composed of four parts; I_1, I_2, I_3, and I_4 represent the moment of inertia of the respective parts about the same axis around which R is taken; and A represents the total area of the section.

It is very easy to apply the above formula when the values of the quantities represented by the different letters are known. All that is required is to substitute the numerical values of the letters, and then perform the indicated operations. Thus, suppose that the values of I_1, I_2, I_3, and I_4 are 4.2, 3.6, 7.5, and 9.8, respectively, and that A is equal to 8 sq. in. Then, the value of R may be found by substituting in the above formula; thus,

$$R=\sqrt{\frac{4.2+3.6+7.5+9.8}{8}}=\sqrt{\frac{25.1}{8}}=\sqrt{3.1375}=1.77 \text{ in.}$$

Attention is called to one or two other facts relating to formulas.

Expressions similar to $\dfrac{\;\dfrac{160}{660}\;}{25}$ sometimes occur, the heavy line indicating that 160 is to be divided by the quotient obtained by dividing 660 by 25. If both lines were light, it would be impossible to tell whether 160 was to be divided by $\frac{660}{25}$ or whether $\frac{160}{660}$ was to be divided by 25. If this latter result were desired, the expression would be written $\dfrac{\;\dfrac{160}{660}\;}{25}$. In every case the heavy line indicates that all above it is to be divided by all below it.

In an expression like $\dfrac{160}{7+\dfrac{660}{25}}$, the heavy line is not neces-

sary, because it is impossible to mistake the operation that has to be performed. But, since $7+\frac{660}{25}=\frac{175+660}{25}$, if $\frac{175+660}{25}$ is substituted for $7+\frac{660}{25}$, the heavy line becomes necessary in order to make the resulting expression clear. Thus,

$$\frac{160}{7+\frac{660}{25}}=\frac{160}{\frac{175+660}{25}}=\frac{160}{\frac{835}{25}}$$

Fractional exponents are sometimes used instead of the radical sign; that is, instead of indicating the square, cube, fourth root, etc. of some quantity, as 37, by $\sqrt{37}$, $\sqrt[3]{37}$, $\sqrt[4]{37}$, etc., those roots are indicated $37^{\frac{1}{2}}$, $37^{\frac{1}{3}}$, $37^{\frac{1}{4}}$, etc. Should the numerator of the fractional exponent be some quantity other than 1, this quantity, whatever it may be, indicates that the quantity affected by the exponent is to be raised to the power indicated by the numerator; the denominator is *always* the index of the root. Hence, instead of expressing the cube root of the square of 37 as $\sqrt[3]{37^2}$, it may be expressed $37^{\frac{2}{3}}$, the denominator being the index of the root; in other words, $\sqrt[3]{37^2}=37^{\frac{2}{3}}$. Likewise, $\sqrt[5]{(1+a^2b)^3}$ may also be written $(1+a^2b)^{\frac{3}{5}}$, a much simpler expression.

Several examples showing how to apply some of the more difficult formulas will now be given.

The area of any segment of a circle that is less than (or equal to) a semicircle is expressed by the formula

$$A=\frac{\pi r^2E}{360}-\frac{c}{2}(r-h),$$

in which A = area of segment; $\pi=3.1416$; r = radius; E = angle obtained by drawing lines from the center to the extremities of arc of segment; c = chord of segment; and h = height of segment.

EXAMPLE.—What is the area of a segment whose chord is 10 in. long, angle subtended by chord is 83.46°, radius is 7.5 in., and height of segment is 1.91 in.?

SOLUTION.—Applying the formula just given,

$$A=\frac{\pi r^2E}{360}-\frac{c}{2}(r-h)=\frac{3.1416\times7.5^2\times83.46}{360}-\frac{10}{2}(7.5-1.91)$$
$$=40.968-27.95=13.018 \text{ sq. in., nearly}$$

The area of any triangle may be found by means of the following formula, in which A = the area, and a, b, and c represent the lengths of the sides:

$$A=\frac{b}{2}\sqrt{a^2-\left(\frac{a^2+b^2-c^2}{2b}\right)^2}$$

EXAMPLE.—What is the area of a triangle whose sides are 21 ft., 46 ft., and 50 ft. long?

SOLUTION.—In order to apply the formula, assume that a represents the side that is 21 ft. long; b, the side that is 50 ft. long; and c, the side that is 46 ft. long. Then substituting in the formula,

$$A=\frac{b}{2}\sqrt{a^2-\left(\frac{a^2+b^2-c^2}{2b}\right)^2}=\frac{50}{2}\sqrt{21^2-\left(\frac{21^2+50^2-46^2}{2\times 50}\right)^2}$$

$$=\frac{50}{2}\sqrt{441-\left(\frac{441+2{,}500-2{,}116}{100}\right)^2}=25\sqrt{441-\left(\frac{825}{100}\right)^2}$$

$$=25\sqrt{441-8.25^2}=25\sqrt{441-68.0625}=25\sqrt{372.9375}$$

$$=25\times 19.312=482.8 \text{ sq. ft., nearly}$$

The above operations have been extended much further than was necessary in order to show every step of the process.

The Rankine-Gordon formula for determining the least load in pounds that will cause a long column to break is

$$P=\frac{SA}{1+q\frac{l^2}{G^2}},$$

in which P = load (pressure) in pounds; S = ultimate strength, in pounds per square inch, of material composing column; A = area of cross-section of column, in square inches; q = a factor (multiplier) whose value depends on the shape of the ends of the column and on the material composing the column; l = length of the column, in inches; G = least radius of gyration of cross-section of column.

The meaning of the term G is explained on page 142.

EXAMPLE.—What is the least load that will break a hollow steel column whose outside diameter is 14 in., inside diameter 11 in., length 20 ft., and whose ends are flat?

SOLUTION.—For steel, $S=150{,}000$, and $q=\frac{1}{25{,}000}$ for flat-

ended steel columns. The area of the cross-section $A = .7854\,(d_1^2 - d_2^2)$, d_1 and d_2 being the outside and inside diameters, respectively; $l = 20 \times 12 = 240$ in.; and $G^2 = \frac{d_1^2 + d_2^2}{16}$. Substituting these values in the formula,

$$P = \frac{SA}{1 + q\frac{l^2}{G^2}} = \frac{150{,}000 \times .7854(14^2 - 11^2)}{1 + \frac{1}{25{,}000} \times \frac{240^2}{\frac{14^2 + 11^2}{16}}}$$

$$= \frac{150{,}000 \times 58.905}{1 + .1163} = \frac{8{,}835.750}{1.1163} = 7{,}915{,}211 \text{ lb.}$$

INVOLUTION AND EVOLUTION

By means of the table on pages 31–48 the square, cube, square root, cube root, and reciprocal of any number may be obtained correct always to five significant figures, and in the majority of cases correct to six significant figures.

In any number, the figures beginning with the first digit (a cipher is not a digit) at the left and ending with the last digit at the right, are called the *significant figures* of the number. Thus, the number 405,800 has the four significant figures, 4, 0, 5, and 8; and the number .000090067 has the five significant figures 9, 0, 0, 6, and 7.

The part of the number consisting of its significant figures is called the *significant part* of the number. Thus, in the number 28,070, the significant part is 2807; in the number .00812, the significant part is 812; and in the number 170.3, the significant part is 1703.

In speaking of the significant figures or of the significant part of a number, the figures are considered in their proper order, from the first digit at the left to the last digit at the right, but no attention is paid to the position of the decimal point. Hence, *all numbers that differ only in the position of the decimal point have the same significant part.* For example, .002103, 21.03, 21,030, and 210,300 have the same significant figures 2, 1, 0, and 3, and the same significant part 2103.

The *integral part* of a number is the part to the left of the decimal point.

It will be more convenient to explain first how to use the table for finding square and cube roots.

Square Root.—First point off the given number into periods of two figures each, beginning with the decimal point and proceeding to the left and right. The following numbers are thus pointed off: 12703, 1'27'03; 12.703, 12.70'30; 220000, 22'00'00; .000442, .00'04'42.

Having pointed off the number, move the decimal point so that it will fall between the first and second periods of the significant part of the number. In the preceding numbers, the decimal point will be placed thus: 1.2703, 12.703, 22., 4.42.

If the number has only three (or less) significant figures, find the significant part of the number in the column headed n; the square root will be found in the column headed $\sqrt{n}$ or $\sqrt{10n}$, according to whether the part to the left of the decimal point contains one figure or two figures. Thus, $\sqrt{4.42}=2.1024$, and $\sqrt{22}=\sqrt{10\times 2.20}=4.6904$. The decimal point is located in all cases by reference to the original number after pointing off into periods.

There will be as many figures in the root preceding the decimal point as there are periods preceding the decimal point in the given number; if the number is entirely decimal, the root is entirely decimal, and there will be as many ciphers following the decimal point in the root as there are cipher periods following the decimal point in the given number.

Applying this rule, $\sqrt{220000}=469.04$ and $\sqrt{.000442}$ $=.021024$.

The operation when the given number has more than three significant figures is best explained by an example.

EXAMPLE.—(*a*) $\sqrt{3.1416}=?$ (*b*) $\sqrt{2342.9}=?$

SOLUTION.—(*a*) Since the first period contains only one figure, there is no need of moving the decimal point. Look in the column headed n^2 and find two consecutive numbers, one a little greater and the other a little less than the given number; in the present case, $3.1684=1.78^2$ and $3.1329=1.77^2$. The first three figures of the root are therefore 177. Find the difference between the two numbers between which the given number falls, and the difference between the smaller

number and the given number; divide the second difference by the first difference, carrying the quotient to three decimal places and increasing the second figure by 1 if the third is 5 or a greater digit. The two figures of the quotient thus determined will be the fourth and fifth figures of the root. In the present example, dropping decimal points, in the remainders, $3.1684-3.1329=355$, the first difference; $3.1416-3.1329=87$, the second difference; $87\div355=.245+$, or .25. Hence, $\sqrt{3.1416}=1.7725$.

(*b*) $\sqrt{2342.9}=?$ Pointed off into periods, the number appears as 23′42.90, and by moving the decimal point, the number appears as 23.4290; the first three figures of the root are 484; the first difference is $23.5225-23.4256=969$; the second difference is $23.4290-23.4256=34$; $34\div969=.035+$, or .04. Hence, $\sqrt{2342.9}=48.404$.

Cube Root.—The *cube root* of a number is found in the same manner as the square root, except the given number is pointed off into periods of three figures each. The following numbers would be pointed off thus: 3141.6, 3′141.6; 67296428, 67′296′428; 601426.314, 601′426.314; .0000000217, .000′000′021′700.

Having pointed off, move the decimal point so that it will fall between the first and second periods of the significant part of the number, as in square root. In the above numbers the decimal point will be placed thus: 3.1416, 67.296,428, 601.426314, and 21.7.

If the given number has but three (or less) significant figures, find the significant part of the number in the column headed n; the cube root will be found in the column headed $\sqrt[3]{n}$, $\sqrt[3]{10n}$, or $\sqrt[3]{100n}$, according to whether one, two, or three figures precede the decimal point after it has been moved. Thus, the cube root of 21.7 will be found opposite 2.17, in column headed $\sqrt[3]{10n}$, while the cube root of 2.17 will be found in the column headed $\sqrt[3]{n}$, and the cube root of 217 in the column headed $\sqrt[3]{100n}$, all on the same line. If the given number contains more than three significant figures, proceed exactly as described for square root, but use the column headed n^3.

EXAMPLE.—(*a*) $\sqrt[3]{.0000062417}=?$ (*b*) $\sqrt[3]{50932676}=?$

SOLUTION.—(*a*) Pointed off into periods, the number appears as 000′006′241′700, and by moving the decimal point the number appears as 6.2417. The number falls between $6.22950 = 1.84^3$ and $6.33163 = 1.85^3$; the first difference is 10213; the second difference is $6.24170 - 6.22950 = 1220$; $1220 \div 10213 = .119+$, or .12, the fourth and fifth figures of the root. The decimal point is located by the rule previously given; hence, $\sqrt[3]{.0000062417} = .018412$.

(*b*) $\sqrt[3]{50932676} = ?$ As the number contains more than six significant figures, reduce it to six significant figures by replacing all after the sixth figure with ciphers, increasing the sixth figure by 1 when the seventh is 5 or a greater digit. In other words, the first five figures of $\sqrt[3]{50932700}$ and of $\sqrt[3]{50932676}$ are the same. Pointed off into periods, the number appears as 50′932′700, and by moving the decimal point, the number appears as 50.9327, which falls between $50.6530 = 3.70^3$ and $51.0648 = 3.71^3$; the first difference is 4118; the second difference is 2797; $2797 \div 4118 = .679+$, or .68. The integral part of the root evidently contains three figures; hence $\sqrt[3]{50932676} = 370.68$, correct to five figures.

Squares and Cubes.—If the given number contains but three (or less) significant figures, the square or cube is found in the column headed n^2 or n^3, opposite the given number in the column headed n. If the given number contains more than three significant figures, proceed in a manner similar to that described for extracting roots. To square a number, place the decimal point between the first and second significant figures and find in the column headed $\sqrt{n}$ or $\sqrt{10n}$ two consecutive numbers, one of which shall be a little greater and the other a little less than the given number. The remainder of the work is exactly as heretofore described. To locate the decimal point, employ the principle that the square of any number contains either twice as many figures as the number squared or twice as many less one. If the column headed $\sqrt{10n}$ is used, the square will contain twice as many figures, while if the column headed $\sqrt{n}$ is used, the square will contain twice as many figures as the number squared, less one. If the number contains an integral part, the principle

is applied to the integral part only; if the number is wholly decimal, there will be twice as many ciphers following the decimal in the square or twice as many plus one as in the number squared, depending on whether $\sqrt{10n}$ or $\sqrt{n}$ column is used. For example, 273.42^2 will contain five figures in the integral part; 4516.2^2 will contain eight figures in the integral part, all after the fifth being denoted by ciphers; $.0029453 2^2$ will have five ciphers following the decimal point; $.052436^2$ will have two ciphers following the decimal point.

Example.—(*a*) $273.42^2 = ?$ (*b*) $.052436^2 = ?$

Solution.—(*a*) Placing the decimal point between the first and second significant figures, the result is 2.7342; this number occurs between $2.73313 = \sqrt{7.47}$ and $2.73496 = \sqrt{7.48}$ in the column headed $\sqrt{n}$. The first difference is $2.73496 - 2.73313 = 183$; the second difference is $2.73420 - 2.73313 = 107$; and $107 \div 183 = .584+$, or .58. Hence, $273.42^2 = 74{,}758$, correct to five significant figures.

(*b*) Shifting the decimal point to between the first and second significant figures, the number 5.2436 is obtained. This falls between $5.23450 = \sqrt{27.4}$ and $5.24404 = \sqrt{27.5}$. The first difference is 954; the second difference is 910; $910 \div 954 = .953+$, or .95. Hence, $.052436^2 = .0027495$, to five significant figures.

A number is cubed in exactly the same manner, using the column headed $\sqrt[3]{n}$, $\sqrt[3]{10n}$, or $\sqrt[3]{100n}$, according to whether the first period of the significant part of the number contains one, two, or three figures, respe tively. If the number contains an integral part, the number of figures in the integral part of the cube will be three times as many as in the given number if column headed $\sqrt[3]{100n}$ is used; it will be three times as many less 1 if the column headed $\sqrt[3]{10n}$ is used and it will be three times as many less 2 if the column headed $\sqrt[3]{n}$ is used. If the given number is wholly decimal, the cube will have either three times, three times plus one, or three times plus two, as many ciphers following the decimal as there are ciphers following the decimal point in the given number.

Example.—(*a*) $129.684^3 = ?$ (*b*) $.76442^3 = ?$ (*c*) $.032425^3 = ?$

Solution.—(*a*) Placing the decimal point between the

first and second significant figures, the number 1.29684 is found between $1.29664=\sqrt[3]{2.18}$ and $1.29862=\sqrt[3]{2.19}$. The first difference is 198; the second difference is 20; and $20\div 198 = .101+$, or .10. Hence, the first five significant figures are 21810; the number of figures in the integral part of the cube is $3\times 3-2=7$; and $129.684^3=2{,}181{,}000$, correct to five significant figures.

(*b*) 7.64420 occurs between $7.64032=\sqrt[3]{446}$ and $7.64603 = \sqrt[3]{447}$. The first difference is 571; the second difference is 388; and $388\div 571=.679+$, or .68. Hence, the first five significant figures are 44668; the number of ciphers following the decimal point is $3\times 0=0$; and $.76442^3=.44866$, correct to five significant figures.

(*c*) 3.2425 falls between $3.24278=\sqrt[3]{34.1}$ and $3.23961 = \sqrt[3]{34.0}$. The first difference is 317; the second difference is 289; $289\div 317=.911+$, or .91. Hence, the first five significant figures are 34091; the number of ciphers following the decimal point is $3\times 1+1=4$; and $.032425^3=.000034091$, correct to five significant figures.

Reciprocals.—The *reciprocal* of a number is 1 divided by the number. By using reciprocals, division is changed into multiplication, since $a\div b=\frac{a}{b}=a\times\frac{1}{b}$. The table gives the reciprocals of all numbers expressed by three significant figures correct to six significant figures. By proceeding in a manner similar to that just described for powers and roots, the reciprocal of any number correct to five significant figures may be obtained. The decimal point in the result may be located as follows: If the given number has an integral part, the number of ciphers following the decimal point in the reciprocal will be one less than the number of figures in the integral part of the given number; and if the given number is entirely decimal, the number of figures in the integral part of the reciprocal will be one greater than the number of ciphers following the decimal point in the given number. For example, the reciprocal of $3370=.00029\text{-}6736$ and of $.00348=287.356$.

When the number whose reciprocal is desired contains more than three significant figures, express the number to

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10\,n}$	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$\frac{1}{n}$
1.01	1.0201	1 03030	1.00499	3.17805	1.00332	2.16159	4.65701	[redacted]
1.02	1.0404	1.06121	1.00995	3.19374	1.00662	2.16870	4.67233	.980392
1.03	1.0609	1.09273	1.01489	3.20936	1.00990	2.17577	4 68755	.970874
1.04	1.0816	1.12486	1.01980	3.22490	1.01316	2.18278	4.70267	.961539
1.05	1.1025	1.15763	1.02470	3.24037	1.01640	2.18976	4.71769	.952381
1.06	1.1236	1.19102	1.02956	3.25576	1.01961	2.19669	4.73262	.943396
1.07	1 1449	1.22504	1.03441	3.27109	1.02281	2.20358	4.74746	.934579
1.08	1.1664	1.25971	1.03923	3.28634	1.02599	2.21042	4.76220	.925926
1.09	1.1881	1.29503	1.04403	3.30151	1.02914	2.21722	4.77686	.917431
1.10	1.2100	1.33100	1.04881	3.31662	1.03228	2.22398	4.79142	.909091
1.11	1.2321	1 36763	1.05357	3.33167	1.03540	2.23070	4.80590	.900901
1.12	1.2544	1.40493	1.05830	3.34664	1.03850	2.23738	4.82028	.892857
1.13	1.2769	1.44290	1.06301	3.36155	1.04158	2.24402	4.83459	.884956
1.14	1.2996	1.48154	1.06771	3.37639	1.04464	2.25062	4.84881	.877193
1.15	1.3225	1.52088	1.07238	3.39116	1.04769	2.25718	4.86294	.869565
1.16	1.3456	1.56090	1.07703	3.40588	1.05072	2.26370	4.87700	.862069
1.17	1.3689	1.60161	1.08167	3.42053	1.05373	2.27019	4.89097	.854701
1.18	1.3924	1.64303	1.08628	3.43511	1.05672	2.27664	4.90487	.847458
1.19	1.4161	1.68516	1.09087	3.44964	1.05970	2.28305	4.91868	.840336
1.20	1.4400	1.72800	1.09545	3.46410	1.06266	2.28943	4.93242	.833333
1.21	1.4641	1.77156	1.10000	3.47851	1.06560	2.29577	4.94609	.826446
1.22	1.4884	1.81585	1.10454	3.49285	1.06853	2.30208	4.95968	.819672
1.23	1.5129	1.86087	1.10905	3.50714	1.07144	2.30835	4.97319	.813008
1.24	1.5376	1.90662	1.11355	3.52136	1.07434	2.31459	4.98663	.806452
1.25	1.5625	1.95313	1.11803	3.53553	1.07722	2.32080	5.00000	.800000
1.26	1.5876	2.00038	1.12250	3.54965	1.08008	2.32697	5.01330	.793651
1.27	1.6129	2.04838	1.12694	3.56371	1.08293	2.33310	5.02653	.787402
1.28	1.6384	2.09715	1.13137	3.57771	1.08577	2.33921	5.03968	.781250
1.29	1.6641	2.14669	1.13578	3.59166	1.08859	2.34529	5.05277	.775194
1.30	1.6900	2.19700	1.14018	3.60555	1.09139	2.35134	5.06580	.769231
1.31	1.7161	2.24809	1.14455	3.61939	1.09418	2.35735	5.07875	.763359
1.32	1.7424	2.29997	1.14891	3.63318	1.09696	2.36333	5.09164	.757576
1.33	1.7689	2.35264	1.15326	3.64692	1.09972	2.36928	5.10447	.751880
1.34	1.7956	2.40610	1.15758	3.66060	1.10247	2.37521	5.11723	.746269
1.35	1.8225	2.46038	1.16190	3.67423	1.10521	2.38110	5.12993	.740741
1.36	1.8496	2.51546	1.16619	3.68782	1.10793	2.38696	5.14256	.735294
1.37	1.8769	2.57135	1.17047	3.70135	1.11064	2.39280	5.15514	.729927
1.38	1.9044	2.62807	1.17473	3.71484	1.11334	2.39861	5.16765	.724638
1.39	1.9321	2.68562	1.17898	3.72827	1.11602	2.40439	5.18010	.719425
1.40	1.9600	2.74400	1.18322	3.74166	1.11869	2.41014	5.19249	.714286
1.41	1.9881	2.80322	1.18743	3.75500	1.12135	2.41587	5.20483	.709220
1.42	2.0164	2.86329	1.19164	3.76829	1.12399	2.42156	5.21710	.704225
1.43	2.0449	2.92421	1.19583	3.78153	1.12662	2.42724	5.22932	.699301
1.44	2.0736	2.98598	1.20000	3 79473	1.12924	2.43288	5.24148	.694444
1.45	2.1025	3.04863	1.20416	3.80789	1.13185	2.43850	5.25359	.689655
1.46	2.1316	3.11214	1.20830	3.82099	1.13445	2.44409	5.26564	.684932
1.47	2.1609	3.17652	1.21244	3.83406	1.13703	2.44966	5.27763	.680272
1.48	2.1904	3.24179	1.21655	3.84708	1.13960	2.45520	5.28957	.675676
1.49	2.2201	3.30795	1.22066	3.86005	1.14216	2.46072	5.30146	.671141
1.50	2.2500	3.37500	1.22474	3.87298	1.14471	2.46621	5.31329	.666667

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10\,n}$	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$\frac{1}{n}$
1.51	2.2801	3.44295	1.22882	3.88587	1.14725	2.47168	5.32507	.662252
1.52	2.3104	3.51181	1.23288	3.89872	1.14978	2.47713	5.33680	.657895
1.53	2.3409	3.58158	1.23693	3.91152	1.15230	2.48255	5.34848	.653595
1.54	2.3716	3.65226	1.24097	3.92428	1.15480	2.48794	5.36011	.649351
1.55	2.4025	3.72388	1.24499	3.93700	1.15729	2.49332	5.37169	.645161
1.56	2.4336	3.79642	1.24900	3.94968	1.15978	2.49866	5.38321	.641026
1.57	2.4649	3.86989	1.25300	3.96232	1.16225	2.50399	5.39469	.636943
1.58	2.4964	3.94431	1.25698	3.97492	1.16471	2.50930	5.40612	.632911
1.59	2.5281	4.01968	1.26095	3.98748	1.16717	2.51458	5.41750	.628931
1.60	2.5600	4.09600	1.26491	4.00000	1.16961	2.51984	5.42884	.625000
1.61	2.5921	4.17328	1.26886	4.01248	1.17204	2.52508	5.44012	.621118
1.62	2.6244	4.25153	1.27279	4.02492	1.17446	2.53030	5.45136	.617284
1.63	2.6569	4.33075	1.27671	4.03733	1.17687	2.53549	5.46256	.613497
1.64	2.6896	4.41094	1.28062	4.04969	1.17927	2.54067	5.47370	.609756
1.65	2.7225	4.49213	1.28452	4.06202	1.18167	2.54582	5.48481	.606061
1.66	2.7556	4.57430	1.28841	4.07431	1.18405	2.55095	5.49586	.602410
1.67	2.7889	4.65746	1.29228	4.08656	1.18642	2.55607	5.50688	.598802
1.68	2.8224	4.74163	1.29615	4.09878	1.18878	2.56116	5.51785	.595238
1.69	2.8561	4.82681	1.30000	4.11096	1.19114	2.56623	5.52877	.591716
1.70	2.8900	4.91300	1.30384	4.12311	1.19348	2.57128	5.53966	.588235
1.71	2.9241	5.00021	1.30767	4.13521	1.19582	2.57631	5.55050	.584795
1.72	2.9584	5.08845	1.31149	4.14729	1.19815	2.58133	5.56130	.581395
1.73	2.9929	5.17772	1.31529	4.15933	1.20046	2.58632	5.57205	.578035
1.74	3.0276	5.26802	1.31909	4.17133	1.20277	2.59129	5.58277	.574713
1.75	3.0625	5.35938	1.32288	4.18330	1.20507	2.59625	5.59344	.571429
1.76	3.0976	5.45178	1.32665	4.19524	1.20736	2.60118	5.60408	.568182
1.77	3.1329	5.54523	1.33041	4.20714	1.20964	2.60610	5.61467	.564972
1.78	3.1684	5.63975	1.33417	4.21900	1.21192	2.61100	5.62523	.561798
1.79	3.2041	5.73534	1.33791	4.23084	1.21418	2.61588	5.63574	.558659
1.80	3.2400	5.83200	1.34164	4.24264	1.21644	2.62074	5.64622	.555556
1.81	3.2761	5.92974	1.34536	4.25441	1.21869	2.62558	5.65665	.552486
1.82	3.3124	6.02857	1.34907	4.26615	1.22093	2.63041	5.66705	.549451
1.83	3.3489	6.12849	1.35277	4.27785	1.22316	2.63522	5.67741	.546448
1.84	3.3856	6.22950	1.35647	4.28952	1.22539	2.64001	5.68773	.543478
1.85	3.4225	6.33163	1.36015	4.30116	1.22760	2.64479	5.69802	.540541
1.86	3.4596	6.43486	1.36382	4.31277	1.22981	2.64954	5.70827	.537634
1.87	3.4969	6.53920	1.36748	4.32435	1.23201	2.65428	5.71848	.534759
1.88	3.5344	6.64467	1.37113	4.33590	1.23420	2.65900	5.72865	.531915
1.89	3.5721	6.75127	1.37477	4.34741	1.23639	2.66371	5.73879	.529101
1.90	3.6100	6.85900	1.37840	4.35890	1.23856	2.66840	5.74890	.526316
1.91	3.6481	6.96787	1.38203	4.37035	1.24073	2.67307	5.75897	.523560
1.92	3.6864	7.07789	1.38564	4.38178	1.24289	2.67773	5.76900	.520833
1.93	3.7249	7.18906	1.38924	4.39318	1.24505	2.68237	5.77900	.518135
1.94	3.7636	7.30138	1.39284	4.40454	1.24719	2.68700	5.78896	.515464
1.95	3.8025	7.41488	1.39642	4.41588	1.24933	2.69161	5.79889	.512821
1.96	3.8416	7.52954	1.40000	4.42719	1.25146	2.69620	5.80879	.510204
1.97	3.8809	7.64537	1.40357	4.43847	1.25359	2.70078	5.81865	.507614
1.98	3.9204	7.76239	1.40712	4.44972	1.25571	2.70534	5.82848	.505051
1.99	3.9601	7.88060	1.41067	4.46094	1.25782	2.70989	5.83827	.502513
2.00	4.0000	8.00000	1.41421	4.47214	1.25992	2.71442	5.84804	.500000

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10\,n}$	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$\frac{1}{n}$
2.01	4.0401	8.12060	1.41774	4.48330	1.26202	2.71893	5.85777	.497512
2.02	4.0804	8.24241	1.42127	4.49444	1.26411	2.72343	5.86746	.495050
2.03	4.1209	8.36543	1.42478	4.50555	1.26619	2.72792	5.87713	.492611
2.04	4.1616	8.48966	1.42829	4.51664	1.26827	2.73239	5.88677	.490196
2.05	4.2025	8.61513	1.43178	4.52769	1.27033	2.73685	5.89637	.487805
2.06	4.2436	8.74182	1.43527	4.53872	1.27240	2.74129	5.90594	.485437
2.07	4.2849	8.86974	1.43875	4.54973	1.27445	2.74572	5.91548	.483092
2.08	4.3264	8.99891	1.44222	4.56070	1.27650	2.75014	5.92499	.480769
2.09	4.3681	9.12933	1.44568	4.57165	1.27854	2.75454	5.93447	.478469
2.10	4.4100	9.26100	1.44914	4.58258	1.28058	2.75893	5.94392	.476191
2.11	4.4521	9.39393	1.45258	4.59347	1.28261	2.76330	5.95334	.473934
2.12	4.4944	9.52813	1.45602	4.60435	1.28463	2.76766	5.96273	.471698
2.13	4.5369	9.66360	1.45945	4.61519	1.28665	2.77200	5.97209	.469484
2.14	4.5796	9.80034	1.46287	4.62601	1.28866	2.77633	5.98142	.467290
2.15	4.6225	9.93838	1.46629	4.63681	1.29066	2.78065	5.99073	.465116
2.16	4.6656	10.0777	1.46969	4.64758	1.29266	2.78495	6.00000	.462963
2.17	4.7089	10.2183	1.47309	4.65833	1.29465	2.78924	6.00925	.460830
2.18	4.7524	10.3602	1.47648	4.66905	1.29664	2.79352	6.01846	.458716
2.19	4.7961	10.5035	1.47986	4.67974	1.29862	2.79779	6.02765	.456621
2.20	4.8400	10.6480	1.48324	4.69042	1.30059	2.80204	6.03681	.454546
2.21	4.8841	10.7939	1.48661	4.70106	1.30256	2.80628	6.04594	.452489
2.22	4.9284	10.9410	1.48997	4.71169	1.30452	2.81051	6.05505	.450451
2.23	4.9729	11.0896	1.49332	4.72229	1.30648	2.81472	6.06413	.448431
2.24	5.0176	11.2394	1.49666	4.73286	1.30843	2.81892	6.07318	.446429
2.25	5.0625	11.3906	1.50000	4.74342	1.31037	2.82311	6.08220	.444444
2.26	5.1076	11.5432	1.50333	4.75395	1.31231	2.82728	6.09120	.442478
2.27	5.1529	11.6971	1.50665	4.76445	1.31424	2.83145	6.10017	.440529
2.28	5.1984	11.8524	1.50997	4.77493	1.31617	2.83560	6.10911	.438597
2.29	5.2441	12.0090	1.51327	4.78539	1.31809	2.83974	6.11803	.436681
2.30	5.2900	12.1670	1.51658	4.79583	1.32001	2.84387	6.12693	.434783
2.31	5.3361	12.3264	1.51987	4.80625	1.32192	2.84798	6.13579	.432900
2.32	5.3824	12.4872	1.52315	4.81664	1.32382	2.85209	6.14463	.431035
2.33	5.4289	12.6493	1.52643	4.82701	1.32572	2.85618	6.15345	.429185
2.34	5.4756	12.8129	1.52971	4.83735	1.32761	2.86026	6.16224	.427350
2.35	5.5225	12.9779	1.53297	4.84768	1.32950	2.86433	6.17101	.425532
2.36	5.5696	13.1443	1.53623	4.85798	1.33139	2.86838	6.17975	.423729
2.37	5.6169	13.3121	1.53948	4.86826	1.33326	2.87243	6.18846	.421941
2.38	5.6644	13.4813	1.54272	4.87852	1.33514	2.87646	6.19715	.420168
2.39	5.7121	13.6519	1.54596	4.88876	1.33700	2.88049	6.20582	.418410
2.40	5.7600	13.8240	1.54919	4.89898	1.33887	2.88450	6.21447	.416667
2.41	5.8081	13.9975	1.55242	4.90918	1.34072	2.88850	6.22308	.414938
2.42	5.8564	14.1725	1.55563	4.91935	1.34257	2.89249	6.23168	.413223
2.43	5.9049	14.3489	1.55885	4.92950	1.34442	2.89647	6.24025	.411523
2.44	5.9536	14.5268	1.56205	4.93964	1.34626	2.90044	6.24880	.409836
2.45	6.0025	14.7061	1.56525	4.94975	1.34810	2.90439	6.25732	.408163
2.46	6.0516	14.8869	1.56844	4.95984	1.34993	2.90834	6.26583	.406504
2.47	6.1009	15.0692	1.57162	4.96991	1.35176	2.91227	6.27431	.404858
2.48	6.1504	15.2530	1.57480	4.97996	1.35358	2.91620	6.28276	.403226
2.49	6.2001	15.4382	1.57797	4.98999	1.35540	2.92011	6.29119	.401606
2.50	6.2500	15.6250	1.58114	5.00000	1.35721	2.92402	6.29961	.400000

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10\,n}$	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$\frac{1}{n}$
2.51	6.3001	15.8133	1.58430	5.00999	1.35902	2.92791	6.30799	.398406
2.52	6.3504	16.0030	1.58745	5.01996	1.36082	2.93179	6.31636	.396825
2.53	6.4009	16.1943	1.59060	5.02991	1.36262	2.93567	6.32470	.395257
2.54	6.4516	16.3871	1.59374	5.03984	1.36441	2.93953	6.33303	.393701
2.55	6.5025	16.5814	1.59687	5.04975	1.36620	2.94338	6.34133	.392157
2.56	6.5536	16.7772	1.60000	5.05964	1.36798	2.94723	6.34960	.390625
2.57	6.6049	16.9746	1.60312	5.06952	1.36976	2.95106	6.35786	.389105
2.58	6.6564	17.1735	1.60624	5.07937	1.37153	2.95488	6.36610	.387597
2.59	6.7081	17.3740	1.60935	5.08920	1.37330	2.95869	6.37431	.386100
2.60	6.7600	17.5760	1.61245	5.09902	1.37507	2.96250	6.38250	.384615
2.61	6.8121	17.7796	1.61555	5.10882	1.37683	2.96629	6.39068	.383142
2.62	6.8644	17.9847	1.61864	5.11859	1.37859	2.97007	6.39883	.381679
2.63	6.9169	18.1914	1.62173	5.12835	1.38034	2.97385	6.40696	.380228
2.64	6.9696	18.3997	1.62481	5.13809	1.38208	2.97761	6.41507	.378788
2.65	7.0225	18.6096	1.62788	5.14782	1.38383	2.98137	6.42316	.377359
2.66	7.0756	18.8211	1.63095	5.15752	1.38557	2.98511	6.43123	.375940
2.67	7.1289	19.0342	1.63401	5.16720	1.38730	2.98885	6.43928	.374532
2.68	7.1824	19.2488	1.63707	5.17687	1.38903	2.99257	6.44731	.373134
2.69	7.2361	19.4651	1.64012	5.18652	1.39076	2.99629	6.45531	.371747
2.70	7.2900	19.6830	1.64317	5.19615	1.39248	3.00000	6.46330	.370370
2.71	7.3441	19.9025	1.64621	5.20577	1.39419	3.00370	6.47127	.369004
2.72	7.3984	20.1236	1.64924	5.21536	1.39591	3.00739	6.47922	.367647
2.73	7.4529	20.3464	1.65227	5.22494	1.39761	3.01107	6.48715	.366300
2.74	7.5076	20.5708	1.65529	5.23450	1.39932	3.01474	6.49507	.364964
2.75	7.5625	20.7969	1.65831	5.24404	1.40102	3.01841	6.50296	.363636
2.76	7.6176	21.0246	1.66132	5.25357	1.40272	3.02206	6.51083	.362319
2.77	7.6729	21.2539	1.66433	5.26308	1.40441	3.02571	6.51868	.361011
2.78	7.7284	21.4850	1.66733	5.27257	1.40610	3.02934	6.52652	.359712
2.79	7.7841	21.7176	1.67033	5.28205	1.40778	3.03297	6.53434	.358423
2.80	7.8400	21.9520	1.67332	5.29150	1.40946	3.03659	6.54213	.357142
2.81	7.8961	22.1880	1.67631	5.30094	1.41114	3.04020	6.54991	.355872
2.82	7.9524	22.4258	1.67929	5.31037	1.41281	3.04380	6.55767	.354610
2.83	8.0089	22.6652	1.68226	5.31977	1.41448	3.04740	6.56541	.353357
2.84	8.0656	22.9063	1.68523	5.32917	1.41614	3.05098	6.57314	.352113
2.85	8.1225	23.1491	1.68819	5.33854	1.41780	3.05456	6.58084	.350877
2.86	8.1796	23.3937	1.69115	5.34790	1.41946	3.05813	6.58853	.349650
2.87	8.2369	23.6399	1.69411	5.35724	1.42111	3.06169	6.59620	.348432
2.88	8.2944	23.8879	1.69706	5.36656	1.42276	3.06524	6.60385	.347222
2.89	8.3521	24.1376	1.70000	5.37587	1.42440	3.06878	6.61149	.346021
2.90	8.4100	24.3890	1.70294	5.38516	1.42604	3.07232	6.61911	.344828
2.91	8.4681	24.6422	1.70587	5.39444	1.42768	3.07585	6.62671	.343643
2.92	8.5264	24.8971	1.70880	5.40370	1.42931	3.07936	6.63429	.342466
2.93	8.5849	25.1538	1.71172	5.41295	1.43094	3.08287	6.64185	.341297
2.94	8.6436	25.4122	1.71464	5.42218	1.43257	3.08638	6.64940	.340136
2.95	8.7025	25.6724	1.71756	5.43139	1.43419	3.08987	6.65693	.338983
2.96	8.7616	25.9343	1.72047	5.44059	1.43581	3.09336	6.66444	.337838
2.97	8.8209	26.1981	1.72337	5.44977	1.43743	3.09684	6.67194	.336700
2.98	8.8804	26.4636	1.72627	5.45894	1.43904	3.10031	6.67942	.335571
2.99	8.9401	26.7309	1.72916	5.46809	1.44065	3.10378	6.68688	.334448
3.00	9.0000	27.0000	1.73205	5.47723	1.44225	3.10723	6.69433	.333333

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$\frac{1}{n}$
3.01	9.0601	27.2709	1.73494	5.48635	1.44385	3.11068	6.70176	.332226
3.02	9.1204	27.5436	1.73781	5.49545	1.44545	3.11412	6.70917	.331126
3.03	9.1809	27.8181	1.74069	5.50454	1.44704	3.11755	6.71657	.330033
3.04	9.2416	28.0945	1.74356	5.51362	1.44863	3.12098	6.72395	.328947
3.05	9.3025	28.3726	1.74642	5.52268	1 45022	3.12440	6.73132	.327869
3.06	9.3636	28.6526	1.74929	5.53173	1.45180	3.12781	6.73866	.326797
3.07	9.4249	28.9344	1.75214	5.54076	1.45338	3.13121	6.74600	.325733
3.08	9.4864	29.2181	1.75499	5.54977	1.45496	3.13461	6.75331	.324675
3.09	9.5481	29.5036	1.75784	5.55878	1.45653	3.13800	6.76061	.323625
3.10	9.6100	29.7910	1.76068	5.56776	1.45810	3.14138	6.76790	.322581
3.11	9.6721	30.0802	1.76352	5.57674	1.45967	3.14475	6.77517	.321543
3.12	9.7344	30.3713	1.76635	5.58570	1.46123	3.14812	6.78242	.320513
3.13	9.7969	30.6643	1.76918	5.59464	1.46279	3.15148	6.78966	.319489
3.14	9.8596	30.9591	1.77200	5.60357	1.46434	3.15484	6.79688	.318471
3.15	9.9225	31.2559	1.77482	5.61249	1.46590	3.15818	6.80409	.317460
3.16	9.9856	31.5545	1.77764	5.62139	1.46745	3.16152	6.81128	.316456
3.17	10.0489	31.8550	1.78045	5.63028	1.46899	3.16485	6.81846	.315457
3.18	10.1124	32.1574	1.78326	5.63915	1.47054	3.16817	6.82562	.314465
3.19	10.1761	32.4618	1.78606	5.64801	1.47208	3.17149	6.83277	.313480
3.20	10.2400	32.7680	1.78885	5.65685	1.47361	3.17480	6.83990	.312500
3.21	10.3041	33.0762	1.79165	5.66569	1.47515	3.17811	6.84702	.311527
3.22	10.3684	33.3862	1.79444	5.67450	1.47668	3.18140	6.85412	.310559
3.23	10.4329	33.6983	1.79722	5.68331	1.47820	3.18469	6.86121	.309598
3.24	10.4976	34.0122	1.80000	5.69210	1.47973	3.18798	6.86829	.308642
3.25	10.5625	34.3281	1.80278	5.70088	1.48125	3.19125	6.87534	.307692
3.26	10.6276	34.6460	1.80555	5.70964	1.48277	3.19452	6.88239	.306749
3.27	10.6929	34.9658	1.80831	5.71839	1.48428	3.19779	6.88942	.305810
3.28	10.7584	35.2876	1.81108	5.72713	1.48579	3.20104	6.89643	.304878
3.29	10.8241	35.6129	1.81384	5.73585	1.48730	3.20429	6.90344	.303951
3.30	10.8900	35.9370	1.81659	5.74456	1.48881	3.20753	6.91042	.303030
3.31	10.9561	36.2647	1.81934	5.75326	1.49031	3.21077	6.91740	.302115
3.32	11.0224	36.5944	1.82209	5.76194	1.49181	3.21400	6.92436	.301205
3.33	11.0889	36.9260	1.82483	5.77062	1.49330	3.21723	6.93130	.300300
3.34	11.1556	37.2597	1.82757	5.77927	1.49480	3.22044	6.93823	.299401
3.35	11.2225	37.5954	1.83030	5.78792	1.49629	3.22365	6.94515	.298508
3.36	11.2896	37.9331	1.83303	5.79655	1.49777	3.22686	6.95205	.297619
3.37	11.3569	38.2728	1.83576	5.80517	1.49926	3.23005	6.95894	.296736
3.38	11.4244	38.6145	1.83848	5.81378	1.50074	3.23325	6.96582	.295858
3.39	11.4921	38.9582	1.84120	5.82237	1.50222	3.23643	6.97268	.294985
3.40	11.5600	39.3040	1.84391	5.83095	1.50369	3.23961	6.97953	.294118
3.41	11.6281	39.6518	1.84662	5.83952	1.50517	3.24278	6.98637	.293255
3.42	11.6964	40.0017	1.84932	5.84808	1.50664	3.24595	6.99319	.292398
3.43	11.7649	40.3536	1.85203	5.85662	1.50810	3.24911	7.00000	.291545
3.44	11.8336	40.7076	1.85472	5.86515	1.50957	3.25227	7.00680	.290698
3.45	11.9025	41.0636	1.85742	5.87367	1.51103	3.25542	7.01358	289855
3.46	11.9716	41.4217	1.86011	5.88218	1.51249	3.25856	7.02035	.289017
3.47	12.0409	41.7819	1.86279	5.89067	1.51394	3.26169	7.02711	.288184
3.48	12.1104	42.1442	1.86548	5.89915	1.51540	3.26482	7.03385	.287356
3.49	12.1801	42.5085	1.86815	5.90762	1.51685	3.26795	7.04058	.286533
3.50	12.2500	42.8750	1.87083	5.91608	1.51829	3.27107	7.04730	.285714

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10\,n}$	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$\frac{1}{n}$
3.51	12.3201	43.2436	1.87350	5.92453	1.51974	3.27418	7.05400	.284900
3.52	12.3904	43.6142	1.87617	5.93296	1.52118	3.27729	7.06070	.284091
3.53	12.4609	43.9870	1.87883	5.94138	1.52262	3.28039	7.06738	.283286
3.54	12.5316	44.3619	1.88149	5.94979	1.52406	3.28348	7.07404	.282486
3.55	12.6025	44.7389	1.88414	5.95819	1.52549	3.28657	7.08070	.281690
3.56	12.6736	45.1180	1.88680	5.96657	1.52692	3.28965	7.08734	.280899
3.57	12.7449	45.4993	1.88944	5.97495	1.52835	3.29273	7.09397	.280112
3.58	12.8164	45.8827	1.89209	5.98331	1.52978	3.29580	7.10059	.279330
3.59	12.8881	46.2683	1.89473	5.99166	1.53120	3.29887	7.10719	.278552
3.60	12.9600	46.6560	1.89737	6.00000	1.53262	3.30193	7.11379	.277778
3.61	13.0321	47.0459	1.90000	6.00833	1.53404	3.30498	7.12037	.277008
3.62	13.1044	47.4379	1.90263	6.01664	1.53545	3.30803	7.12694	.276243
3.63	13.1769	47.8321	1.90526	6.02495	1.53686	3.31107	7.13349	.275482
3.64	13.2496	48.2285	1.90788	6.03324	1.53827	3.31411	7.14004	.274725
3.65	13.3225	48.6271	1.91050	6.04152	1.53968	3.31714	7.14657	.273973
3.66	13.3956	49.0279	1.91311	6.04979	1.54109	3.32017	7.15309	.273224
3.67	13.4689	49.4309	1.91572	6.05805	1.54249	3.32319	7.15960	.272480
3.68	13.5424	49.8360	1.91833	6.06630	1.54389	3.32621	7.16610	.271739
3.69	13.6161	50.2434	1.92094	6.07454	1.54529	3.32922	7.17258	.271003
3.70	13.6900	50.6530	1.92354	6.08276	1.54668	3.33222	7.17905	.270270
3.71	13.7641	51.0648	1.92614	6.09098	1.54807	3.33522	7.18552	.269542
3.72	13.8384	51.4788	1.92873	6.09918	1.54946	3.33822	7.19197	.268817
3.73	13.9129	51.8951	1.93132	6.10737	1.55085	3.34120	7.19841	.268097
3.74	13.9876	52.3136	1.93391	6.11555	1.55223	3.34419	7.20483	.267380
3.75	14.0625	52.7344	1.93649	6.12372	1.55362	3.34716	7.21125	.266667
3.76	14.1376	53.1574	1.93907	6.13188	1.55500	3.35014	7.21765	.265957
3.77	14.2129	53.5826	1.94165	6.14003	1.55637	3.35310	7.22405	.265252
3.78	14.2884	54.0102	1.94422	6.14817	1.55775	3.35607	7.23043	.264550
3.79	14.3641	54.4399	1.94679	6.15630	1.55912	3.35902	7.23680	.263852
3.80	14.4400	54.8720	1.94936	6.16441	1.56049	3.36198	7.24316	.263158
3.81	14.5161	55.3063	1.95192	6.17252	1.56186	3.36492	7.24950	.262467
3.82	14.5924	55.7430	1.95448	6.18061	1.56322	3.36786	7.25584	.261780
3.83	14.6689	56.1819	1.95704	6.18870	1.56459	3.37080	7.26217	.261097
3.84	14.7456	56.6231	1.95959	6.19677	1.56595	3.37373	7.26848	.260417
3.85	14.8225	57.0666	1.96214	6.20484	1.56731	3.37666	7.27479	.259740
3.86	14.8996	57.5125	1.96469	6.21289	1.56866	3.37958	7.28108	.259067
3.87	14.9769	57.9606	1.96723	6.22093	1.57001	3.38249	7.28736	.258398
3.88	15.0544	58.4111	1.96977	6.22896	1.57137	3.38540	7.29363	.257732
3.89	15.1321	58 8639	1.97231	6.23699	1.57271	3.38831	7.29989	.257069
3.90	15.2100	59.3190	1.97484	6.24500	1.57406	3.39121	7.30614	.256410
3.91	15.2881	59.7765	1.97737	6.25300	1.57541	3.39411	7.31238	.255755
3.92	15.3664	60.2363	1.97990	6.26099	1.57675	3.39700	7.31861	.255102
3.93	15.4449	60.6985	1.98242	6.26897	1.57809	3.39988	7.32483	.254453
3.94	15.5236	61.1630	1.98494	6.27694	1.57942	3.40277	7.33104	.253807
3.95	15.6025	61.6299	1.98746	6.28490	1.58076	3.40564	7.33723	.253165
3.96	15.6816	62.0991	1.98997	6.29285	1.58209	3.40851	7.34342	.252525
3.97	15.7609	62.5708	1.99249	6.30079	1.58342	3.41138	7.34960	.251889
3.98	15.8404	63.0448	1.99499	6.30872	1.58475	3.41424	7.35576	.251256
3.99	15.9201	63.5212	1.99750	6.31664	1.58608	3.41710	7.36192	.250627
4.00	16.0000	64.0000	2.00000	6.32456	1.58740	3.41995	7.36806	.250000

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10\,n}$	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$\frac{1}{n}$
4.01	16.0801	64.4812	2.00250	6.33246	1.58872	3.42280	7.37420	.249377
4.02	16.1604	64.9648	2.00499	6.34035	1.59004	3.42564	7.38032	.248756
4.03	16.2409	65.4508	2.00749	6.34823	1.59136	3.42848	7.38644	.248139
4.04	16.3216	65.9393	2.00998	6.35610	1.59267	3.43131	7.39254	.247525
4.05	16.4025	66.4301	2.01246	6.36396	1.59399	3.43414	7.39864	.246914
4.06	16.4836	66.9234	2.01494	6.37181	1.59530	3.43697	7.40472	.246305
4.07	16.5649	67.4191	2.01742	6.37966	1.59661	3.43979	7.41080	.245700
4.08	16.6464	67.9173	2.01990	6.38749	1.59791	3.44260	7.41686	.245098
4.09	16.7281	68.4179	2.02237	6.39531	1.59922	3.44541	7.42291	.244499
4.10	16.8100	68.9210	2.02485	6.40312	1.60052	3.44822	7.42896	.243902
4.11	16.8921	69.4265	2.02731	6.41093	1.60182	3.45102	7.43499	.243309
4.12	16.9744	69.9345	2.02978	6.41872	1.60312	3.45382	7.44102	.242718
4.13	17.0569	70.4450	2.03224	6.42651	1.60441	3.45661	7.44703	.242131
4.14	17.1396	70.9579	2.03470	6.43428	1.60571	3.45939	7.45304	.241546
4.15	17.2225	71.4734	2.03715	6.44205	1.60700	3.46218	7.45904	.240964
4.16	17.3056	71.9913	2.03961	6.44981	1.60829	3.46496	7.46502	.240385
4.17	17.3889	72.5117	2.04206	6.45755	1.60958	3.46773	7.47100	.239808
4.18	17.4724	73.0346	2.04450	6.46529	1.61086	3.47050	7.47697	.239234
4.19	17.5561	73.5601	2.04695	6.47302	1.61215	3.47327	7.48292	.238664
4.20	17.6400	74.0880	2.04939	6.48074	1.61343	3.47603	7.48887	.238095
4.21	17.7241	74.6185	2.05183	6.48845	1.61471	3.47878	7.49481	.237530
4.22	17.8084	75.1514	2.05426	6.49615	1.61599	3.48154	7.50074	.236967
4.23	17.8929	75.6870	2.05670	6.50385	1.61726	3.48428	7.50666	.236407
4.24	17.9776	76.2250	2.05913	6.51153	1.61853	3.48703	7.51257	.235849
4.25	18.0625	76.7656	2.06155	6.51920	1.61981	3.48977	7.51847	.235294
4.26	18.1476	77.3088	2.06398	6.52687	1.62108	3.49250	7.52437	.234742
4.27	18.2329	77.8545	2.06640	6.53452	1.62234	3.49523	7.53025	.234192
4.28	18.3184	78.4028	2.06882	6.54217	1.62361	3.49796	7.53612	.233645
4.29	18.4041	78.9536	2.07123	6.54981	1.62487	3.50068	7.54199	.233100
4.30	18.4900	79.5070	2.07364	6.55744	1.62613	3.50340	7.54784	.232558
4.31	18.5761	80.0630	2.07605	6.56506	1.62739	3.50611	7.55369	.232019
4.32	18.6624	80.6216	2.07846	6.57267	1.62865	3.50882	7.55953	.231482
4.33	18.7489	81.1827	2.08087	6.58027	1.62991	3.51153	7.56535	.230947
4.34	18.8356	81.7465	2.08327	6.58787	1.63116	3.51423	7.57117	.230415
4.35	18.9225	82.3129	2.08567	6.59545	1.63241	3.51692	7.57698	.229885
4.36	19.0096	82.8819	2.08806	6.60303	1.63366	3.51962	7.58279	.229358
4.37	19.0969	83.4535	2.09045	6.61060	1.63491	3.52231	7.58858	.228833
4.38	19.1844	84.0277	2.09284	6.61816	1.63616	3.52499	7.59436	.228311
4.39	19.2721	84.6045	2.09523	6.62571	1.63740	3.52767	7.60014	.227790
4.40	19.3600	85.1840	2.09762	6.63325	1.63864	3.53035	7.60590	.227273
4.41	19.4481	85.7661	2.10000	6.64078	1.63988	3.53302	7.61166	.226757
4.42	19.5364	86.3509	2.10238	6.64831	1.64112	3.53569	7.61741	.226244
4.43	19.6249	86.9383	2.10476	6.65582	1.64236	3.53835	7.62315	.225734
4.44	19.7136	87.5284	2.10713	6.66333	1.64359	3.54101	7.62888	.225225
4.45	19.8025	88.1211	2.10950	6.67083	1.64483	3.54367	7.63461	.224719
4.46	19.8916	88.7165	2.11187	6.67832	1.64606	3.54632	7.64032	.224215
4.47	19.9809	89.3146	2.11424	6.68581	1.64729	3.54897	7.64603	.223714
4.48	20.0704	89.9154	2.11660	6.69328	1.64851	3.55162	7.65172	.223214
4.49	20.1601	90.5188	2.11896	6.70075	1.64974	3.55426	7.65741	.222717
4.50	20.2500	91.1250	2.12132	6.70820	1.65096	3.55689	7 66309	.222222

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$\frac{1}{n}$
4.51	20.3401	91.7339	2.12368	6.71565	1.65219	3.55953	7.66877	.221730
4.52	20.4304	92.3454	2.12603	6.72309	1.65341	3.56215	7.67443	.221239
4.53	20.5209	92.9597	2.12838	6.73053	1.65462	3.56478	7.68009	.220751
4.54	20.6116	93.5767	2.13073	6.73795	1.65584	3.56740	7.68573	.220264
4.55	20.7025	94.1964	2.13307	6.74537	1.65706	3.57002	7.69137	.219780
4.56	20.7936	94.8188	2.13542	6.75278	1.65827	3.57263	7.69700	.219298
4.57	20.8849	95.4440	2.13776	6.76018	1.65948	3.57524	7.70262	.218818
4.58	20.9764	96.0719	2.14009	6.76757	1.66069	3.57785	7.70824	.218341
4.59	21.0681	96.7026	2.14243	6.77495	1.66190	3.58045	7.71384	.217865
4.60	21.1600	97.3360	2.14476	6.78233	1.66310	3.58305	7.71944	.217391
4.61	21.2521	97.9722	2.14709	6.78970	1.66431	3.58564	7.72503	.216920
4.62	21.3444	98.6111	2.14942	6.79706	1.66551	3.58823	7.73061	.216450
4.63	21.4369	99.2528	2.15174	6.80441	1.66671	3.59082	7.73619	.215983
4.64	21.5296	99.8973	2.15407	6.81175	1.66791	3.59340	7.74175	.215517
4.65	21.6225	100.545	2.15639	6.81909	1.66911	3.59598	7.74731	.215054
4.66	21.7156	101.195	2.15870	6.82642	1.67030	3.59856	7.75286	.214592
4.67	21.8089	101.848	2.16102	6.83374	1.67150	3.60113	7.75840	.214133
4.68	21.9024	102.503	2.16333	6.84105	1.67269	3.60370	7.76394	.213675
4.69	21.9961	103.162	2.16564	6.84836	1.67388	3.60626	7.76946	.213220
4.70	22.0900	103.823	2.16795	6.85565	1.67507	3.60883	7.77498	.212766
4.71	22.1841	104.487	2.17025	6.86294	1.67626	3.61138	7.78049	.212314
4.72	22.2784	105.154	2.17256	6.87023	1.67744	3.61394	7.78599	.211864
4.73	22.3729	105.824	2.17486	6.87750	1.67863	3.61649	7.79149	.211417
4.74	22.4676	106.496	2.17715	6.88477	1.67981	3.61904	7.79697	.210971
4.75	22.5625	107.172	2.17945	6.89202	1.68099	3.62158	7.80245	.210526
4.76	22.6576	107.850	2.18174	6.89928	1.68217	3.62412	7.80793	.210084
4.77	22.7529	108.531	2.18403	6.90652	1.68334	3.62665	7.81339	.209644
4.78	22.8484	109.215	2.18632	6.91375	1.68452	3.62919	7.81885	.209205
4.79	22.9441	109.902	2.18861	6.92098	1.68569	3.63171	7.82429	.208768
4.80	23.0400	110.592	2.19089	6.92820	1.68687	3.63424	7.82974	.208333
4.81	23.1361	111.285	2.19317	6.93542	1.68804	3.63676	7.83517	.207900
4.82	23.2324	111.980	2.19545	6.94262	1.68920	3.63928	7.84059	.207469
4.83	23.3289	112.679	2.19773	6.94982	1.69037	3.64180	7.84601	.207039
4.84	23.4256	113.380	2.20000	6.95701	1.69154	3.64431	7.85142	.206612
4.85	23.5225	114.084	2.20227	6.96419	1.69270	3.64682	7.85683	.206186
4.86	23.6196	114.791	2.20454	6.97137	1.69386	3.64932	7.86222	.205761
4.87	23.7169	115.501	2.20681	6.97854	1.69503	3.65182	7.86761	.205339
4.88	23.8144	116.214	2.20907	6.98570	1.69619	3.65432	7.87299	.204918
4.89	23.9121	116.930	2.21133	6.99285	1.69734	3.65682	7.87837	.204499
4.90	24.0100	117.649	2.21359	7.00000	1.69850	3.65931	7.88374	.204082
4.91	24.1081	118.371	2.21585	7.00714	1.69965	3.66179	7.88909	.203666
4.92	24.2064	119.095	2.21811	7.01427	1.70081	3.66428	7.89445	.203252
4.93	24.3049	119.823	2.22036	7.02140	1.70196	3.66676	7.89979	.202840
4.94	24.4036	120.554	2.22261	7.02851	1.70311	3.66924	7.90513	.202429
4.95	24.5025	121.287	2.22486	7.03562	1.70426	3.67171	7.91046	.202020
4.96	24.6016	122.024	2.22711	7.04273	1.70540	3.67418	7.91578	.201613
4.97	24.7009	122.763	2.22935	7.04982	1.70655	3.67665	7.92110	.201207
4.98	24.8004	123.506	2.23159	7.05691	1.70769	3.67911	7.92641	.200803
4.99	24.9001	124.251	2.23383	7.06399	1.70884	3.68157	7.93171	.200401
5.00	25.0000	125.000	2.23607	7.07107	1.70998	3.68403	7.93701	.200000

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10\,n}$	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$\frac{1}{n}$
5.01	25.1001	125.752	2.23830	7.07814	1.71112	3.68649	7.94229	.199601
5.02	25.2004	126.506	2.24054	7.08520	1.71225	3.68894	7.94757	.199203
5.03	25.3009	127.264	2.24277	7.09225	1.71339	3.69138	7.95285	.198807
5.04	25.4016	128.024	2.24499	7.09930	1.71452	3.69383	7.95811	.198413
5.05	25.5025	128.788	2.24722	7.10634	1.71566	3.69627	7.96337	.198020
5.06	25.6036	129.554	2.24944	7.11337	1.71679	3.69871	7.96863	.197629
5.07	25.7049	130.324	2.25167	7.12039	1.71792	3.70114	7.97387	.197239
5.08	25.8064	131.097	2.25389	7.12741	1.71905	3.70358	7.97911	.196850
5.09	25.9081	131.872	2.25610	7.13442	1.72017	3.70600	7.98434	.196464
5.10	26.0100	132.651	2.25832	7.14143	1.72130	3.70843	7.98957	.196078
5.11	26.1121	133.433	2.26053	7.14843	1.72242	3.71085	7.99479	.195695
5.12	26.2144	134.218	2.26274	7.15542	1.72355	3.71327	8.00000	.195313
5.13	26.3169	135.006	2.26495	7.16240	1.72467	3.71566	8.00520	.194932
5.14	26.4196	135.797	2.26716	7.16938	1.72579	3.71810	8.01040	.194553
5.15	26.5225	136.591	2.26936	7.17635	1.72691	3.72051	8.01559	.194175
5.16	26.6256	137.388	2.27156	7.18331	1.72802	3.72292	8.02078	.193798
5.17	26.7289	138.188	2.27376	7.19027	1.72914	3.72532	8.02596	.193424
5.18	26.8324	138.992	2.27596	7.19722	1.73025	3.72772	8.03113	.193050
5.19	26.9361	139.798	2.27816	7.20417	1.73137	3.73012	8.03629	.192678
5.20	27.0400	140.608	2.28035	7.21110	1.73248	3.73251	8.04145	.192308
5.21	27.1441	141.421	2.28254	7.21803	1.73359	3.73490	8.04660	.191939
5.22	27.2484	142.237	2.28473	7.22496	1.73470	3.73729	8.05175	.191571
5.23	27.3529	143.056	2.28692	7.23187	1.73580	3.73968	8.05689	.191205
5.24	27.4576	143.878	2.28910	7.23878	1.73691	3.74206	8.06202	.190840
5.25	27.5625	144.703	2.29129	7.24569	1.73801	3.74443	8.06714	.190476
5.26	27.6676	145.532	2.29347	7.25259	1.73912	3.74681	8.07226	.190114
5.27	27.7729	146.363	2.29565	7.25948	1.74022	3.74918	8.07737	.189753
5.28	27.8784	147.198	2.29783	7.26636	1.74132	3.75158	8.08248	.189394
5.29	27.9841	148.036	2.30000	7.27324	1.74242	3.75392	8.08758	.189036
5.30	28.0900	148.877	2.30217	7.28011	1.74351	3.75629	8.09267	.188679
5.31	28.1961	149.721	2.30434	7.28697	1.74461	3.75865	8.09776	.188324
5.32	28.3024	150.569	2.30651	7.29383	1.74570	3.76100	8.10284	.187970
5.33	28.4089	151.419	2.30868	7.30068	1.74680	3.76336	8.10791	.187617
5.34	28.5156	152.273	2.31084	7.30753	1.74789	3.76571	8.11298	.187266
5.35	28.6225	153.130	2.31301	7.31437	1.74898	3.76806	8.11804	.186916
5.36	28.7296	153.991	2.31517	7.32120	1.75007	3.77041	8.12310	.186567
5.37	28.8369	154.854	2.31733	7.32803	1.75116	3.77275	8.12814	.186220
5.38	28.9444	155.721	2.31948	7.33485	1.75224	3.77509	8.13319	.185874
5.39	29.0521	156.591	2.32164	7.34166	1.75333	3.77740	8.13822	.185529
5.40	29.1600	157.464	2.32379	7.34847	1.75441	3.77976	8.14325	.185185
5.41	29.2681	158.340	2.32594	7.35527	1.75549	3.78210	8.14828	.184843
5.42	29.3764	159.220	2.32809	7.36206	1.75657	3.78442	8.15329	.184502
5.43	29.4849	160.103	2.33024	7.36885	1.75765	3.78675	8.15831	.184162
5.44	29.5936	160.989	2.33238	7.37564	1.75873	3.78907	8.16331	.183824
5.45	29.7025	161.879	2.33452	7.38241	1.75981	3.79139	8.16831	.183486
5.46	29.8116	162.771	2.33666	7.38918	1.76088	3.79371	8.17330	.183150
5.47	29.9209	163.667	2.33880	7.39594	1.76196	3.79603	8.17829	.182815
5.48	30.0304	164.567	2.34094	7.40270	1.76303	3.79834	8.18327	.182482
5.49	30.1401	165.469	2.34307	7.40945	1.76410	3.80065	8.18824	.182149
5.50	30.2500	166.375	2.34521	7.41620	1.76517	3.80295	8.19321	.181818

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$\frac{1}{n}$
5.51	30.3601	167.284	2.34734	7.42294	1.76624	3.80526	8.19818	.181488
5.52	30.4704	168.197	2.34947	7.42967	1.76731	3.80756	8.20313	.181159
5.53	30.5809	169.112	2.35160	7.43640	1.76838	3.80986	8.20808	.180832
5.54	30.6916	170.031	2.35372	7.44312	1.76944	3.80115	8.21303	.180505
5.55	30.8025	170.954	2.35584	7.44983	1.77051	3.81444	8.21797	.180180
5.56	30.9136	171.880	2.35797	7.45654	1.77157	3.81673	8.22290	.179856
5.57	31.0249	172.809	2.36008	7.46324	1.77263	3.81902	8.22783	.179533
5.58	31.1364	173.741	2.36220	7.46994	1.77369	3.82130	8.23275	.179212
5.59	31.2481	174.677	2.36432	7.47663	1.77475	3.82358	8.23766	.178891
5.60	31.3600	175.616	2.36643	7.48331	1.77581	3.82586	8.24257	.178571
5.61	31.4721	176.558	2.36854	7.48999	1.77686	3.82814	8.24747	.178253
5.62	31.5844	177.504	2.37065	7.49667	1.77792	3.83041	8.25237	.177936
5.63	31.6969	178.454	2.37276	7.50333	1.77897	3.83268	8.25726	.177620
5.64	31.8096	179.406	2.37487	7.50999	1.78003	3.83495	8.26215	.177305
5.65	31.9225	180.362	2.37697	7.51665	1.78108	3.83721	8.26703	.176991
5.66	32.0356	181.321	2.37908	7.52330	1.78213	3.83948	8.27190	.176678
5.67	32.1489	182.284	2.38118	7.52994	1.78318	3.84174	8.27677	.176367
5.68	32.2624	183.250	2.38328	7.53658	1.78422	3.84400	8.28164	.176056
5.69	32.3761	184.220	2.38537	7.54321	1.78527	3.84625	8.28649	.175747
5.70	32.4900	185.193	2.38747	7.54983	1.78632	3.84850	8.29134	.175439
5.71	32.6041	186.169	2.38956	7.55645	1.78736	3.85075	8.29619	.175131
5.72	32.7184	187.149	2.39165	7.56307	1.78840	3.85300	8.30103	.174825
5.73	32.8329	188.133	2.39374	7.56968	1.78944	3.85524	8.30587	.174520
5.74	32.9476	189.119	2.39583	7.57628	1.79048	3.85748	8.31069	.174216
5.75	33.0625	190.109	2.39792	7.58288	1.79152	3.85972	8.31552	.173913
5.76	33.1776	191.103	2.40000	7.58947	1.79256	3.86196	8.32034	.173611
5.77	33.2929	192.100	2.40208	7.59605	1.79360	3.86419	8.32515	.173310
5.78	33.4084	193.101	2.40416	7.60263	1.79463	3.86612	8.32995	.173010
5.79	33.5241	194.105	2.40624	7.60920	1.79567	3.86865	8.33476	.172712
5.80	33.6400	195.112	2.40832	7.61577	1.79670	3.87088	8.33955	.172414
5.81	33.7561	196.123	2.41039	7.62234	1.79773	3.87310	8.34434	.172117
5.82	33.8724	197.137	2.41247	7.62889	1.79876	3.87532	8.34913	.171821
5.83	33.9889	198.155	2.41454	7.63544	1.79979	3.87754	8.35390	.171527
5.84	34.1056	199.177	2.41661	7.64199	1.80082	3.87975	8.35868	.171233
5.85	34.2225	200.202	2.41868	7.64853	1.80185	3.88197	8.36345	.170940
5.86	34.3396	201.230	2.42074	7.65506	1.80288	3.88418	8.36821	.170649
5.87	34.4569	202.262	2.42281	7.66159	1.80390	3.88639	8.37297	.170358
5.88	34.5744	203 297	2.42487	7.66812	1.80492	3.88859	8.37772	.170068
5.89	34.6921	204.336	2.42693	7.67463	1.80595	3.89082	8.38247	.169779
5.90	34.8100	205.379	2.42899	7.68115	1.80697	3.89300	8.38721	.169492
5.91	34.9281	206.425	2.43105	7.68765	1.80799	3.89520	8.39194	.169205
5.92	35.0464	207.475	2.43311	7.69415	1.80901	3.89739	8.39667	.168919
5.93	35.1649	208.528	2.43516	7.70065	1.81003	3.89958	8.40140	.168634
5.94	35.2836	209.585	2.43721	7.70714	1.81104	3.90177	8.40612	.168350
5.95	35.4025	210.645	2.43926	7.71362	1.81206	3.90396	8.41083	.168067
5.96	35.5216	211.709	2.44131	7.72010	1.81307	3.90615	8.41554	.167785
5.97	35.6409	212.776	2.44336	7.72658	1.81409	3.90833	8.42025	.167504
5.98	35.7604	213.847	2.44540	7.73305	1.81510	3.91051	8.42494	.167224
5.99	35.8801	214.922	2.44745	7.73951	1.81611	3.91269	8.42964	.166945
6.00	36.0000	216.000	2.44949	7.74597	1.81712	3.91487	8.43433	.166667

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10\,n}$	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$\frac{1}{n}$
6.01	36.1201	217.082	2.45153	7.75242	1.81813	3.91704	8.43901	.166389
6.02	36.2404	218.167	2.45357	7.75887	1.81914	3.91921	8.44369	.166113
6.03	36.3609	219.256	2.45561	7.76531	1.82014	3.92138	8.44836	.165838
6.04	36.4816	220.349	2.45764	7.77174	1.82115	3.92355	8.45303	.165563
6.05	36.6025	221.445	2.45967	7.77817	1.82215	3.92571	8.45769	.165289
6.06	36.7236	222.545	2.46171	7.78460	1.82316	3.92787	8.46235	.165017
6.07	36.8449	223.649	2.46374	7.79102	1.82416	3.93003	8.46700	.164745
6.08	36.9664	224.756	2.46577	7.79744	1.82516	3.93219	8.47165	.164474
6.09	37.0881	225.867	2.46779	7.80385	1.82616	3.93434	8.47629	.164204
6.10	37.2100	226.981	2.46982	7.81025	1.82716	3.93650	8.48093	.163934
6.11	37.3321	228.099	2.47184	7.81665	1.82816	3.93865	8.48556	.163666
6.12	37.4544	229.221	2.47386	7.82304	1.82915	3.94079	8.49018	.163399
6.13	37.5769	230.346	2.47588	7.82943	1.83015	3.94294	8.49481	.163132
6.14	37.6996	231.476	2.47790	7.83582	1.83115	3.94508	8.49942	.162866
6.15	37.8225	232.608	2.47992	7.84219	1.83214	3.94722	8.50404	.162602
6.16	37.9456	233.745	2.48193	7.84857	1.83313	3.94936	8.50864	.162338
6.17	38.0689	234.885	2.48395	7.85493	1.83412	3.95150	8.51324	.162075
6.18	38.1924	236.029	2.48596	7.86130	1.83511	3.95363	8.51784	.161812
6.19	38.3161	237.177	2.48797	7.86766	1.83610	3.95576	8.52243	.161551
6.20	38.4400	238.328	2.48998	7.87401	1.83709	3.95789	8.52702	.161290
6.21	38.5641	239.483	2.49199	7.88036	1.83808	3.96002	8.53160	.161031
6.22	38.6884	240.642	2.49399	7.88670	1.83906	3.96214	8.53618	.160772
6.23	38.8129	241.804	2.49600	7.89303	1.84005	3.96426	8.54075	.160514
6.24	38.9376	242.971	2.49800	7.89937	1.84103	3.96639	8.54532	.160256
6.25	39.0625	244.141	2.50000	7.90569	1.84202	3.96850	8.54988	.160000
6.26	39.1876	245.314	2.50200	7.91202	1.84300	3.97062	8.55444	.159744
6.27	39.3129	246.492	2.50400	7.91833	1.84398	3.97273	8.55899	.159490
6.28	39.4384	247.673	2.50599	7.92465	1.84496	3.97484	8.56354	.159236
6.29	39.5641	248.858	2.50799	7.93095	1.84594	3.97695	8.56808	.158983
6.30	39.6900	250.047	2.50998	7.93725	1.84691	3.97906	8.57262	.158730
6.31	39.8161	251.240	2.51197	7.94355	1.84789	3.98116	8.57715	.158479
6.32	39.9424	252.436	2.51396	7.94984	1.84887	3.98326	8.58168	.158228
6.33	40.0689	253.636	2.51595	7.95613	1.84984	3.98536	8.58620	.157978
6.34	40.1956	254.840	2.51794	7.96241	1.85082	3.98746	8.59072	.157729
6.35	40.3225	256.048	2.51992	7.96869	1.85179	3.98956	8.59524	.157480
6.36	40.4496	257.259	2.52190	7.97496	1.85276	3.99165	8.59975	.157233
6.37	40.5769	258.475	2.52389	7.98123	1.85373	3.99374	8.60425	.156986
6.38	40.7044	259.694	2.52587	7.98749	1.85470	3.99583	8.60875	.156740
6.39	40.8321	260.917	2.52784	7.99375	1.85567	3.99792	8.61325	.156495
6.40	40.9600	262.144	2.52982	8.00000	1.85664	4.00000	8.61774	.156250
6.41	41.0881	263.375	2.53180	8.00625	1.85760	4.00208	8.62222	.156006
6.42	41.2164	264.609	2.53377	8.01249	1.85857	4.00416	8.62671	.155763
6.43	41.3449	265.848	2.53574	8.01873	1.85953	4.00624	8.63118	.155521
6.44	41.4736	267.090	2.53772	8.02496	1.86050	4.00832	8.63566	.155280
6.45	41.6025	268.336	2.53969	8.03119	1.86146	4.01039	8.64012	.155039
6.46	41.7316	269.586	2.54165	8.03741	1.86242	4.01246	8.64459	.154799
6.47	41.8609	270.840	2.54362	8.04363	1.86338	4.01453	8.64904	.154560
6.48	41.9904	272.098	2.54558	8.04984	1.86434	4.01660	8.65350	.154321
6.49	42.1201	273.359	2.54755	8.05605	1.86530	4.01866	8.65795	.154083
6.50	42.2500	274.625	2.54951	8.06226	1.86626	4.02073	8.66239	.153846

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10\,n}$	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$\frac{1}{n}$
6.51	42.3801	275.894	2.55147	8.06846	1.86721	4.02279	8.66683	.153610
6.52	42.5104	277.168	2.55343	8.07465	1.86817	4.02485	8.67127	.153374
6.53	42.6409	278.445	2.55539	8.08084	1.86912	4.02690	8.67570	.153139
6.54	42.7716	279.726	2.55734	8.08703	1.87008	4.02896	8.68012	.152905
6.55	42.9025	281.011	2.55930	8.09321	1.87103	4.03101	8.68455	.152672
6.56	43.0336	282.300	2.56125	8.09938	1.87198	4.03306	8.68896	.152439
6.57	43.1649	283.593	2.56320	8.10555	1.87293	4.03511	8.69338	.152207
6.58	43.2964	284.890	2.56515	8.11172	1.87388	4.03715	8.69778	.151976
6.59	43.4281	286.191	2.56710	8.11788	1.87483	4.03920	8.70219	.151745
6.60	43.5600	287.496	2.56905	8.12404	1.87578	4.04124	8.70659	.151515
6.61	43.6921	288.805	2.57099	8.13019	1.87672	4.04328	8.71098	.151286
6.62	43.8244	290.118	2.57294	8.13634	1.87767	4.04532	8.71537	.151057
6.63	43.9569	291.434	2.57488	8.14248	1.87862	4.04735	8.71976	.150830
6.64	44.0896	292.755	2.57682	8.14862	1.87956	4.04939	8.72414	.150602
6.65	44.2225	294.080	2.57876	8.15475	1.88050	4.05142	8.72852	.150376
6.66	44.3556	295.408	2.58070	8.16088	1.88144	4.05345	8.73289	.150150
6.67	44.4889	296.741	2.58263	8.16701	1.88239	4.05548	8.73726	.149925
6.68	44.6224	298.078	2.58457	8.17313	1.88333	4.05750	8.74162	.149701
6.69	44.7561	299.418	2.58650	8.17924	1.88427	4.05953	8.74598	.149477
6.70	44.8900	300.763	2.58844	8.18535	1.88520	4.06155	8.75034	.149254
6.71	45.0241	302.112	2.59037	8.19146	1.88614	4.06357	8.75469	.149031
6.72	45.1584	303.464	2.59230	8.19756	1.88708	4.06558	8.75904	.148810
6.73	45.2929	304.821	2.59422	8.20366	1.88801	4.06760	8.76338	.148588
6.74	45.4276	306.182	2.59615	8.20975	1.88895	4.06961	8.76772	.148368
6.75	45.5625	307.547	2.59808	8.21584	1.88988	4.07163	8.77205	.148148
6.76	45.6976	308.916	2.60000	8.22192	1.89081	4.07364	8.77638	.147929
6.77	45.8329	310.289	2.60192	8.22800	1.89175	4.07564	8.78071	.147711
6.78	45.9684	311.666	2.60384	8.23408	1.89268	4.07765	8.78503	.147493
6.79	46.1041	313.047	2.60576	8.24015	1.89361	4.07965	8.78935	.147275
6.80	46.2400	314.432	2.60768	8.24621	1.89454	4.08166	8.79366	.147059
6.81	46.3761	315.821	2.60960	8.25227	1.89546	4.08365	8.79797	.146843
6.82	46.5124	317.215	2.61151	8.25833	1.89639	4.08565	8.80227	.146628
6.83	46.6489	318.612	2.61343	8.26438	1.89732	4.08765	8.80657	.146413
6.84	46.7856	320.014	2.61534	8.27043	1.89824	4.08964	8.81087	.146199
6.85	46.9225	321.419	2.61725	8.27647	1.89917	4.09164	8.81516	.145985
6.86	47.0596	322.829	2.61916	8.28251	1.90009	4.09362	8.81945	.145773
6.87	47.1969	324.243	2.62107	8.28855	1.90102	4.09561	8.82373	.145560
6.88	47.3344	325.661	2.62298	8.29458	1.90194	4.09760	8.82801	.145349
6.89	47.4721	327.083	2.62488	8.30060	1.90286	4.09958	8.83229	.145138
6.90	47.6100	328.509	2.62679	8.30662	1.90378	4.10157	8.83656	.144928
6.91	47.7481	329.939	2.62869	8.31264	1.90470	4.10355	8.84082	.144718
6.92	47.8864	331.374	2.63059	8.31865	1.90562	4.10552	8.84509	.144509
6.93	48.0249	332.813	2.63249	8.32466	1.90653	4.10750	8.84934	.144300
6.94	48.1636	334.255	2.63439	8.33067	1.90745	4.10948	8.85360	.144092
6.95	48.3025	335.702	2.63629	8.33667	1.90837	4.11145	8.85785	.143885
6.96	48.4416	337.154	2.63818	8.34266	1.90928	4.11342	8.86210	.143678
6.97	48.5809	338.609	2.64008	8.34865	1.91019	4.11539	8.86634	.143472
6.98	48.7204	340.068	2.64197	8.35464	1.91111	4.11736	8.87058	.143267
6.99	48.8601	341.532	2.64386	8.36062	1.91202	4.11932	8.87481	.143062
7.00	49.0000	343.000	2.64575	8.36660	1.91293	4.12129	8.87904	.142857

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10\,n}$	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$\frac{1}{n}$
7.01	49.1401	344.472	2.64764	8.37257	1.91384	4.12325	8.88327	.142653
7.02	49.2804	345.948	2.64953	8.37854	1.91475	4.12521	8.88749	.142450
7.03	49.4209	347.429	2.65141	8.38451	1.91566	4.12716	8.89171	.142248
7.04	49.5616	348.914	2.65330	8.39047	1.91657	4.12912	8.89592	.142046
7.05	49.7025	350.403	2.65518	8.39643	1.91747	4.13107	8.90013	.141844
7.06	49.8436	351.896	2.65707	8.40238	1.91838	4.13303	8.90434	.141643
7.07	49.9849	353.393	2.65895	8.40833	1.91929	4.13498	8.90854	.141443
7.08	50.1264	354.895	2.66083	8.41427	1.92019	4.13695	8.91274	.141243
7.09	50.2681	356.401	2.66271	8.42021	1.92109	4.13887	8.91693	.141044
7.10	50.4100	357.911	2.66458	8.42615	1.92200	4.14082	8.92112	.140845
7.11	50.5521	359.425	2.66646	8.43208	1.92290	4.14276	8.92531	.140647
7.12	50.6944	360.944	2.66833	8.43801	1.92380	4.14470	8.92949	.140449
7.13	50.8369	362.467	2.67021	8.44393	1.92470	4.14664	8.93367	.140253
7.14	50.9796	363.994	2.67208	8.44985	1.92560	4.14858	8.93784	.140056
7.15	51.1225	365.526	2.67395	8.45577	1.92650	4.15051	8.94201	.139860
7.16	51.2656	367.062	2.67582	8.46168	1.92740	4.15245	8.94618	.139665
7.17	51.4089	368.602	2.67769	8.46759	1.92829	4.15438	8.95034	.139470
7.18	51.5524	370.146	2.67955	8.47349	1.92919	4.15631	8.95450	.139276
7.19	51.6961	371.695	2.68142	8.47939	1.93008	4.15824	8.95866	.139082
7.20	51.8400	373.248	2.68328	8.48528	1.93098	4.16017	8.96281	.138889
7.21	51.9841	374.805	2.68514	8.49117	1.93187	4.16209	8.96696	.138696
7.22	52.1284	376.367	2.68701	8.49706	1.93277	4.16402	8.97110	.138504
7 23	52.2729	377.933	2.68887	8.50294	1.93366	4.16594	8.97524	.138313
7.24	52.4176	379.503	2.69072	8.50882	1.93455	4.16786	8.97938	.138122
7.25	52.5625	381.078	2.69258	8.51469	1.93544	4.16978	8.98351	.137931
7.26	52.7076	382.657	2.69444	8.52056	1.93633	4.17169	8.98764	.137741
7.27	52.8529	384.241	2.69629	8.52643	1.93722	4.17361	8.99176	.137552
7.28	52.9984	385.828	2.69815	8.53229	1.93810	4.17552	8.99588	.137363
7.29	53.1441	387.420	2.70000	8.53815	1.93899	4.17743	9.00000	.137174
7.30	53.2900	389.017	2.70185	8.54400	1.93988	4.17934	9.00411	.136986
7.31	53.4361	390.618	2.70370	8.54985	1.94076	4.18125	9.00822	.136799
7.32	53.5824	392.223	2.70555	8.55570	1.94165	4.18315	9.01233	.136612
7.33	53.7289	393.833	2.70740	8.56154	1.94253	4.18506	9.01643	.136426
7.34	53.8756	395.447	2.70924	8.56738	1.94341	4.18696	9.02053	.136240
7.35	54.0225	397.065	2.71109	8.57321	1.94430	4.18886	9.02462	.136054
7.36	54.1696	398.688	2.71293	8.57904	1.94518	4.19076	9.02871	.135870
7.37	54.3169	400.316	2.71477	8.58487	1.94606	4.19266	9.03280	.135685
7.38	54.4644	401.947	2.71662	8.59069	1.94694	4.19455	9.03689	.135501
7.39	54.6121	403.583	2.71846	8.59651	1.94782	4.19644	9.04097	.135318
7.40	54.7600	405.224	2.72029	8.60233	1.94870	4.19834	9.04504	.135135
7.41	54.9081	406.869	2.72213	8.60814	1.94957	4.20023	9.04911	.134953
7.42	55.0564	408.518	2.72397	8.61394	1.95045	4.20212	9.05318	.134771
7.43	55.2049	410.172	2.72580	8.61974	1.95132	4.20400	9.05725	.134590
7.44	55.3536	411.831	2.72764	8.62554	1.95220	4.20589	9.06131	.134409
7.45	55.5025	413.494	2.72947	8.63134	1.95307	4.20777	9.06537	.134228
7.46	55.6516	415.161	2.73130	8.63713	1.95395	4.20965	9.06942	.134048
7.47	55.8009	416.833	2.73313	8.64292	1.95482	4.21153	9.07347	.133869
7.48	55.9504	418.509	2.73496	8.64870	1.95569	4.21341	9.07752	.133690
7.49	56.1001	420.190	2.73679	8.65448	1.95656	4.21529	9.08156	.133511
7.50	56.2500	421.875	2.73861	8.66025	1.95743	4.21716	9.08560	.133333

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$\frac{1}{n}$
7.51	56.4001	423.565	2.74044	8.66603	1.95830	4.21904	9.08964	.133156
7.52	56.5504	425.259	2.74226	8.67179	1.95917	4.22091	9.09367	.132979
7.53	56.7009	426.958	2.74408	8.67756	1.96004	4.22278	9.09770	.132802
7.54	56.8516	428.661	2.74591	8.68332	1.96091	4.22465	9.10173	.132626
7.55	57.0025	430.369	2.74773	8.68907	1.96177	4.22651	9.10575	.132450
7.56	57.1536	432.081	2.74955	8.69483	1.96264	4.22838	9.10977	.132275
7.57	57.3049	433.798	2.75136	8.70057	1.96350	4.23024	9.11378	.132100
7.58	57.4564	435.520	2.75318	8.70632	1.96437	4.23210	9.11779	.131926
7.59	57.6081	437.245	2.75500	8.71206	1.96523	4.23396	9.12180	.131752
7.60	57.7600	438.976	2.75681	8.71780	1.96610	4.23582	9.12581	.131579
7.61	57.9121	440.711	2.75862	8.72353	1.96696	4.23768	9.12981	.131406
7.62	58.0644	442.451	2.76043	8.72926	1.96782	4.23954	9.13380	.131234
7.63	58.2169	444.195	2.76225	8.73499	1.96868	4.24139	9.13780	.131062
7.64	58.3696	445.994	2.76405	8.74071	1.96954	4.24324	9.14179	.130890
7.65	58.5225	447.697	2.76586	8.74643	1.97040	4.24509	9.14577	.130719
7.66	58.6756	449.455	2.76767	8.75214	1.97126	4.24694	9.14976	.130548
7.67	58.8289	451.218	2.76948	8.75785	1.97211	4.24879	9.15374	.130378
7.68	58.9824	452.985	2.77128	8.76356	1.97297	4.25063	9.15771	.130208
7.69	59.1361	454.757	2.77308	8.76926	1.97383	4.25248	9.16169	.130039
7.70	59.2900	456.533	2.77489	8.77496	1.97468	4.25432	9.16566	.129870
7.71	59.4441	458.314	2.77669	8.78066	1.97554	4.25616	9.16962	.129702
7.72	59.5984	460.100	2.77849	8.78635	1.97639	4.25800	9.17359	.129534
7.73	59.7529	461.890	2.78029	8.79204	1.97724	4.25984	9.17754	.129366
7.74	59.9076	463.685	2.78209	8.79773	1.97809	4.26168	9.18150	.129199
7.75	60.0625	465.484	2.78388	8.80341	1.97895	4.26351	9.18545	.129032
7.76	60.2176	467.289	2.78568	8.80909	1.97980	4.26534	9.18940	.128866
7.77	60.3729	469.097	2.78747	8.81476	1.98065	4.26717	9.19335	.128700
7.78	60.5284	470.911	2.78927	8.82043	1.98150	4.26900	9.19729	.128535
7.79	60.6841	472.729	2.79106	8.82610	1.98234	4.27083	9.20123	.128370
7.80	60.8400	474.552	2.79285	8.83176	1.98319	4.27266	9.20516	.128205
7.81	60.9961	476.380	2.79464	8.83742	1.98404	4.27448	9.20910	.128041
7.82	61.1524	478.212	2.79643	8.84308	1.98489	4.27631	9.21303	.127877
7.83	61.3089	480.049	2.79821	8.84873	1.98573	4.27813	9.21695	.127714
7.84	61.4656	481.890	2.80000	8.85438	1.98658	4.27995	9.22087	.127551
7.85	61.6225	483.737	2.80179	8.86002	1.98742	4.28177	9.22479	.127389
7.86	61.7796	485.588	2.80357	8.86566	1.98826	4.28359	9.22871	.127227
7.87	61.9369	487.443	2.80535	8.87130	1.98911	4.28540	9.23262	.127065
7.88	62.0944	489.304	2.80713	8.87694	1.98995	4.28722	9.23653	.126904
7.89	62.2521	491.169	2.80891	8.88257	1.99079	4.28903	9.24043	.126743
7.90	62.4100	493.039	2.81069	8.88819	1.99163	4.29084	9.24433	.126582
7.91	62.5681	494.914	2.81247	8.89382	1.99247	4.29265	9.24823	.126422
7.92	62.7264	496.793	2.81425	8.89944	1.99331	4.29446	9.25213	.126263
7.93	62.8849	498.677	2.81603	8.90505	1.99415	4.29627	9.25602	.126103
7.94	63.0436	500.566	2.81780	8.91067	1.99499	4.29807	9.25991	.125945
7.95	63.2025	502.460	2.81957	8.91628	1.99582	4.29987	9.26380	.125786
7.96	63.3616	504.358	2.82135	8.92188	1.99666	4.30168	9.26768	.125628
7.97	63.5209	506.262	2.82312	8.92749	1.99750	4.30348	9.27156	.125471
7.98	63.6804	508.170	2.82489	8.93308	1.99833	4.30528	9.27544	.125313
7.99	63.8401	510.082	2.82666	8.93868	1.99917	4.30707	9.27931	.125156
8.00	64.0000	512.000	2.82843	8.94427	2.00000	4.30887	9.28318	.125000

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$\frac{1}{n}$
8.01	64.1601	513.922	2.83019	8.94986	2.00083	4.31066	9.28704	.124844
8.02	64.3204	515.850	2.83196	8.95545	2.00167	4.31246	9.29091	.124688
8.03	64.4809	517.782	2.83373	8.96103	2.00250	4.31425	9.29477	.124533
8.04	64.6416	519.718	2.83549	8.96660	2.00333	4.31604	9.29862	.124378
8.05	64.8025	521.660	2.83725	8.97218	2.00416	4.31783	9.30248	.124224
8.06	64.9636	523.607	2.83901	8.97775	2.00499	4.31961	9.30633	.124070
8.07	65.1249	525.558	2.84077	8.98332	2.00582	4.32140	9.31018	.123916
8.08	65.2864	527.514	2.84253	8.98888	2.00664	4.32318	9.31402	.123762
8.09	65.4481	529.475	2.84429	8.99444	2.00747	4.32497	9.31786	.123609
8.10	65.6100	531.441	2.84605	9.00000	2.00830	4.32675	9.32170	.123457
8.11	65.7721	533.412	2.84781	9.00555	2.00912	4.32853	9.32553	.123305
8.12	65.9344	535.387	2.84956	9.01110	2.00995	4.33031	9.32936	.123153
8.13	66.0969	537.368	2.85132	9.01665	2.01078	4.33208	9.33319	.123001
8.14	66.2596	539.353	2.85307	9.02219	2.01160	4.33386	9.33702	.122850
8.15	66.4225	541.343	2.85482	9.02774	2.01242	4.33563	9.34084	.122699
8.16	66.5856	543.338	2.85657	9.03327	2.01325	4.33741	9.34466	.122549
8.17	66.7489	545.339	2.85832	9.03881	2.01407	4.33918	9.34847	.122399
8.18	66.9124	547.343	2.86007	9.04434	2.01489	4.34095	9.35229	.122249
8.19	67.0761	549.353	2.86182	9.04986	2.01571	4.34272	9.35610	.122100
8.20	67.2400	551.368	2.86356	9.05539	2.01653	4.34448	9.35990	.121951
8.21	67.4041	553.388	2.86531	9.06091	2.01735	4.34625	9.36370	.121803
8.22	67.5684	555.412	2.86705	9.06642	2.01817	4.34801	9.36751	.121655
8.23	67.7329	557.442	2.86880	9.07193	2.01899	4.34977	9.37130	.121507
8.24	67.8976	559.476	2.87054	9.07744	2.01980	4.35153	9.37510	.121359
8.25	68.0625	561.516	2.87228	9.08295	2.02062	4.35329	9.37889	.121212
8.26	68.2276	563.560	2.87402	9.08845	2.02144	4.35505	9.38268	.121065
8.27	68.3929	565.609	2.87576	9.09395	2.02225	4.35681	9.38646	.120919
8.28	68.5584	567.664	2.87750	9.09945	2.02307	4.35856	9.39024	.120773
8.29	68.7241	569.723	2.87924	9.10494	2.02388	4.36032	9.39402	.120627
8.30	68.8900	571.787	2.88097	9.11043	2.02469	4.36207	9.39780	.120482
8.31	69.0561	573.856	2.88271	9.11592	2.02551	4.36382	9.40157	.120337
8.32	69.2224	575.930	2.88444	9.12140	2.02632	4.36557	9.40534	.120192
8.33	69.3889	578.010	2.88617	9.12688	2.02713	4.36732	9.40911	.120048
8.34	69.5556	580.094	2.88791	9.13236	2.02794	4.36907	9.41287	.119904
8.35	69.7225	582.183	2.88964	9.13783	2.02875	4.37081	9.41663	.119761
8.36	69.8896	584.277	2.89137	9.14330	2.02956	4.37255	9.42039	.119617
8.37	70.0569	586.376	2.89310	9.14877	2.03037	4.37430	9.42414	.119474
8.38	70.2244	588.480	2.89482	9.15423	2.03118	4.37604	9.42789	.119332
8.39	70.3921	590.590	2.89655	9.15969	2.03199	4.37778	9.43164	.119190
8.40	70.5600	592.704	2.89828	9.16515	2.03279	4.37952	9.43539	.119048
8.41	70.7281	594.823	2.90000	9.17061	2.03360	4.38126	9.43913	.118906
8.42	70.8964	596.948	2.90172	9.17606	2.03440	4.38299	9.44287	.118765
8.43	71.0649	599.077	2.90345	9.18150	2.03521	4.38473	9.44661	.118624
8.44	71.2336	601.212	2.90517	9.18695	2.03601	4.38646	9.45034	.118483
8.45	71.4025	603.351	2.90689	9.19239	2.03682	4.38819	9.45407	.118343
8.46	71.5716	605.496	2.90861	9.19783	2.03762	4.38992	9.45780	.118203
8.47	71.7409	607.645	2.91033	9.20326	2.03842	4.39165	9.46152	.118064
8.48	71.9104	609.800	2.91204	9.20869	2.03923	4.39338	9.46525	.117925
8.49	72.0801	611.960	2.91376	9.21412	2.04003	4.39511	9.46897	.117786
8.50	72.2500	614.125	2.91548	9.21954	2.04083	4.39683	9.47268	.117647

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$\frac{1}{n}$
8.51	72.4201	616.295	2.91719	9.22497	2.04163	4.39855	9.47640	.117509
8.52	72.5904	618.470	2.91890	9.23038	2.04243	4.40028	9.48011	.117371
8.53	72.7609	620.650	2.92062	9.23580	2.04323	4.40200	9.48381	.117233
8.54	72.9316	622.836	2.92233	9.24121	2.04402	4.40372	9.48752	.117096
8.55	73.1025	625.026	2.92404	9.24662	2.04482	4.40543	9.49122	.116959
8.56	73.2736	627.222	2.92575	9.25203	2.04562	4.40715	9.49492	.116822
8.57	73.4449	629.423	2.92746	9.25743	2.04641	4.40887	9.49861	.116686
8.58	73.6164	631.629	2.92916	9.26283	2.04721	4.41058	9.50231	.116550
8.59	73.7881	633.840	2.93087	9.26823	2.04801	4.41229	9.50600	.116414
8.60	73.9600	636.056	2.93258	9.27362	2.04880	4.41400	9.50969	.116279
8.61	74.1321	638.277	2.93428	9.27901	2.04959	4.41571	9.51337	.116144
8.62	74.3044	640.504	2.93598	9.28440	2.05039	4.41742	9.51705	.116009
8.63	74.4769	642.736	2.93769	9.28978	2.05118	4.41913	9.52073	.115875
8.64	74.6496	644.973	2.93939	9.29516	2.05197	4.42084	9.52441	.115741
8.65	74.8225	647.215	2.94109	9.30054	2.05276	4.42254	9.52808	.115607
8.66	74.9956	649.462	2.94279	9.30591	2.05355	4.42425	9.53175	.115473
8.67	75.1689	651.714	2.94449	9.31128	2.05434	4.42595	9.53542	.115340
8.68	75.3424	653.972	2.94618	9.31665	2.05513	4.42765	9.53908	.115207
8.69	75.5161	656.235	2.94788	9.32202	2.05592	4.42935	9.54274	.115075
8.70	75.6900	658.503	2.94958	9.32738	2.05671	4.43105	9.54640	.114943
8.71	75.8641	660.776	2.95127	9.33274	2.05750	4.43274	9.55006	.114811
8.72	76.0384	663.055	2.95296	9.33809	2.05828	4.43444	9.55371	.114679
8.73	76.2129	665.339	2.95466	9.34345	2.05907	4.43614	9.55736	.114548
8.74	76.3876	667.628	2.95635	9.34880	2.05986	4.43783	9.56101	.114417
8.75	76.5625	669.922	2.95804	9.35414	2.06064	4.43952	9.56466	.114286
8.76	76.7376	672.221	2.95973	9.35949	2.06143	4.44121	9.56830	.114155
8.77	76.9129	674.526	2.96142	9.36483	2.06221	4.44290	9.57194	.114025
8.78	77.0884	676.836	2.96311	9.37017	2.06299	4.44459	9.57557	.113895
8.79	77.2641	679.151	2.96479	9.37550	2.06378	4.44627	9.57921	.113766
8.80	77.4400	681.472	2.96648	9.38083	2.06456	4.44796	9.58284	.113636
8.81	77.6161	683.798	2.96816	9.38616	2.06534	4.44964	9.58647	.113507
8.82	77.7924	686.129	2.96985	9.39149	2.06612	4.45133	9.59009	.113379
8.83	77.9689	688.465	2.97153	9.39681	2.06690	4.45301	9.59372	.113250
8.84	78.1456	690.807	2.97321	9.40213	2.06768	4.45469	9.59734	.113122
8.85	78.3225	693.154	2.97489	9.40744	2.06846	4.45637	9.60095	.112994
8.86	78.4996	695.506	2.97658	9.41276	2.06924	4.45805	9.60457	.112867
8.87	78.6769	697.864	2.97825	9.41807	2.07002	4.45972	9.60818	.112740
8.88	78.8544	700.227	2.97993	9.42338	2.07080	4.46140	9.61179	.112613
8.89	79.0321	702.595	2.98161	9.42868	2.07157	4.46307	9.61540	.112486
8.90	79.2100	704.969	2.98329	9.43398	2.07235	4.46474	9.61900	.112360
8.91	79.3881	707.348	2.98496	9.43928	2.07313	4.46642	9.62260	.112233
8.92	79.5664	709.732	2.98664	9.44458	2.07390	4.46809	9 62620	.112108
8.93	79.7449	712.122	2.98831	9.44987	2.07468	4.46976	9.62980	.111982
8.94	79.9236	714.517	2.98998	9.45516	2.07545	4.47142	9.63339	.111857
8.95	80.1025	716.917	2.99166	9.46044	2.07622	4.47309	9.63698	.111732
8.96	80.2816	719.323	2.99333	9.46573	2.07700	4.47476	9.64057	.111607
8.97	80.4609	721.734	2.99500	9.47101	2.07777	4.47642	9.64415	.111483
8.98	80.6404	724.151	2.99666	9.47629	2.07854	4.47808	9.64774	.111359
8.99	80.8201	726.573	2.99833	9.48156	2.07931	4.47974	9.65132	.111235
9.00	81.0000	729.000	3.00000	9.48683	2.08008	4.48140	9.65489	.111111

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10\,n}$	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$\frac{1}{n}$
9.01	81.1801	731.433	3.00167	9.49210	2.08085	4.48306	9.65847	.110988
9.02	81.3604	733.871	3.00333	9.49737	2.08162	4.48472	9.66204	.110865
9.03	81.5409	736.314	3.00500	9.50263	2.08239	4.48638	9.66561	.110742
9.04	81.7216	738.763	3.00666	9.50789	2.08316	4.48803	9.66918	.110620
9.05	81.9025	741.218	3.00832	9.51315	2.08393	4.48968	9.67274	.110497
9.06	82.0836	743.677	3.00998	9.51840	2.08470	4.49134	9.67630	.110375
9.07	82.2649	746.143	3.01164	9.52365	2.08546	4.49299	9.67986	.110254
9.08	82.4464	748.613	3.01330	9.52890	2.08623	4.49464	9.68342	.110132
9.09	82.6281	751.089	3.01496	9.53415	2.08699	4.49629	9.68697	.110011
9.10	82.8100	753.571	3.01662	9.53939	2.08776	4.49794	9.69052	.109890
9.11	82.9921	756.058	3.01828	9.54463	2.08852	4.49959	9.69407	.109770
9.12	83.1744	758.551	3.01993	9.54987	2.08929	4.50123	9.69762	.109649
9.13	83.3569	761.048	3.02159	9.55510	2.09005	4.50288	9.70116	.109529
9.14	83.5396	763.552	3.02324	9.56033	2.09081	4.50452	9.70470	.109409
9.15	83.7225	766.061	3.02490	9.56556	2.09158	4.50616	9.70824	.109290
9.16	83.9056	768.575	3.02655	9.57079	2.09234	4.50780	9.71177	.109170
9.17	84.0889	771.095	3.02820	9.57601	2.09310	4.50945	9.71531	.109051
9.18	84.2724	773.621	3.02985	9.58123	2.09386	4.51108	9.71884	.108933
9.19	84.4561	776.152	3.03150	9.58645	2.09462	4.51272	9.72236	.108814
9.20	84.6400	778.688	3.03315	9.59166	2.09538	4.51436	9.72589	.108696
9.21	84.8241	781.230	3.03480	9.59687	2.09614	4.51599	9.72941	.108578
9.22	85.0084	783.777	3.03645	9.60208	2.09690	4.51763	9.73293	.108460
9.23	85.1929	786.330	3.03809	9.60729	2.09765	4.51926	9.73645	.108342
9.24	85.3776	788.889	3.03974	9.61249	2.09841	4.52089	9.73996	.108225
9.25	85.5625	791.453	3.04138	9.61769	2.09917	4.52252	9.74348	.108108
9.26	85.7476	794.023	3.04302	9.62289	2.09992	4.52415	9.74699	.107991
9.27	85.9329	796.598	3.04467	9.62808	2.10068	4.52578	9.75049	.107875
9.28	86.1184	799.179	3.04631	9.63328	2.10144	4.52740	9.75400	.107759
9.29	86.3041	801.765	3.04795	9.63846	2.10219	4.52903	9.75750	.107643
9.30	86.4900	804.357	3.04959	9.64365	2.10294	4.53065	9.76100	.107527
9.31	86.6761	806.954	3.05123	9.64883	2.10370	4.53228	9.76450	.107411
9.32	86.8624	809.558	3.05287	9.65401	2.10445	4.53390	9.76799	.107296
9.33	87.0489	812.166	3.05450	9.65919	2.10520	4.53552	9.77148	.107181
9.34	87.2356	814.781	3.05614	9.66437	2.10595	4.53714	9.77497	.107066
9.35	87.4225	817.400	3.05778	9.66954	2.10671	4.53876	9.77846	.106952
9.36	87.6096	820.026	3.05941	9.67471	2.10746	4.54038	9.78195	.106838
9.37	87.7969	822.657	3.06105	9.67988	2.10821	4.54199	9.78543	.106724
9.38	87.9844	825.294	3.06268	9.68504	2.10896	4.54361	9.78891	.106610
9.39	88.1721	827.936	3.06431	9.69020	2.10971	4.54522	9.79239	.106496
9.40	88.3600	830.584	3.06594	9.69536	2.11045	4.54684	9.79586	.106383
9.41	88.5481	833.238	3.06757	9.70052	2.11120	4.54845	9.79933	.106270
9.42	88.7364	835.897	3.06920	9.70567	2.11195	4.55006	9.80280	.106157
9.43	88.9249	838.562	3.07083	9.71082	2.11270	4.55167	9.80627	.106045
9.44	89.1136	841.232	3.07246	9.71597	2.11344	4.55328	9.80974	.105932
9.45	89.3025	843.909	3.07409	9.72111	2.11419	4.55488	9.81320	.105820
9.46	89.4916	846.591	3.07571	9.72625	2.11494	4.55649	9.81666	.105708
9.47	89.6809	849.278	3.07734	9.73139	2.11568	4.55809	9.82012	.105597
9.48	89.8704	851.971	3.07896	9.73653	2.11642	4.55970	9.82357	.105485
9.49	90.0601	854.670	3.08058	9.74166	2.11717	4.56130	9.82703	.105374
9.50	90.2500	857.375	3.08221	9.74679	2.11791	4.56290	9.83048	.105263

n	n^2	n^3	$\sqrt{n}$	$\sqrt{10\,n}$	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$\frac{1}{n}$
9.51	90.4401	860.085	3.08383	9.75192	2.11865	4.56450	9.83392	.105153
9.52	90.6304	862.801	3.08545	9.75705	2.11940	4.56610	9.83737	.105042
9.53	90.8209	865.523	3.08707	9.76217	2.12014	4.56770	9.84081	.104932
9.54	91.0116	868.251	3.08869	9.76729	2.12088	4.56930	9.84425	.104822
9.55	91.2025	870.984	3.09031	9.77241	2.12162	4.57089	9.84769	.104712
9.56	91.3936	873.723	3.09192	9.77753	2.12236	4.57249	9.85113	.104603
9.57	91.5849	876.467	3.09354	9.78264	2.12310	4.57408	9.85456	.104493
9.58	91.7764	879.218	3.09516	9.78775	2.12384	4.57568	9.85799	.104384
9.59	91.9681	881.974	3.09677	9.79285	2.12458	4.57727	9.86142	.104275
9.60	92.1600	884.736	3.09839	9.79796	2.12532	4.57886	9.86485	.104167
9.61	92.3521	887.504	3.10000	9.80306	2.12605	4.58045	9.86827	.104058
9.62	92.5444	890.277	3.10161	9.80816	2.12679	4.58203	9.87169	.103950
9.63	92.7369	893.056	3.10322	9.81326	2.12753	4.58362	9.87511	.103842
9.64	92.9296	895.841	3.10483	9.81835	2.12826	4.58521	9.87853	.103734
9.65	93.1225	898.632	3.10644	9.82344	2.12900	4.58679	9.88195	.103627
9.66	93.3156	901.429	3.10805	9.82853	2.12974	4.58838	9.88536	.103520
9.67	93.5089	904.231	3.10966	9.83362	2.13047	4.58996	9.88877	.103413
9.68	93.7024	907.039	3.11127	9.83870	2.13120	4.59154	9.89217	.103306
9.69	93.8961	909.853	3.11288	9.84378	2.13194	4.59312	9.89558	.103199
9.70	94.0900	912.673	3.11448	9.84886	2.13267	4.59470	9.89898	.103093
9.71	94.2841	915.499	3.11609	9.85393	2.13340	4.59628	9.90238	.102987
9.72	94.4784	918.330	3.11769	9.85901	2.13414	4.59786	9.90578	.102881
9.73	94.6729	921.167	3.11929	9.86408	2.13487	4.59943	9.90918	.102775
9.74	94.8676	924.010	3.12090	9.86914	2.13560	4.60101	9.91257	.102669
9.75	95.0625	926.859	3.12250	9.87421	2.13633	4.60258	9.91596	.102564
9.76	95.2576	929.714	3.12410	9.87927	2.13706	4.60416	9.91935	.102459
9.77	95.4529	932.575	3.12570	9.88433	2.13779	4.60573	9.92274	.102354
9.78	95.6484	935.441	3.12730	9.88939	2.13852	4.60730	9.92612	.102250
9.79	95.8441	938.314	3.12890	9.89444	2.13925	4.60887	9.92950	.102145
9.80	96.0400	941.192	3.13050	9.89949	2.13997	4.61044	9.93288	.102041
9.81	96.2361	944.076	3.13209	9.90454	2.14070	4.61200	9.93626	.101937
9.82	96.4324	946.966	3.13369	9.90959	2.14143	4.61357	9.93964	.101833
9.83	96.6289	949.862	3.13528	9.91464	2.14216	4.61513	9.94301	.101729
9.84	96.8256	952.764	3.13688	9.91968	2.14288	4.61670	9.94638	.101626
9.85	97.0225	955.672	3.13847	9.92472	2.14361	4.61826	9.94975	.101523
9.86	97.2196	958.585	3.14006	9.92975	2.14433	4.61983	9.95311	.101420
9.87	97.4169	961.505	3.14166	9.93479	2.14506	4.62139	9.95648	.101317
9.88	97.6144	964.430	3.14325	9.93982	2.14578	4.62295	9.95984	.101215
9.89	97.8121	967.362	3.14484	9.94485	2.14651	4.62451	9.96320	.101112
9.90	98.0100	970.299	3.14643	9.94987	2.14723	4.62607	9.96655	.101010
9.91	98.2081	973.242	3.14802	9.95490	2.14795	4.62762	9.96991	.100908
9.92	98.4064	976.191	3.14960	9.95992	2.14867	4.62918	9.97326	.100807
9.93	98.6049	979.147	3.15119	9.96494	2.14940	4.63073	9.97661	.100705
9.94	98.8036	982.108	3.15278	9.96995	2.15012	4.63229	9.97996	.100604
9.95	99.0025	985.075	3.15436	9.97497	2.15084	4.63384	9.98331	.100503
9.96	99.2016	988.048	3.15595	9.97998	2.15156	4.63539	9.98665	.100402
9.97	99.4009	991.027	3.15753	9.98499	2.15228	4.63694	9.98999	.100301
9.98	99.6004	994.012	3.15911	9.98999	2.15300	4.63849	9.99333	.100200
9.99	99.8001	997.003	3.16070	9.99500	2.15372	4.64004	9.99667	.100100
10.00	100.000	1000.00	3.16228	10.0000	2.15443	4.64159	10.0000	.100000

six significant figures (adding ciphers, if necessary, to make six figures) and find between what two numbers in the column headed $\frac{1}{n}$ the significant figures of the given number fall; then proceed exactly as previously described to determine the fourth and fifth figures.

EXAMPLE.—(*a*) The reciprocal of 379.426 = ? (*b*) $\frac{1}{.0004692} = ?$

SOLUTION.—(*a*) .379426 falls between $.378788 = \frac{1}{2.64}$ and $.380228 = \frac{1}{2.63}$. The first difference is $380228 - 378788 = 1440$; the second difference is $380228 - 379426 = 802$; $802 \div 1440 = .557$, or .56. Hence, the first five significant figures are 26356, and the reciprocal of 379.426 is .0026356, to five significant figures.

(*b*) .469200 falls between $.469484 = \frac{1}{2.13}$ and $.467290 = \frac{1}{2.14}$. The first difference is 2194; the second difference is 284; $284 \div 2194 = .129+$, or .13. Hence, $\frac{1}{.0004692} = 2131.3$, correct to five significant figures.

MENSURATION

In the **following** formulas, unless otherwise stated, the letters have the meanings here given:

D = larger diameter;
d = smaller diameter;
R = radius corresponding to D;
r = radius corresponding to d;
p = perimeter of circumference;
C = area of convex surface = area of flat surface that can be rolled into the shape shown;
S = area of entire surface = C + area of the end or ends;
A = area of plane figure;
π = 3.1416, nearly = ratio of any circumference to its diameter;
V = volume of solid.

The other letters used will be found on the cuts.

CIRCLE

$$p=\pi d=3.1416d$$
$$p=2\pi r=6.2832r$$
$$p=2\sqrt{\pi A}=3.5449\sqrt{A}$$
$$p=\frac{2A}{r}=\frac{4A}{d}$$
$$d=\frac{p}{\pi}=\frac{p}{3.1416}=.3183p$$
$$d=2\sqrt{\frac{A}{\pi}}=1.1284\sqrt{A}$$
$$r=\frac{p}{2\pi}=\frac{p}{6.2832}=.1592p$$
$$r=\sqrt{\frac{A}{\pi}}=.5642\sqrt{A}$$
$$A=\frac{\pi d^2}{4}=.7854d^2$$
$$A=\pi r^2=3.1416r^2$$
$$A=\frac{pr}{2}=\frac{pd}{4}$$

TRIANGLES

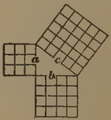
$D=B+C$ $\qquad$ $E+B+C=180°$

$B=D-C$ $\qquad$ $E'+B+C=180°$

$E'=E$ $\qquad$ $B'=B$

The above letters refer to angles.

For a right triangle, c being the hypotenuse,



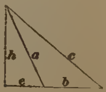
$$c=\sqrt{a^2+b^2}$$
$$a=\sqrt{c^2-b^2}$$
$$b=\sqrt{c^2-a^2}$$

c = length of side opposite an acute angle of an oblique-angled triangle.

$$c=\sqrt{a^2+b^2-2be}$$
$$h=\sqrt{a^2-e^2}$$

c = length of side opposite an obtuse angle of an oblique-angled triangle.

$$c=\sqrt{a^2+b^2+2be}$$
$$h=\sqrt{a^2-e^2}$$



For a triangle inscribed in a semicircle; i. e., any right triangle,

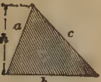
$$c : b :: a : h$$

$$h = \frac{ab}{c} = \frac{ce}{a}$$

$$a : b + e = e : a = h : c$$

$$b : h :: h : e$$

For any triangle,

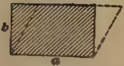


$$A = \frac{bh}{2} = \tfrac{1}{2}bh$$

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2}$$

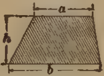
RECTANGLE AND PARALLELOGRAM

$$A = ab$$



TRAPEZOID

$$A = \tfrac{1}{2}h(a + b)$$



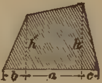
TRAPEZIUM

Divide into two triangles and a trapezoid.

$$A = \tfrac{1}{2}bh' + \tfrac{1}{2}a(h' + h) + \tfrac{1}{2}ch;$$

or, $A = \tfrac{1}{2}[bh' + ch + a(h' + h)]$

Or, divide into two triangles by drawing a diagonal. Consider the diagonal as the base of both triangles, call its length l; call the altitudes of the triangles h_1 and h_2; then



$$A = \tfrac{1}{2}l(h_1 + h_2)$$

ELLIPSE

$$p^* = \pi\sqrt{\frac{D^2 + d^2}{2} - \frac{(D - d)^2}{8.8}}$$

$$A = \frac{\pi}{4}Dd = .7854Dd$$

SECTOR

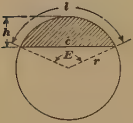
$$A = \tfrac{1}{2}lr$$

$$A = \frac{\pi r^2 E}{360} = .008727r^2E$$

l = length of arc

*The perimeter of an ellipse cannot be exactly determined without a very elaborate calculation, and this formula is merely an approximation giving fairly close results.

SEGMENT



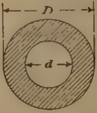
$$A=\tfrac{1}{2}[lr-c(r-h)]$$

$$A=\frac{\pi r^2E}{360}-\frac{c}{2}(r-h)$$

$$l=\frac{\pi rE}{180}=.0175rE$$

$$E=\frac{180l}{\pi r}=57.2956\frac{l}{r}$$

RING



$$A=\frac{\pi}{4}(D^2-d^2)$$

CHORD

c = length of chord

$$r=\frac{c^2+4h^2}{8h}=\frac{e^2}{2h}$$

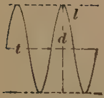
$$c=2\sqrt{2hr-h^2}$$

$$l=\frac{8e-c}{3}\text{, approximately}$$

HELIX

Tc construct a helix:

l = length of helix;
n = number of turns;
t = pitch.



$$t=\sqrt{\frac{l^2}{n^2}-\pi^2d^2}$$

$$l=n\sqrt{\pi^2d^2+t^2}$$

$$n=\frac{l}{\sqrt{\pi^2a^2+t^2}}$$

CYLINDER

$$C=\pi dh$$

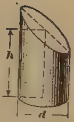
$$S=2\pi rh+2\pi r^2$$

$$=\pi dh+\frac{\pi}{2}d^2$$

$$V=\pi r^2h=\frac{\pi}{4}d^2h$$

$$V=\frac{p^2h}{4\pi}=.0796p^2h$$

FRUSTRUM OF CYLINDER



$h = \frac{1}{2}$ sum of greatest and least heights

$C = ph = \pi dh$

$S = \pi dh + \frac{\pi}{4}d^2 +$ area of elliptical top

$V = Ah = \frac{\pi}{4}d^2h$

CONE

$$C = \frac{1}{2}\pi dl = \pi rl$$

$$S = \pi rl + \pi r^2 = \pi r\sqrt{r^2 + h^2} + \pi r^2$$

$$V = \frac{\pi d^2}{4} \times \frac{h}{3} = \frac{.7854d^2h}{3} = \frac{p^2h}{12\pi}$$

FRUSTUM OF CONE

$$C = \frac{1}{2}l(P + p) = \frac{\pi}{2}l(D + d)$$

$$S = \frac{\pi}{2}[l(D + d) + \frac{1}{2}(D^2 + d^2)]$$

$$V = \frac{\pi}{4}(D^2 + Dd + d^2) \times \frac{1}{3}h$$

$$= .2618h(D^2 + Dd + d^2)$$

SPHERE

$$S = \pi d^2 = 4\pi r^2 = 12.5664r^2$$

$$V = \frac{1}{6}d^3 = \frac{4}{3}\pi r^3 = .5236d^3 = 4.1888r^3$$

CIRCULAR RING

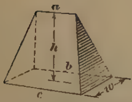
$D =$ mean diameter;

$R =$ mean radius.

$S = 4\pi^2Rr = 9.8696Dd$

$V = 2\pi^2Rr^2 = 2.4674Dd^2$

WEDGE



$$V = \frac{1}{6}wh(a + b + c)$$

PRISMOID

A prismoid is a solid having two parallel plane ends, the edges of which are connected by plane triangular or quadrilateral surfaces.

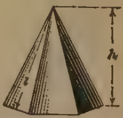
A = area of one end;
a = area of other end;
m = area of section midway between ends;
l = perpendicular distance between ends.

$$V=\tfrac{1}{6}l(A+a+4m)$$

The area m is not in general a mean between the areas of the two ends, but its sides are means between the corresponding lengths of the ends.

Approximately, $$V=\frac{A+a}{2}l$$

REGULAR PYRAMID



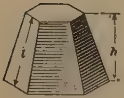
P = perimeter of base;
A = area of base.

$$C=\tfrac{1}{2}Pl$$
$$S=\tfrac{1}{2}Pl+A$$
$$V=\frac{Ah}{3}$$

To obtain area of base, divide it into triangles, and find their sum.

The formula for V applies to any pyramid whose base is A and altitude h.

FRUSTUM OF REGULAR PYRAMID



a = area of upper base;
A = area of lower base;
p = perimeter of upper base;
P = perimeter of lower base.

$$C=\tfrac{1}{2}l(P+p)$$
$$S=\tfrac{1}{2}l(P+p)+A+a$$
$$V=\tfrac{1}{3}h\left(A+a+\sqrt{Aa}\right)$$

The formula for V applies to the frustum of any pyramid.

LENGTH OF SPIRAL

$$l=\pi n\left(\frac{D+d}{2}\right)$$
$$l=\frac{\pi}{t}(R^2-r^2)$$

n = number of coils;
l = length of spiral;
t = pitch.

PRISM OR PARALLELOPIPED

$$C = ph$$
$$S = ph + 2A$$
$$V = Ah$$

For prisms with regular polygon as bases, p = length of one side × number of sides.

To obtain area of base, if it is a polygon, divide it into triangles, and find sum of partial areas.

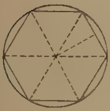
FRUSTUM OF PRISM

If a section perpendicular to the edges is a triangle, square, parallelogram, or *regular* polygon,

$$V = \frac{\text{sum of lengths of edges}}{\text{number of edges}} \times \text{area of right section.}$$

REGULAR POLYGONS

Divide the polygon into equal triangles and find the sum of the partial areas. Otherwise, square the length of one side and multiply by proper number from the following table:



Name	*No. Sides*	*Multiplier*
Triangle	3	.433
Square	4	1.000
Pentagon	5	1.720
Hexagon	6	2.598
Heptagon	7	3.634
Octagon	8	4.828
Nonagon	9	6.182
Decagon	10	7.694

IRREGULAR AREAS

Divide the area into trapezoids, triangles, parts of circles, etc., and find the sum of the partial areas.

If the figure is very irregular, the approximate area may be found as follows: Divide the figure into trapezoids by equidistant parallel lines b, c, d, etc. The lengths of these lines being measured, then, calling a the first and n the last length, and y the width of strips,

$$\text{area} = y\left(\frac{a+n}{2} + b + c + \text{etc.} + m\right)$$

LOADS IN STRUCTURES

The loads that a structure has to carry may be divided into five classes namely, the dead load, the live load, the accidental load, the snow load, and the wind load.

The *dead load* is the weight of the materials of which the structure is composed, as the weight of the beams, columns, walls, floors, and the like.

The *live load* is the weight of the various articles in the building that are not part of the structure itself, such as furniture, material stored, if the building is a storehouse, the occupants of the building, light machinery, etc.

The *accidental load* is one whose application is doubtful or whose magnitude is great and its effect local. As examples of such loads may be classed the load that would be generated if the ropes of an elevator broke and the safety device was suddenly brought into action, and the load that would be created on a stairway in a mill if there was a panic among the employes, as during a fire. As an accidental load may also be classed such loads as would be caused by an especially heavy office safe or piece of machinery. Many engineers no longer consider an accidental load. The load that would come from an extra-large crowd, as on a stairway in a fire, they class as part of the live load. Such loads as would occur from a large safe, they usually class as part of the dead load, because the safe would be too heavy to be moved. Therefore, only that part of the structure in the vicinity of the safe need be strengthened.

The *snow* and *wind loads* are, as the names imply, loads that are caused respectively by snow and wind.

DEAD LOADS

WEIGHT OF ROOF TRUSSES

The dead load must be calculated for each member of a structure by means of tables that give the weights of building materials, such as those on pages 59–61. However, it

is often desirable to know the approximate weight of roof trusses before the actual size of the members has been calculated. In such cases, the following empirical formula may be used:

$$W = aDL\left(1+\frac{L}{10}\right),$$

in which W is the approximate weight of the truss, in pounds; a, a constant—for wood .5, for steel .75; D, the distance, in feet, from center to center of trusses; and L, the span of the truss, in feet.

It is sometimes desirable to know the approximate weight of roof truss per square foot of roof surface. It will be noted that this is not the same thing as per square foot of horizontal surface, since the roof slants. If x is the angle between the rafter and the chord, L the span of the truss in feet, and w the approximate weight of the truss per square foot of roof surface, then

$$w = \frac{a(10+L)\cos x}{10}$$

in which a is .5 for wooden trusses and .75 for steel trusses.

EXAMPLE.—Determine the weight, per square foot, that must be added to the weight of a roof covering to provide for the weight of the principals, the steel trusses in this case having a span of 72 ft. and a rise of 18 ft.

SOLUTION.—The angle between the rafter and the chord may be found from the formula $\tan x = \frac{\text{rise}}{\frac{\text{span}}{2}} = \frac{18}{36} = .5$, corresponding to an angle x of 26° 34′. Substituting the values of a, L, and $\cos x$ in the preceding formula,

$$w = \frac{.75\ (10+72)\cos 26° 34'}{10} = \frac{.75 \times 82 \times .8944}{10} = 5.5 \text{ lb.}$$

The following table has been calculated from the preceding formula. This table gives the weight that must be added to a square foot of roof covering to provide for the weight of the principals, or trusses. The slope of the roof is in each case given in terms of its "pitch," which is here taken as the ratio of rise to span. Thus, a $\frac{1}{3}$ pitch is such a slope that the rise of the truss is one-third the span of the truss.

APPROXIMATE WEIGHT OF ROOF TRUSSES

$$w = \frac{a(10+L)\cos x}{10}$$

Character of Truss	Span Feet	Pounds per Square Foot of Roof Surface: 1 Pitch	$\frac{3}{4}$ Pitch	$\frac{2}{3}$ Pitch	$\frac{1}{2}$ Pitch	$\frac{1}{3}$ Pitch	$\frac{1}{4}$ Pitch	$\frac{1}{6}$ Pitch
Wood	30	.8944	1.109	1.200	1.414	1.66	1.79	1.90
	35	1.0062	1.248	1.350	1.591	1.87	2.01	2.13
	40	1.1180	1.387	1.500	1.768	2.08	2.24	2.37
	45	1.2298	1.525	1.650	1.945	2.29	2.46	2.61
	50	1.3416	1.664	1.800	2.121	2.50	2.68	2.85
	55	1.4534	1.802	1.950	2.298	2.70	2.91	3.08
	60	1.5652	1.941	2.100	2.475	2.91	3.13	3.32
	65	1.6770	2.080	2.250	2.652	3.12	3.35	3.56
	70	1.7888	2.218	2.400	2.828	3.33	3.58	3.79
	75	1.9006	2.357	2.550	3.005	3.54	3.80	4.03
	80	2.0124	2.496	2.700	3.182	3.74	4.02	4.27
Iron or steel	30	1.3416	1.664	1.800	2.121	2.50	2.68	2.85
	35	1.5093	1.872	2.025	2.386	2.81	3.02	3.20
	40	1.6770	2.080	2.250	2.652	3.12	3.35	3.56
	45	1.8447	2.288	2.475	2.917	3.43	3.69	3.91
	50	2.0124	2.496	2.700	3.182	3.74	4.02	4.27
	55	2.1801	2.704	2.925	3.447	4.06	4.36	4.62
	60	2.3478	2.912	3.150	3.712	4.37	4.70	4.98
	65	2.5155	3.120	3.375	3.977	4.68	5.03	5.34
	70	2.6832	3.328	3.600	4.243	4.99	5.37	5.69
	75	2.8509	3.536	3.825	4.508	5.30	5.70	6.05
	80	3.0186	3.744	4.050	4.773	5.62	6.04	6.40

WEIGHT OF BUILDING MATERIALS PER CUBIC FOOT

Name of Material	Average Weight	
	Pounds per Cubic Inch	Pounds per Cubic Foot
Asphalt-pavement composition		130
Bluestone		160
Brick, best pressed		150
Brick, common and hard		125
Brick, paving		150
Brick, soft, inferior		100
Brickwork, in lime mortar (average)		120
Brickwork, in cement mortar (average)		130
Brickwork, pressed brick, thin joints		140
Cement, Portland, packed		100 to 120
Cement, natural, packed		75 to 95
Concrete, cinder		105
Concrete, gravel		140
Concrete, slag		135
Concrete, stone		140
Concrete, reinforced (average)		150
Earth, dry and loose		72 to 80
Earth, dry and moderately rammed		90 to 100
Firebrick		150
Granite		165 to 170
Gravel		117 to 125
Iron, cast	.260	450
Iron, wrought	.277	480
Limestone		146 to 168
Marble		168
Masonry, squared granite or limestone		165
Masonry, granite or limestone rubble		150
Masonry, granite or limestone dry rubble		138
Masonry, sandstone		145
Mineral wool		12
Mortar, hardened		90 to 100
Quicklime, ground, loose, or small lumps		53
Quicklime, ground, thoroughly shaken		75
Sand, pure quartz, dry		90 to 106

WEIGHT OF BUILDING MATERIALS PER CUBIC FOOT
(Continued)

Name of Material	Average Weight: Pounds Per Cubic Inch	Average Weight: Pounds Per Cubic Foot
Sandstone, building, dry		139 to 151
Slate		160 to 180
Snow, fresh fallen		5 to 12
Steel, structural	.283	489.6
Terra cotta		110
Terra-cotta masonry work		112
Tile		110 to 120

WEIGHT OF BUILDING MATERIALS PER SQUARE FOOT

Name of Material		Average Weight Pounds per Square Foot
Corrugated ($2\frac{1}{2}$-in.) galvanized iron	No. 16	2.91
	No. 18	2.36
	No. 20	1.82
	No. 22	1.54
	No. 24	1.27
	No. 26	.99
	No. 27	.93
	No. 28	.86
Corrugated galvanized iron, No. 20, average amount of side lap, unboarded		$2\frac{1}{4}$
Copper roofing, 16-oz., standing seam		$1\frac{1}{4}$
Felt and pitch, without sheathing		3
Glass, $\frac{1}{8}$ in. thick		$1\frac{3}{4}$
Hemlock sheathing, 1 in. thick		2
Lead, about $\frac{1}{8}$ in. thick		6 to 8
Lath-and-plaster ceiling (ordinary)		6 to 8
Mackite, 1 in. thick, with plaster		10

WEIGHT OF BUILDING MATERIALS PER SQUARE FOOT
(*Continued*)

Name of Material	Average Weight Pounds Per Square Foot
Neponset roofing felt, two layers	½
Spruce sheathing, 1 in. thick	2
Slate, single thickness, ⅛ in. thick	1.81
Slate, single thickness, $\frac{3}{16}$ in. thick	2.71
Slate, single thickness, ¼ in. thick	3.62
Slate, single thickness, ⅜ in. thick	5.43
Slate, single thickness, ½ in. thick	7.25
Slate, single thickness, ⅝ in. thick	9.06
Slate, single thickness, ¾ in. thick	10.87
Shingles, common, 6 in. × 18 in., 5 in. to weather	2
Skylight of glass, $\frac{3}{16}$ to ½ in., including frame	4 to 10
Slag roof, four-ply	4
Steel roofing, standing seam	1
Tiles, Spanish, 14½ in. × 10½ in., 7¼ in. to weather	8½
Tiles, plain, 10½ in. × 6¼ in. × ⅝ in., 5¼ in. to weather	18
White-pine sheathing, 1 in. thick	2
Yellow-pine sheathing, 1 in. thick	3
Gravel roof and four-ply felt	5½
Gravel roof and five-ply felt	6
Roofing, three-ply ready (asphalt, rubberoid, etc.)	6 to 10
Purlins, wooden, with 12- to 16-ft. span	2
Chestnut or maple sheathing, 1 in. thick	4
Ash, hickory, or oak sheathing, 1 in. thick	5
Sheet iron, $\frac{1}{16}$ in. thick	3
Thatch	6.5

LIVE LOADS

If the live load consists of heavy material, such as merchandise in a warehouse, and if the amount of this material and its location are known, the live load is usually calculated from the table of weights of materials given on pages 64–81. If, however, the structure is to be used as a dwelling, a hotel,

or a warehouse for general merchandise the character of which is yet unknown, it is customary to assume the live load to be a certain number of pounds per square foot of floor space and to design the structure for this load. The following table gives the loads per square foot of floor space often employed:

LIVE LOADS PER SQUARE FOOT IN BUILDINGS

Character of Building	Pounds
Dwellings	70
Offices	70
Hotels and apartment houses	70
Theaters	120
Churches	120
Ballrooms and drill halls	120
Factories	from 150 up
Warehouses	from 150 to 250 up

The load of 70 lb. will probably never be realized in dwellings; but inasmuch as a city house may at times be used for some purpose other than that of a dwelling, it is not generally advisable to use a lighter load. In a country house, a hotel, or a building of like character, a live load of 40 lb. per sq. ft. of floor surface is ample for all rooms not used for public assembly. For assembly rooms, a live load of 100 lb. will be sufficient. If the desks and chairs are fixed, as in a schoolroom or a church, a live load of more than from 40 to 50 lb. will never be attained. Retail stores should have floors proportioned for a live load of 100 lb. and upwards. Wholesale stores, machine shops, etc. should have the floors proportioned for a live load of not less than 150 lb. per sq. ft. The floors of printing houses and binderies should be proportioned for a live load of at least 250 lb. per sq. ft. Special provision should be made in such floor systems for heavy presses, trimmers, and cutters, and the beams should be proportioned for twice the static load likely to occur from such machines. The static load in factories seldom exceeds from 40 to 50 lb. per sq. ft. of floor

surface; therefore, in the majority of cases, a live load of 100 lb. is ample. The conservative rule is, in general, to assume loads not less than those just given, and to proportion the beams so as to avoid excessive deflection. Stiffness is as important a factor as strength.

ALLOWABLE LIVE LOADS ON FLOORS IN DIFFERENT CITIES

Character of Building	Pounds per Square Foot			
	New York	Chicago	Philadelphia	Boston
Buildings for public assembly	90	100	120	150
Buildings for ordinary stores, light manufacturing, and light storage	120	100	120	
Dwellings, apartment houses, tenement houses, and lodging houses	60	40	70	50
Office buildings, first floor	150	100	100	100
Office buildings, above first floor	75	100	100	100
Public buildings, except schools				150
Roofs, pitch less than 20°	50	25	30	25*
Roofs, pitch more than 20°	30	25	30	25*
Schools or places of instruction	75			80
Stables or carriage houses less than 500 sq. ft. in area	75	40		
Stables or carriage houses more than 500 sq. ft. in area	75	100		
Stores for heavy materials, warehouses, and factories	150		150	250
Sidewalks	300			

NOTE.—In the table the values given for roofs are for snow and wind loads. In the last column, the roof loads marked with the asterisk (*) do not include the wind load; the building laws of Boston require that a proper allowance for the wind load exerting a pressure of 30 lb. per sq. ft. of vertical surface shall be made in designing roofs.

In proportioning the live loads on floors, the engineer cannot always exercise his own judgment, for if the building is to be erected in a large city, the live load must comply with the building laws. As such laws are not uniform in the several cities the table on page 63 is given to show the stipulated live loads in the four largest cities in the United States.

WEIGHTS OF MERCHANDISE, IN BULK, FOR CALCULATING LIVE LOADS

Name of Material	Measurements		Approximate Weights		
	Floor Area, Square Feet	Contents, Cubic Feet	Total Pounds	Pounds per Square Foot	Pounds per Cubic Foot
Cotton, etc.:					
Bale of commercial cotton	8.10	44.20	515	64	12
Bale of compressed cotton	4.10	21.60	550	134	25
Bale of American Cotton Co.	4.00	11.00	263	66	24
Bale of Planters Compress Co.	2.30	7.20	254	110	35
Bale of jute	2.40	9.90	300	125	30
Bale of jute lashings	2.60	10.50	450	172	43
Bale of manila	3.20	10.90	280	88	26
Bale of hemp	8.70	34.70	700	81	20
Bale of sisal	5.30	17.00	400	75	24
Cotton Goods:					
Bale of unbleached jeans	4.00	12.50	300	75	24
Piece of duck	1.10	2.30	75	68	33
Bale of brown sheetings	3.60	10.10	235	65	23
Case of bleached sheetings	4.80	11.40	330	69	29
Case of quilts	7.20	19.00	295	41	16
Bale of print cloths	4.00	9.30	175	44	19
Case of prints	4.50	13.40	420	93	31
Bale of tickings	3.30	8.80	325	99	37
Burlaps			130		30
Jute bagging	1.40	5.30	100	71	19
Grain:					
Wheat in bags	4.20	4.20	165	39	39
Flour in barrels on side	4.10	5.40	218	53	40

WEIGHTS OF MERCHANDISE, IN BULK, FOR CALCULATING LIVE LOADS—(*Continued*)

Name of Material	Measurements: Floor Area, Square Feet	Measurements: Contents, Cubic Feet	Approximate Weights: Total Pounds	Approximate Weights: Pounds per Square Foot	Approximate Weights: Pounds per Cubic Foot
Grain—Continued					
Flour in barrels on end...	3.10	7.10	218	70	31
Corn in bags.............	3.60	3.60	112	31	31
Corn meal in barrels.....	3.70	5.90	218	59	37
Oats in bags.............	3.30	3.60	96	29	27
Bale of clover hay........	5.00	20.00	284	57	14
Bale of clover hay, derrick compressed	1.75	5.25	125	71	24
Bale of straw	1.75	5.25	100	57	19
Bale of tow	1.75	5.25	150	86	29
Bale of excelsior	1.75	5.25	100	57	19
Rags in Bales:					
White linen.............	8.50	39.50	910	107	23
White cotton............	9.20	40.00	715	78	18
Brown cotton............	7.60	30.00	442	58	15
Paper shavings..........	7.50	34.00	507	68	15
Sacking.................	16.00	65.00	450	28	7
Woolen..................	7.50	30.00	600	80	20
Jute butts...............	2.80	11.00	400	143	36
Wool:					
Bale, East India.........	3.00	12.00	340	113	28
Bale, Australian.........	5.80	22.00	385	66	15
Bale, South American....	7.00	34.00	1,000	143	29
Bale, Oregon............	6.90	33 00	482	70	15
Bale, California..........	7.5C	33.00	550	73	17
Bag of wool..............	5.00	30.00	200	40	7
Sack of scoured wool.....					5
Woolen Goods:					
Case of flannels..........	5.50	12.70	220	40	17
Case of flannels, heavy ...	7.10	15.20	330	46	22
Case of dress goods.......	5.50	22.00	460	84	21
Case of cassimeres........	10.50	28.00	550	52	20
Case of underwear........	7.30	21.00	350	48	17
Case of blankets.........	10.30	35.00	450	44	13
Case of horse blankets....	4.00	14.00	250	63	18

WEIGHT OF MERCHANDISE, IN BULK, FOR CALCULATING LIVE LOADS—(*Continued*)

Name of Material	Measurements		Approximate Weights		
	Floor Area, Square Feet	Contents, Cubic Feet	Total Pounds	Pounds per Square Foot	Pounds per Cubic Foot
Miscellaneous:					
Box of tin	2.7	.5	139	51	278
Crate of crockery	9.9	39.6	1,600	162	40
Casket of crockery	13.4	42.5	600	45	14
Bale of leather	7.3	12.2	190	26	16
Bale of goat skins	11.2	16.7	300	27	18
Bale of raw hides	6.0	30.0	400	67	13
Bale of raw hides, compressed	6.0	30.0	700	117	23
Bale of sole leather	12.6	8.9	200	16	22
Barrel of granulated sugar	3.0	7.5	317	106	42
Barrel of brown sugar	3.0	7.5	339	113	45
Hogshead of bleaching powder	11.8	39.2	1,200	102	31
Hogshead of soda ash	10.8	29.2	1,800	167	62
Box of indigo	3.0	9.0	385	128	43
Box of sumac	1.6	4.1	160	100	39
Caustic soda in iron drum	4.3	6.8	600	140	88
Barrel of starch	3.0	10.5	250	83	24
Barrel of pearl alum	3.0	10.5	350	117	33
Box of extract logwood	1.1	.8	57	52	71
Barrel of lime	3.6	4.5	225	63	50
Barrel of Portland cement		3.8	376		100 to 120
Barrel of natural cement		3.8	282		75 to 95
Barrel of slag cement		3.8	330		80 to 100
Barrel of English Portland cement	3.8	5.5	400	105	73
Barrel of plaster	3.7	6.1	325	88	53
Barrel of rosin	3.0	9.0	430	143	48
Barrel of lard oil	4.3	12.3	422	98	34
Books in library					30
Crowd of men					134 to 157

WEIGHTS OF VARIOUS METALS

Name of Metal	Average Weight per Cubic Foot Pounds	Name of Metal	Average Weight per Cubic Foot Pounds
Aluminum	167	Lead, commercial cast	712
Antimony	418	Magnesium	109
Bismuth	613	Manganese	499
Brass, cast	504	Mercury, at 60° F.	846
Brass, rolled	524	Molybdenum	538
Bronze	546	Nickel	548
Chromium	456	Platinum, hammered	1,270
Cobalt	560	Platinum, rolled	1,415
Copper, cast	552	Platinum, wire	1,313
Copper, rolled	555	Silver, hammered	657
Gold, cast, 24 carat	1,204	Silver, pure	653
Gold, pure hammered	1,217	Steel	490
Iridium	1,400	Tin	458
Iron, cast	450	Tungsten	1,232
Iron, wrought	480	Zinc	437

AVERAGE WEIGHTS OF MISCELLANEOUS MATERIALS

Name of Material	Average Weight per Cubic Foot Pounds	Name of Material	Average Weight per Cubic Foot Pounds
Acid, acetic	66	Cement, natural, packed	75 to 95
Acid, fluoric	94	Cement, natural, loose	45 to 65
Acid, muriatic (hydrochloric)	75	Cement, slag, packed	80 to 100
Acid, nitric	76	Cement, slag, loose	55 to 75
Acid, phosphoric	97	Chalk	156
Acid, sulphuric	115	Champagne	62
Alabaster, white	171	Charcoal from birch	34
Alabaster, yellow	169	Charcoal from fir	28
Alcohol, commercial	52	Charcoal from oak	21
Alcohol, grain	49.6	Charcoal from pine	18
Alcohol, wood	49.9	Chrome ore dust, well shaken	160
Ammonia, 28 per cent	56	Clay, ordinary	120 to 150
Antimony	418	Clay, potters', dry	119
Asbestos, starry	192	Clinker	85
Ashes	40	Coal, anthracite, broken	54
Asphalt, pure	80	Coal, anthracite, moderately shaken	58
Basalt	181	Coal, anthracite, solid	93
Beer, lager	65	Coal, bituminous, broken, loose	50
Borax	107	Coal bituminous, slaked	53
Cement, Portland, packed	100 to 120	Coal, bituminous, solid	84
Cement, Portland, loose	70 to 90	Coal, cannel, solid	79

Coke, loose	23 to 32	Hornblende	203
Coral, red	169	Ice	57
Cork	15	India rubber	58
Corundum	244	Isinglass	70
Cotton yarn, in skeins	11	Ivory	114
Crowd of men	134 to 157	Leather, sole, in piles	17
Earth, common loam, loose	72 to 80	Magnesia, carbonate	150
Earth, common loam, shaken	82 to 92	Magnesite, calcined	110
Earth, common loam, rammed moderately	90 to 100	Mastic	67
		Mica	183
Earth, like soft, flowing mud	108	Millstone	155
Earth, like dense mud	125	Naphtha	53
Emery	250	Niter	119
Ether, sulphuric	45	Oil, linseed	59
Feldspar	166	Oil, olive	57
Flint	162	Oil, turpentine	54
Glass, common	156 to 172	Oil, whale	58
Glass, flint	180 to 196	Ore, hard iron (magnetite)	312
Gneiss, common	168	Ore, soft iron (hematite)	306
Gneiss, in loose piles	96	Paper, calendered, book	50
Grindstone	134	Paper, leather-board	59
Gum arabic	91	Paper manila	37
Gun metal	528	Paper, news	38
Gunpowder, loose	56	Paper, strawboard	33
Gunpowder, shaken	63	Paper, supercalendered, book	69
Gunpowder, solid	105	Paper wrapping	10
Gutta percha	61	Paper, writing	64
Gypsum	143	Paving stone	150
Hematite ore	306	Pearl, oriental	165

AVERAGE WEIGHTS OF MISCELLANEOUS MATERIALS—(*Continued*)

Name of Material	Average Weight per Cubic Foot Pounds	Name of Material	Average Weight per Cubic Foot Pounds
Peat, dry, compressed	20 to 30	Shales	162
Petroleum	55	Slate	174
Phosphorus	110	Soil, common	124
Pitch	72	Soapstone	170
Plaster of Paris, cast	80	Spelter, or zinc	437
Plumbago	140	Spermaceti	59
Porphyry	170	Sugar	100
Pumice stone	57	Sulphur	125
Quartz, common, pure	165	Talc, block	181
Rosin	69	Tallow, sheep or ox	58
Rope, Manila	42	Tar	63
Rottenstone	124	Trap	170
Saltpeter	131	Turf, or peat	20 to 30
Salt, coarse	45	Vinegar	68
Salt, West India, well-dried	74	Whalebone	81
Sand	90 to 106	Wines	62

AVERAGE WEIGHTS OF FARM PRODUCTS

Name of Material	Weight per Cubic Foot Pounds	Name of Material	Weight per Cubic Foot Pounds
Apples	38	Cheese	30
Apples, dried	20	Cherries	40
Apple seed	32	Chestnuts	43
Barley	38	Chufa	43
Beans, white	48	Cider	64
Beans, castor, shelled	37	Clover seed	48
Beeswax	61	Corn on the cob, husked	56
Beets	44	Corn on the cob, unhusked	58
Beggarweed seed	50	Corn, shelled	45
Blackberries	32	Corn meal, bolted	37
Blueberries	34	Corn meal, unbolted	38
Blue-grass seed	11	Cottonseed	25
Bran	16	Cranberries	29
Brome grass	11	Currants	32
Broom-corn seed	30	Eggs	68
Buckwheat	39	Fat of beef	58
Butter	59	Fat of hogs	59
Cabbage	40	Fat of mutton	58
Canary seed	48	Flaxseed (linseed)	45
Cantaloupe, melon	40	Gooseberries	34
Carrots	40	Grapes with stems	38

AVERAGE WEIGHTS OF FARM PRODUCTS—(*Continued*)

Name of Material	Weight per Cubic Foot Pounds	Name of Material	Weight per Cubic Foot Pounds
Guavas	43	Lime	64
Hay, alfalfa, in bales	12.5 to 14.3	Malt	27
Hay, alfalfa, in rectangular double-compressed bales	25.53	Meal	37
Hay, alfalfa, in cylindrical double-compressed bales	36.36	Middlings, coarse	38
Hay, clover, in bales	14	Middlings, fine	32
Hay, clover, compressed	24	Milk	65
Hay, clover, in mow	4.6	Millet	40
Hair, plastering	6	Millet, Japanese barnyard	28
Hemp seed	35	Mustard	24
Hickory nuts	40	Oats	26
Hominy	49	Onions	45
Honey	91	Orchard-grass seed	11
Horseradish	40	Osage-orange seed	26
Hungarian grass seed	39	Parsnips	38
Indian corn, or maize	45	Peaches	40
Italian rye-grass seed	16	Peaches, dried and peeled	26
Johnson grass	22	Peanuts	18
Kaffir corn	45	Pears	39
Kale	24	Peas	48
Land plaster	80	Plums	42
Lard	59	Plums, dried	22
		Popcorn	56
		Popcorn, on the cob	34

Potatoes, white	48	Silage (at top)	19
Potatoes, sweet	41	Silage (at 36 ft.)	61
Prunes, dried	22	Sorghum seed	37
Prunes, green	36	Spelt, or speltz	34
Quinces	38	Spinach	24
Rape seed	40	Straw	19
Raspberries	32	Strawberries	32
Redtop	11	Sugar-cane seed	46
Rhubarb	40	Tares	49
Rice corn	45	Timothy seed	36
Rice, rough	35	Tomatoes	44
Rutabagas	45	Turnips	44
Rye	45	Velvet-grass seed	6
Rye meal	40	Walnuts	40
Sage	3	Wheat	48

WEIGHT OF WOODS, DRY

Name of Tree	Average Weight per Cubic Foot Pounds	Name of Tree	Average Weight per Cubic Foot Pounds
Alder	42	Birch, yellow	40
Apple	47	Blue beech (ironwood)	45
Arbor vitæ	19	Blue gum (fever tree)	43 to 69
Ash, black	39	Box elder, or ash-leaved maple	26
Ash, blue	44	Boxwood, Brazilian, red	64
Ash, green	39	Boxwood, Dutch	83
Ash, Oregon	35	Boxwood, French	57
Ash, red	38	Buckeye, Ohio	28
Ash, white	39	Buckeye, sweet	27
Aspen	27	Bullet tree	65
Bamboo	22	Butternut	25
Basswood	28	Buttonwood, or sycamore	35
Bay tree	51	Cabacalli	56
Beech	42	Catalpa, or Indian bean	27
Bethabara	76	Catalpa, hardy	25
Birch, paper, or white	37	Cedar California white	25
Birch, red	35	Cedar, canoe	23
Birch, sweet	47	Cedar, incense	25

NOTE.—The weight of wood depends largely on the amount of moisture it contains. The weights in this and the following tables are for very dry wood and not for green wood. See separate tables for Philippine and Australian woods on pages **79–81**. The percentage of moisture contained in the wood must be added to the values given in these tables.

Cedar, Indian	82	Fir, balsam	23
Cedar, juniper	35	Fir, great silver	22
Cedar, Palestine	38	Fir, red, or California	29
Cedar, Port Oxford	28	Fir, red, or noble	28
Cedar, red	30	Fir, white	22
Cedar, white, or post	23	Greenheart	72
Cedar, white (arbor vitæ)	19	Gum, cotton	32
Cedar, wild	37	Gum, sour	39
Cedar, yellow	29	Gum, sweet	37
Cherry, wild black	36	Hackmatack (American larch)	38
Chestnut	28	Hawthorn	57
Chinkapin	36	Hazel	38
Citron	45	Hemlock	26
Cocoa wood	65	Hemlock, Western	28
Cocobolo	55	Hickory, mocker nut	53
Cottonwood	24	Hickory, pecan	49
Cottonwood, black	23	Hickory, pignut	56
Cucumber tree	29	Hickory, shagbark, or shellbark	51
Cypress, bald	29	Holly	36
Cypress, Spanish	40	Hornbeam	47
Dagame	56	Ironwood, or blue beech	45
Dogwood	50	Ironwood, or hop hornbeam	51
Ebony	76	Joshua tree	23
Elder tree	43	Jasmine, Spanish	48
Elm, cork	45	Jucaro Prieto	67
Elm, slippery	43	Juneberry	54
Elm, white	34	Karri	63
Elm, wing	46	Kauri	37
Filbert tree	38	Kranji	64

WEIGHT OF WOODS, DRY—(*Continued*)

Name of Tree	Average Weight per Cubic Foot Pounds	Name of Tree	Average Weight per Cubic Foot Pounds
Laburnum	57	Maple, sugar, or hard	43
Lancewood	53	Mastic tree	53
Larch	38	Medlar	59
Larch, tamarack	46	Mesquit	47
Laurel, California	40	Missel tree	59
Laurel, Madroña	43	Mora	57
Lemon	45	Mulberry, red or black	36
Lignum vitæ	83	Oak, black	45
Linden	38	Oak, bur	46
Locust, black, or yellow	45	Oak, chestnut	46
Locust, honey	42	Oak, cow	46
Logwood	58	Oak, English	51
Madroña	43	Oak, live, California	51
Mahoe	41	Oak, live (found in the Southern States)	59
Mahogany	45	Oak, pin	43
Mahogany, Mexican	32	Oak, post	50
Mahogany, Spanish	53	Oak, red	45
Mahogany, white	33	Oak, Spanish	43
Maple, Oregon	30	Oak, white (North-Central and Eastern United States)	50
Maple, red	38		
Maple, silver, or soft	32		

Oak, white (Pacific Coast from British Columbia into California)	46	Poon	36
Orange, Osage	48	Poplar, or large-tooth aspen	28
Orange tree	44	Poplar, yellow, or tulip tree	26
Paddlewood	52	Quebracho	82
Palm, Washington	32	Quince tree	44
Palmetto, cabbage	27	Redwood	26
Pear	41	Roller wood	52
Persimmon	49	Rosewood	68
Pine, bull	29	Sal	60
Pine, Cuban	39	Sassafras	31
Pine, Kauri	60	Shadblow	54
Pine, loblolly	33	Shadbush	54
Pine, long-leaf, or Georgia	38	Spruce, black	28
Pine, northern	34	Spruce, Douglas	32
Pine, Norway	31	Spruce, Norway	29
Pine, Oregon	32	Spruce, single (balsam fir)	23
Pine, pitch	32	Spruce, Sitka	26
Pine, short-leaf, or Carolina	32	Spruce, white (Northern United States)	25
Pine, sugar	22	Spruce, white (Rocky Mountains and British Columbia)	21
Pine, white (North-Central and Northeastern States)	24	Sycamore, or buttonwood	35
Pine, white (Pacific States and British Columbia)	24	Sycamore, California	30
Pine, white (Rocky Mountains)	27	Tamarack	38
Pingow	47	Teak	50
Plum tree	49	Tonka	64
Pockwood	81	Tooart	67
Pomegranate tree	85	Tulip tree	26
		Tulip wood	61

WEIGHT OF WOODS, DRY—(*Continued*)

Name of Tree	Average Weight per Cubic Foot Pounds	Name of Tree	Average Weight per Cubic Foot Pounds
Vine tree	83	Wasahba	76
Walnut, black	38	Whitewood	26
Walnut, Circassian	35	Willow, black	27
Walnut, English	36	Yarura	52
Walnut, Italian	42	Yew, Dutch	49
Walnut, Persian	36	Yew, Spanish	50
Walnut, white	25	Yucca, or joshua tree	23

WEIGHT OF PHILIPPINE WOODS, DRY

Name of Tree	Average Weight per Cubic Foot Pounds	Name of Tree	Average Weight per Cubic Foot Pounds
Acle	37	Liusin	44
Amuguis	43	Lumbayao	35
Apitong	41	Macaasin	44
Aranga	54	Malasantol	40
Balacat	33	Malugay	40
Balacbacan	34	Mayapis	25
Bansalaguin	53	Molave	49
Banuyo	33	Narra	36
Batitinan	49	Palo Maria	39
Betis	49	Sacat	37
Calantas	27	Sasalit	55
Dungon	49	Supa	45
Guijo	43	Tanguile	30
Ipil	47	Tindalo	48
Lauan	29	Yacal	52

WEIGHT OF AUSTRALIAN WOODS, DRY

Name of Tree	Average Weight per Cubic Foot Pounds
Acacia dealbata (silver wattle)	57
Acacia decurrens (common wattle)	47
Acacia implexa	44
Acacia melanoxylon (blackwood; lightwood)	47
Acacia mollissima (silver wattle)	50
Acacia pycnantha (golden wattle)	52
Acacia salicina	48
Araucaria cunninghamii (pine)	45
Aster argophyllus (musk tree)	40
Banksia integrifolia (coast honeysuckle tree)	50
Banksia marginata (common honeysuckle tree)	38
Banksia serrata (heath honeysuckle tree)	50
Callitris verrucosa (dessert sandarac pine, or cypress)	43

NOTE.—On account of the unsettled nomenclature of Australian woods, this table gives botanical names, with common names, so far as possible, in parenthesis after the botanical name.

WEIGHT OF AUSTRALIAN WOODS, DRY—(*Continued*)

Name of Tree	Average Weight per Cubic Foot Pounds
Castanaspermum australe (black bean)	57
Casuarina torulusa (forest oak)	66
Casuarina quadravalvis (drooping she oak)	61
Cedrela australis (cedar)	28
Ceratopetalum apetalum (coachwood)	42
Dacrydium cupressinum (rimu)	38
Dissiliaria baloghioides (teak)	60
Dysoxylon muelleri (red bean)	46
Eucalyptus amygdalina regnans (mountain ash or peppermint tree)	60
Eucalyptus botryoides (blue gum, Gippsland mahogany, or bastard mahogany)	60
Eucalyptus corymbosa (bloodwood)	58
Eucalyptus corynocalyx (sugar gum)	69
Eucalyptus diversicolor (karri)	61
Eucalyptus globulus (blue gum)	57
Eucalyptus gomphocephala (tuart)	66
Eucalyptus goniocalyx (bastard box, spotted gum)	72
Eucalyptus haewastowa (spotted gum)	69
Eucalyptus hemiphloia (canary wood, white box, or gray box)	48
Eucalyptus largiflorens (slaty gum)	77
Eucalyptus leucoxylon (iron bark, red flowering, or black iron bark)	70
Eucalyptus longifolia (wollybutt tree)	69
Eucalyptus maculata (spotted gum)	63
Eucalyptus marginata (jarrah)	54
Eucalyptus melliodora (yellow box)	69
Eucalyptus microcorys (tallow wood)	59
Eucalyptus obliqua (messmate, stringy bark)	57
Eucalyptus pilularis (blackbutt, or flintwood)	53
Eucalyptus piperita (blackbutt, white stringy bark tree)	69
Eucalyptus resinifera (mahogany)	68
Eucalyptus robusta (swamp mahogany)	67
Eucalyptus rostrata (red gum tree)	56
Eucalyptus saligna (gray gum)	61
Eucalyptus siderophloia (iron bark)	68
Eucalyptus sieberiana (iron bark, gumtop stringy bark, mountain ashes)	56

WEIGHT OF AUSTRALIAN WOODS, DRY—(*Continued*)

Name of Tree	Average Weight per Cubic Foot Pounds
Eucalyptus tereticarnis (flooded gum)	68
Eucalyptus viminalis (manna gum tree, drooping gum, or white gum tree)	43
Eugenia smithii (myrtle)	57
Exocarpus cupressiformis (native cherry tree)	50
Fagus cunninghamii (evergreen beech or native myrtle)	45
Hakea leucoptera (water tree)	51
Heterodendron oleifolium	53
Lomatia fraseri	42
Melalenca decussata	59
Melalenca parviflora	62
Myporum insulare	51
Myrsine variabilis	45
Panax murrayi (palm panax)	22
Pimelea microcephala	55
Pittosporum bicolor (white wood)	48
Pomaderris apetala (hazel)	48
Prostanthera lasianthas (mint tree)	51
Santalum acuminatum (native peach or quandong)	52
Santalum persicarium (native sandalwood)	47
Senecio bedfordii (native dogwood)	56
Syncarpia laurifolia (turpentine)	63
Tristania conferta (brush or white box)	67
Tristania neriifolia (water gum)	63
Viminaria denudata	39

SNOW AND WIND LOADS

In calculating the weights on roofs, the snow load must be included. When the rise of the roof is under 12 in. per ft. of horizontal distance, the snow load is estimated at 20 lb. per sq. ft.; for roofs of a rise of more than 12 in. per ft., assume the snow load to be 8 lb. per sq. ft. In northern climates, such as that of Canada, Michigan, and New England, snow loads 50% greater than the preceding should be assumed.

The wind pressure depends on the velocity with which the air is moving, United States government tests have

determined that the pressure per square foot on a vertical surface is approximately represented by the formula

$$p = .00492V^2,$$

in which p is the pressure, in pounds per square foot, of vertical surface, and V is the velocity of wind, in miles per hour.

Careful records, extending over a period of years, show that the velocity of the wind seldom attains 100 mi. per hr.—probably not more than once in the lifetime of a structure.

The following table was calculated by means of the preceding formula. Though the table indicates that for 100 mi. an hr. the pressure per square foot is nearly 50 lb., modern practice often allows only 40 lb. per sq. ft. for large surfaces.

VELOCITY AND FORCE OF WIND, IN POUNDS PER SQUARE FOOT, ON A VERTICAL SURFACE

Strength of Wind	Miles per Hour	Feet per Minute	Feet per Second	Force in Pounds per Square Foot
Hardly perceptible........	1	88	1.47	.005
	2	176	2.93	.020
Just perceptible..........	3	264	4.40	.044
	4	352	5.78	.079
Gentle breeze.............	5	440	7.33	.123
	10	880	14.67	.492
Pleasant breeze...........	15	1,320	22.00	1.107
Brisk gale................	20	1,760	29.33	1.968
	25	2,200	36.67	3.075
High wind.................	30	2,640	44.00	4.428
	35	3,080	51.33	6.027
	40	3,520	58.67	7.872
Very high wind............	45	3,960	66.00	9.963
Storm.....................	50	4,400	73.33	12.300
	60	5,280	88.00	17.712
Great storm...............	70	6,160	102.67	24.108
	80	7,040	117.33	31.488
Hurricane or cyclone......	100	8,800	146.67	49.200

Curved and flat surfaces not in a vertical plane are subjected to less pressure than flat vertical surfaces. The pres-

sure on a cylindrical surface is about one-half the pressure on a flat surface having the same width as the diameter of the cylinder and the same height.

If p', Fig. 1, represents the direction and strength of the wind pressure against the roof *abc*, it is the normal component p that must be ascertained in order to calculate the total pressure normal to the roof, or to determine the stresses in the members of a roof frame or truss. The other component p_p is acting upwards and in a direction parallel with the slope. The latter force is not taken into consideration. The wind, which is usually supposed to exert a horizontal pressure of 40 lb., strikes the roof at an angle; consequently, the pressure p, normal to the slope, is less than 40 lb.

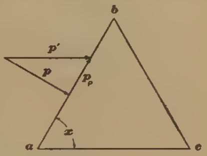


FIG. 1

The full discussion of the relation between p' and p is somewhat more complex than the one given here, however. What has been said shows in a general way why p is more nearly equal to p' when a roof is steep than when a roof is flat. In the design of roof trusses, a horizontal wind pressure of 40 lb. is usually assumed.

NORMAL WIND PRESSURE FROM HORIZONTAL PRESSURE OF 40 LB. PER SQ. FT.

Horizontal Rise per Foot Inches	Angle of Slope With Horizontal	Pitch, Proportion of Rise to Span	Wind Pressure Normal to Slope Pounds per Square Foot
4	18° 26′	$\frac{1}{6}$	23.00
4.8	21° 48′	$\frac{1}{5}$	26.11
6	26° 34′	$\frac{1}{4}$	29.82
8	33° 41′	$\frac{1}{3}$	33.93
12	45° 0′	$\frac{1}{2}$	37.71
16	53° 8′	$\frac{2}{3}$	39.02
18	56° 19′	$\frac{3}{4}$	39.33
24	63° 26′	1	39.75

All necessary data for calculating the wind pressure on a roof with any one of the customary pitches and a horizontal wind pressure of 40 lb. per sq. ft. are given in the table on page 83.

The diagram shown in Fig. 2 facilitates the finding of the normal pressure p for the usual slopes and for horizontal wind pressures of 20, 30, and 40 lb. per sq. ft.

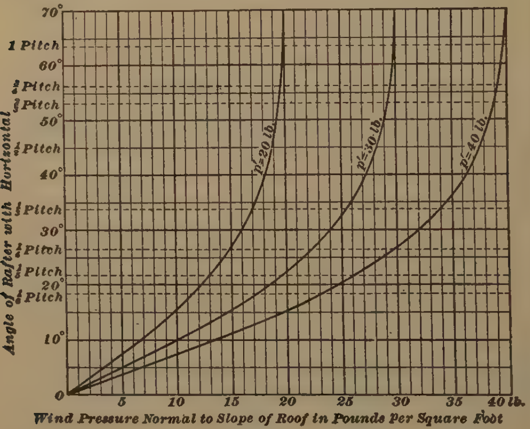


FIG. 2

The values of the normal pressure for a given slope and a horizontal wind pressure of 20, 30, or 40 lb. may be found as follows: Assume that the normal pressures on a roof having an angle with the horizontal of 40° is to be determined. Proceed along the horizontal line marked *40°* until it intersects the curve marked *20 lb.*, which represents a horizontal wind pressure of 20 lb. The point of intersection indicates the normal pressure p, the value of which is found by drawing an imaginary vertical line to the base line, which is marked off in pounds of pressure per square foot. It is

found that the normal pressure p amounts to 18.2 lb. per sq. ft. Proceeding in the same manner, it is found that for horizontal pressures of 30 and 40 lb., the normal pressures are 27.3 and 36.4 lb. per sq. ft., respectively.

In Fig. 3, the normal force p has been resolved into its two components, p_h and p_v, the former acting in a horizontal direction and the latter in a vertical one. The force p_h tends to push the roof in a direction parallel with the wind, while the force p_v tends to depress the roof or, in some cases, to press it sidewise. In open sheds, where the wind is liable to strike the inner, far side of the shed roof, the effect of the force p_v must be considered, as its tendency would be to lift the roof.

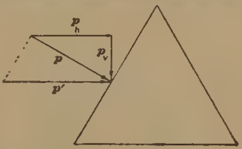


FIG. 3

DISPOSITION OF LOADS

In warehouses where all floors are likely at any one time to be fully loaded, the beams, girders, columns, and foundations are always proportioned for the entire live and dead loads. However, where the building exceeds four or five stories in height and is used for any other purpose except storage, as, for instance, a modern office building, it is customary to assume that certain members, while proportioned for the entire dead load, carry only a certain percentage of the live load.

In an office building, or similar structure, it is highly improbable that all the floors or all parts of the same floor will be fully loaded at the same time, and in view of this fact it is considered good practice, while proportioning the floor-beams for the full live load, to calculate only say 90% of the live load on the girders and columns. It is customary to proportion the columns supporting the roof and the top floor for the full live load. The live loads on the columns, in each successive tier, from the floor above is reduced 10% until 50% of the live load is reached, when such reduced loads are used for all the remaining floors to the basement.

The economy obtained by this disposition of the live load is observed from the following table, which gives the distribution of the assumed live loads on the columns in the several tiers of an eighteen-story office building.

REDUCTION OF LIVE LOADS FROM FLOOR TO FLOOR

Floors	a	$a_1 = .90a$	Σa	Σa_1	$\frac{\Sigma a - \Sigma a_1}{\Sigma a}$
Roof	20	20.00	20	20.00	
18	60	60.00	80	80.00	
17	60	54.00	140	134.00	4.3
16	60	48.60	200	182.60	8.7
15	60	43.74	260	226.34	12.9
14	60	39.37	320	265.71	17.0
13	60	35.43	380	301.14	20.8
12	60	31.89	440	333.03	24.3
11	60	30.00	500	363.03	27.4
10	60	30.00	560	393.03	29.8
9	60	30.00	620	423.03	31.8
8	60	30.00	680	453.03	33.4
7	60	30.00	740	483.03	34.7
6	60	30.00	800	513.03	35.9
5	60	30.00	860	543.03	36.9
4	60	30.00	920	573.03	37.7
3	60	30.00	980	603.03	38.5
2	60	30.00	1,040	633.03	39.1
1	60	30.00	1,100	663.03	39.7

The following may serve to explain the data given in the table: a represents the live load on each floor, in pounds per square foot; a_1 the live load on each floor, in pounds per square foot, reduced by 10%, as $a_1 = .90\,a$; Σa, the sum of all live loads, in pounds per square foot, on a column from all floors above, if no reduction is made; and Σa_1, the sum of all live loads, in pounds per square foot, on a column from all floors above, if 10% reduction is made.

The theoretical percentage of saving resulting from the reduction of 10% on the upper floors is found by the formula $\frac{\Sigma a - \Sigma a_1}{\Sigma a}$. These percentages of saving are given in the last column of the table.

It should be understood that each column of a building supports a given floor area, and that the load coming on each column will depend on the extent of this area multiplied by the live load, in pounds per square foot of floor. Each column carries not alone this load, but also the loads transmitted directly from column to column. Thus, the column supporting the fifteenth floor supports also four other columns above with all their loads.

While this system of graduating the live loads on the columns from floor to floor is generally practiced, the amount of reduction at each floor is a matter that depends on the judgment of the designer. The percentage of reduction is often fixed by city building laws, with which the designer must comply.

MECHANICS

FORCES

Two forces may be compared when the three following facts about each force are known: (1) The point of application, or point at which the force acts; (2) the direction of the force or line along which it acts; and (3) the magnitude of the force when compared with a given standard.

In engineering work in America, the unit of force is always taken as the *pound*.

Representation of a Force.—A force may be represented by a line. Thus, in Fig. 1, let A be the *point of application* of the force, let the length of the line AB represent its *magnitude* and *line of action* to any convenient selected scale, as, for instance, 1 in. equals 10 lb., and let the arrowhead indicate the *direction* in which the force acts. Then the line AB fulfils the three required conditions in regard to point of application, direction, and intensity, and the force is fully represented.

Fig. 1

COMPOSITION OF FORCES

If several forces act on a body, and if they are replaced by a single force that has the same effect in moving the body through space as the several forces combined, the single force is called the *resultant* of the several forces; and, conversely, the several forces are called the *components* of the single force. The process of finding the resultant when the various components are known is called the *composition of forces*.

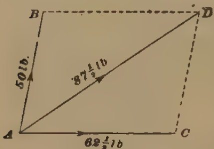


FIG. 2

Parallelogram of Forces.—When two forces act on a body at the same time and at the same point, but at different angles, their final effect may be obtained as follows:

In Fig. 2, let A be the common *point of application* of the two forces, and let AB and AC represent the *magnitude* and *direction* of the forces. Let, for instance, the line AB represent the distance that the force AB would cause the body to move in a certain length of time; similarly, let AC represent the distance that the force AC would cause the body to move in the same length of time, when both forces are acting separately. A fundamental law of mechanics states that the motion is proportional to the force applied, and, therefore, while AB and AC represent the magnitude of the forces to some scale, they are also proportional to the distances these forces would move the same body in the same length of time. The force AB, acting alone, would carry the body to B. If the force AC were now to act on the body, it would carry it along the line BD, parallel to AC, to a point D, at a distance from B equal to AC. Join C and D, then CD is parallel to AB and $ABDC$ is a parallelogram. Draw the diagonal AD. The body will stop at D, whether the forces act separately or together, but if they act together, the path of the body will be along AD, the diagonal of the parallelogram. Moreover, the length of the line AD represents the magnitude of a force, which, acting at A in the direction AD, would cause the body to move from A to D; in other words, AD, measured to the same scale as AB and AC, represents the magnitude

and direction of the combined effect of the two forces AB and AC.

The force represented by the line AD is the resultant of the forces AB and AC. Suppose that the scale used was 50 lb. to the inch; then, if $AB = 50$ lb. and $AC = 62\frac{1}{2}$ lb., the length of AB would be $50 \div 50 = 1$ in., and the length of AC would be $62.5 \div 50 = 1\frac{1}{4}$ in. If the line AD measures $1\frac{3}{4}$ in., the magnitude of the resultant, which it represents, would be $1\frac{3}{4} \times 50 = 87\frac{1}{2}$ lb.

Therefore, a force of $87\frac{1}{2}$ lb., acting on a body at A, in the direction AD, will produce the same result as the combined effects of a force of 50 lb. acting in the direction AB and a force of $62\frac{1}{2}$ lb. acting in the direction AC.

Triangle of Forces.—Let Fig. 3 represent a parallelogram of forces. AB and AC are the two component forces and AD is the resultant. Now, to find this resultant AD without drawing the entire parallelogram, first lay off AB, and then from the point B lay off BD equal to AC and parallel to its line of action; then, draw AD, which completes the triangle ABD and makes it unnecessary to draw AC and CD. Or, the force AC could be drawn first, and then from C, the line CD, thus completing the triangle ACD and not drawing AB and BD. In either case, it is seen that, if desired, the resultant of two forces can be found by drawing a triangle instead of a parallelogram.

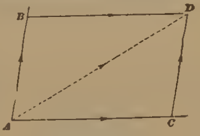


FIG. 3

Resultant of Several Forces.—When three or more forces act on a body at a given point, their resultant may be found as follows: Find the resultant of any two forces; treat this resultant as a single force, and combine it with a third force to find a second resultant. Combine this second resultant with a fourth force, to find a third resultant, etc. After all the forces have been thus combined, the last resultant will be the resultant of all the forces, both in magnitude and direction. The order in which the forces are taken is immaterial.

EXAMPLE.—Find the resultant of all the forces acting on the point O, Fig. 4, the length of the lines being proportional to the magnitude of the forces.

SOLUTION.—Draw OE parallel and equal to AO, and EF parallel and equal to BO; then OF is the resultant of these two forces, and its direction is from O to F. Consider OF as replacing OE and EF, and draw FG parallel and equal to CO; OG will be the resultant of OF and FG; but OF is the resultant of OE and EF; hence, OG is the resultant of OE, EF, and FG, and likewise of AO, BO, and CO. The line FG, parallel to CO, could not be drawn from the point O to the right of OE, for in that case it would be opposed in direction to OF; but FG must have the same direction as OF.

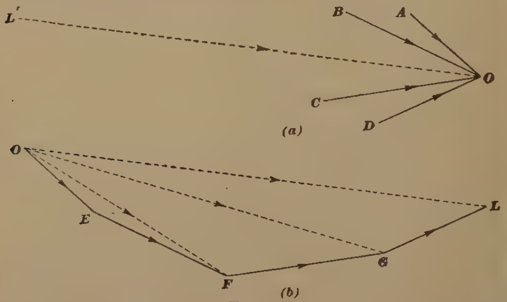


FIG. 4

For the same reason, draw GL parallel and equal to DO. Join O and L, and OL will be the resultant of all the forces AO, BO, CO, and DO (both in magnitude and direction) acting at the point O. If $L'O$ is drawn parallel and equal to OL, and having the same direction, it will represent the effect produced on the body by the combined action of the forces AO, BO, CO, and DO. For brevity, the terms forces AO, BO, etc. and resultants OF, OG, and OL have been used in this solution. It should be remembered, however, that these are merely lines that represent the forces in magnitude and direction.

In Fig. 4 the forces OE, EF, etc., all point in the same direction; that is, a body at O acted on by these forces in succession would move from O to L. The resultant therefore acts from O to L, and not from L to O. This line of reasoning will give the direction of the resultant in a force diagram. In Fig. 4, the forces were taken in the order AO, BO, CO, and DO. However, the magnitude and direction of the resultant OL would be the same, no matter in what order the forces were taken.

RESOLUTION OF FORCES

Since two forces can be combined to form a single resultant force, a single force may also be treated as if it were the resultant of two forces whose joint action on a body will be the same as that of a single force. Thus, in Fig. 5, the force OA may be resolved into two forces, OB' and $B'A$.

It will be observed that one resultant force may have an innumerable number of combinations of components. Instead of OB' and $B'A$, Fig. 5, OB'' or $B''A$ or OB''' and $B'''A$ may be taken as components. It is customary, however, to make OB' and $B'A$ perpendicular to each other, as shown in the illustration.

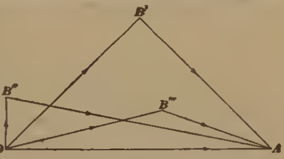


FIG. 5

Frequently, the position, magnitude, and direction of a certain force are known, and it is desired to know the effect of the force in some direction other than that in which it acts. Thus, in Fig. 6, suppose that OA represents, to some scale, the magnitude, direction, and line of action of a force acting on a body at A, and that it is desired to know what effect OA produces in the direction BA. From A draw a line AB in the required direction; from O draw a line perpendicular to AB. Then BA is the component required.

FIG. 6

It is necessary, of course, that OB be at right angles to BA, so that all the effect of OA in the required direction may be represented by BA and none of it by OB. Thus, OB' and $B'A$, although components of OA, and although one of them is in the required direction, would not be a correct solution of the problem because, besides $B'A$, OA exerts some more effect in the line BA, namely, a part of OB' is in that direction with an amount BB'.

EXAMPLE.—If a body weighing 200 lb. rests on an inclined plane whose angle of inclination to the horizontal is 18°, what force does it exert perpendicular to the plane, and what force does it exert parallel to the plane, tending to slide it downwards?

SOLUTION.—Let ABC, Fig. 7, be the plane, the angle A being 18°, and let W be the weight. Draw a vertical line

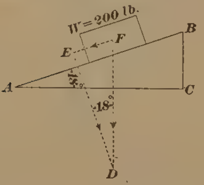


FIG. 7

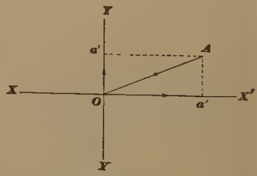


FIG. 8

$FD = 200$ lb., to represent the magnitude of the weight. Through F draw FE parallel to AB, and through D draw DE perpendicular to EF, the two lines intersecting at E. FD is now resolved into two components, one FE tending to pull the weight down the incline, and the other ED acting as a perpendicular pressure on the plane. On measuring FE with the same scale by which the weight FD was laid off, it is found to be about 61.8 lb., and the perpendicular pressure ED on the plane is found to measure 190.2 lb.

As it is often necessary to resolve a force into two components at right angles to each other, a simple method of solution is employed. In Fig. 8, let OA be the force it is desired to resolve into two components at right angles to

each other. Through O as a center draw an indefinite line XX' in the required direction of one of the components, and through the same center O draw the indefinite line YY' in the direction of the other component. Then XX' and YY' are at right angles to each other, since the components to be found are to be at right angles to each other. The line XX' is commonly called the XX' *axis*, or simply the X *axis*, and the line YY', the YY' *axis* or the Y *axis*. From A draw a line perpendicular to the X axis, as Aa', and also a line perpendicular to the Y axis, as Aa''. Then Oa' is the component force in the direction of XX' and Oa'' is the other component in the direction of YY'.

When the components Oa' and Oa'' are horizontal and vertical, as in the present case, they are called, respectively, the *horizontal component* and the *vertical component* of the force OA.

MOMENTS OF FORCES

DEFINITIONS AND MEASUREMENTS

In Fig. 1, W is a weight that tends to fall—that is, to act downwards—with a force of 10,000 lb. It is well known that if some fixed point, as a, not in the line along which the weight W acts, is connected with the line of action of W by a rigid arm, so that W pulls on one end of this arm while the other end is firmly held at a, the pull of W will tend to turn, or rotate, the arm about the point a.

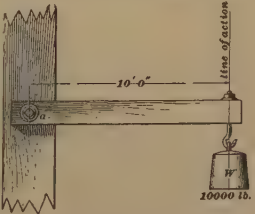


FIG. 1

It is also known that the tendency to rotate is directly proportional to the magnitude of the force, provided the arm remains of the same length, and directly proportional to the length of the arm, if the force remains constant. In general, therefore, the rotative effect is proportional to the

product of the magnitude of the force and the length of the lever arm. This product is called the *moment* of the force with respect to the point in question. Thus, in Fig. 1, the moment of the force W with respect to the point a is the product obtained by multiplying the magnitude, 10,000 lb., by the perpendicular distance, 10 ft., from the point a to the line of action of W.

The point a, Fig. 1, that is assumed as the center around which there is a tendency to rotate, is called the *center*, or *origin, of moments.*

The perpendicular distance from the center of moments to the line along which the force acts, is the *lever arm* of the force, or the *leverage* of the force.

Since the unit of force is the pound, and the ordinary unit of length is the foot, the unit of moment will be a derived unit, the *foot-pound* (abbreviated to *ft.-lb.*), and moments will usually be expressed in foot-pounds. In Fig. 1, for example, the moment of the force W with respect to the point a is $10{,}000 \times 10 = 100{,}000$ ft.-lb.

The moment of a force may be expressed in *inch-pounds* (*in.-lb.*), *foot-pounds*, or *foot-tons* (*ft.-T.*), depending on the unit of measurement used to designate the magnitude of the force and the length of its lever arm. For instance, if the magnitude of a force is measured in pounds, and the lever arm through which it acts in inches, the moment will be in inch-pounds; again, if a force of 10 T. acts through a lever arm of 20 ft., the moment of the force is $10 \times 20 = 200$ ft.-T.

Positive and Negative Moments.—In order to distinguish between the directions in which there is a tendency to produce rotation, the signs + and − may be used. Thus if a force tends to produce right-hand rotation, that is, rotation in the same direction as the hands of a clock, it is called *positive*, and its moment takes the sign +; a force that tends to produce rotation in the opposite direction is called *negative*, and its moment takes the sign −. The selection of one direction for positive and another for negative is merely an arbitrary distinction to show that the directions are opposite. It has, however, been adopted by engineers, and should always be used as given.

Resultant Moments.—In Fig. 2 is shown a lever composed of two arms at right angles to each other, and free to turn about the center C. A force A acts on the horizontal arm in such a manner that it tends to produce left-hand rotation, its moment being $10 \times 5 = 50$ ft.-lb., which, since it tends to produce left-hand rotation, will be called negative. Another force B, whose moment with respect to the center C is $12 \times 3 = 36$ ft.-lb., tends to produce right-hand rotation.

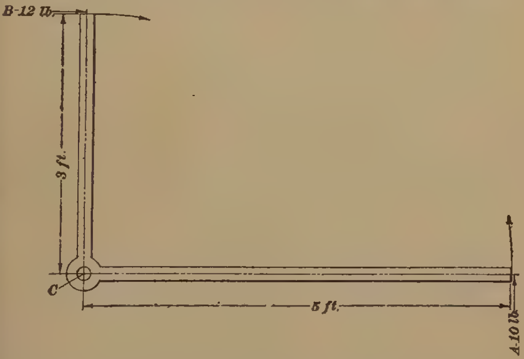


FIG. 2

Therefore, $-50 + 36 = -14$ ft.-lb., which is the *resultant moment*, and has the same turning effect as the two moments already given.

If, instead of the two forces just considered, there is a body that is acted on by any number of forces whose moments about a given center are known, the resultant moment of these forces will be the algebraic sum of the moments of the given forces.

CENTER OF GRAVITY

The *center of gravity* of a body, or of a system of bodies, or forces, is that point at which the body or system may be balanced, or it is the point at which the whole weight

of the body or bodies may be considered as concentrated. If the body or system were suspended from any other point than the center of gravity, and in such a manner as to be free to turn about the point of suspension, it would rotate until the center of gravity reached a position directly under the point of suspension.

Center of Gravity of Plane Figures.—If the plane figure has one axis of symmetry, this axis passes through its center of gravity. If the figure has *two* axes of symmetry, its center of gravity is at their point of intersection.

The center of gravity of a *triangle* lies on a line drawn from a vertex to the middle point of the opposite side, and at a distance from that side equal to one-third the length of the line; or it is at the intersection of lines drawn from the vertexes to the middle of the opposite sides. The perpendicular distance of the center of gravity of a triangle from the base is equal to one-third the altitude.

The center of gravity of a *parallelogram* is at the intersection of its two diagonals; consequently, it is midway between its sides.

The center of gravity of an *irregular four-sided figure* may be found as follows: First divide it, by a diagonal, into two triangles and join the centers of gravity of the triangles by a straight line; then, by means of the other diagonal, divide the figure into two other triangles, and join their centers of gravity by another straight line; the center of gravity of the figure is at the intersection of the lines joining the centers of gravity of the two sets of triangles.

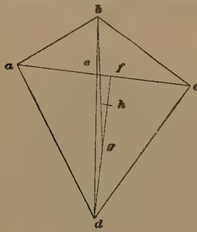


FIG. 3

Another method by which to locate the center of gravity of an irregular four-sided figure is illustrated in Fig. 3. Draw the diagonals *ac* and *bd*, and from their intersection *e*, measure the distance to any vertex, as *ae*. From the opposite vertex, lay off this distance, as at *cf* Then from *f*, draw a line to

one of the other vertexes, as *fd*, and bisect this line as at *g*. Connect *g* and *b* and lay off one-third of its length from *g* at the point *h*. This point is the center of gravity of the figure.

The distance of the center of gravity of the *surface of a half circle* from the center is equal to the radius multiplied by .424.

Neutral Axis.—The *neutral axis* of a flat or plane figure may be considered as any line passing through the center of gravity. This definition of neutral axis is correct for use in the mechanics of ordinary materials. However, in speaking of the neutral axis of reinforced-concrete beams, something else is meant. This will be explained later. The location of the horizontal neutral axis, that is, a horizontal line through the center of gravity of a plane figure, is of great importance in engineering. Of course, if the center of gravity is located, the horizontal neutral axis can be located easily; but, as a rule, the horizontal neutral axis is located without first locating the center of gravity.

Locating the Neutral Axis by Means of the Principle of Moments.—A convenient method of locating the neutral axis is based on the principle that the moment of any plane figure, with respect to any given line as an axis or origin of moments, is equal to the product of its area by the perpendicular distance from the center of gravity of the figure to the given axis.

Let M represent the moment; A, the area of the figure or section; and c the perpendicular distance from center of gravity of figure to given axis. Then,

$$M = A \times c, \text{ and } c = \frac{M}{A}$$

If necessary, the figure may be subdivided; then the moment of the figure is equal to the sum of the moments of its separate parts with respect to the same axis. Designating the areas of the subdivisions by the letters a, a', a'', etc., and their moments by m, m', m'', etc., then the preceding formula becomes

$$c = \frac{m + m' + m'', \text{ etc.}}{a + a' + a'', \text{ etc.}}$$

Built-Up Section.—Fig. 4 shows a section of the rafter member of a large roof truss formed of a $\frac{3}{8}'' \times 16''$ web-plate and a $\frac{3}{8}'' \times 12''$ flange plate, the two joined by two $4'' \times 4'' \times \frac{1}{2}''$ angles. It is desired to know the distance from the neutral axis of the section to the top edge of the flange plate. By means of the principles given, the centers of gravity of the two rectangular plates are easily located as shown. The centers of gravity of the angles might also be located by applying the rule given in the preceding paragraph; this, however, is unnecessary because the position of the center of gravity

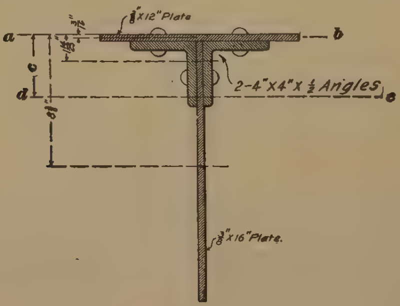


FIG. 4

can be obtained directly by referring to the tables of the properties of rolled sections furnished by the various steel manufacturers. By referring to any of these tables, the center of gravity of a $4'' \times 4'' \times \frac{1}{2}''$ angle is found to be 1.18 in. from the back of a flange, thus giving the distance $1.18 + .375 = 1.555$, or about $1\frac{1}{3}\frac{8}{2}$, in. from the top edge of the flange plate to the axis through the centers of gravity of the angles. From the same tables it is also found that the area of the section of a $4'' \times 4'' \times \frac{1}{2}''$ angle is 3.75 sq. in.

The area of the section of the flange plate is $\frac{3}{8} \times 12 = 4.5$ sq. in., and of the web-plate, $\frac{3}{8} \times 16 = 6$ sq. in.; the area of the whole section is therefore $2 \times 3.75 + 4.5 + 6 = 18$ sq. in.

The moments of the areas of the separate sections, with respect to the line *ab*, are as follows:

Flange plate,	$4.5 \times \frac{3}{16} =$	.84
Two angles,	$2 \times 3.75 \times 1\frac{14}{25} =$	11.70
Web-plate,	$6 \times 8\frac{3}{8} =$	50.25
	Total,	62.79

The distance c from the top edge of the flange plate to the neutral axis de of the section is therefore $62.79 \div 18 = 3.48$ in.

FORCES ACTING ON BEAMS

STYLES OF BEAMS

A beam that has two points of support, one at each end, as shown in Fig. 1, is known as a *simple beam*. The distance between the points of support is called the *span*.

FIG. 1

A beam that has more than two supports, as shown in Fig 2, is known as a *continuous beam*. The point of support of the middle support is considered to be at its center, as at *a*. Continuous beams, except in reinforced-concrete work, have not found much favor among engineers in the last few years.

FIG. 2

FIG. 3

A **cantilever beam** is one with only one support, which is at the middle, or any part of a beam that projects out beyond its support. A common form of cantilever is shown in Fig. 3. This beam projects from a wall or some other solid structure and has no support at its outer end.

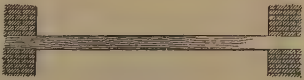


FIG. 4

When a beam is rigidly held, or fixed, at both ends, as shown in Fig. 4, it is called a *restrained beam*, or, more commonly, a beam *fixed at both ends*.

LOADS ON BEAMS

The forces due to the weights that a beam supports are known as *loads*. If the whole load is applied at one point, or practically one point, it is called a *concentrated load;* if it extends over a portion of the beam, it is called a *distributed load;* and if it is equally distributed over the beam, so that each unit of length has the same load, it is called a *uniform load*.

There are certain methods by which such loads may be represented graphically. These methods may best be illustrated by referring to Fig. 5, which shows a simple beam.

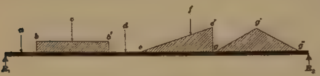


FIG. 5

Starting at the left, the point of the arrow shown at R_1 is the point of support. The arrow a represents a concentrated load. At bb' is shown a uniformly distributed load, of a certain number of pounds per foot of beam, that extends from b to b'. At c, d, and f are shown other concentrated loads similar to the one at a. From e to e' is a distributed load, represented by the shaded triangle under ee', that starts from nothing at e and increases to a given number of pounds per foot of beam at e'. At gg'' is a distributed load, starting from nothing at g and increasing to a maximum at g', and then decreasing to nothing at g''. At R_2 is shown the right-hand support.

REACTIONS

A beam with any loads it may carry is held up by the supports; that is, the beam presses on its supports at the ends. The supports resist this pressure and prevent the beam from falling. This upward force exerted by each support is known as the *reaction*.

There are two facts susceptible to proof in regard to a beam: (1) The resultant of all the forces acting on a body

or beam must be zero, and (2) the resultant moment of all the forces about any point must be zero.

The forces acting on a beam are the forces due to loads on the beam and the weight of the beam itself, if that is considered, and then there are the reactions of the supports on the beam. Now, all the loads act vertically downwards and the reactions act vertically upwards. They are therefore parallel; and as they are opposite, the sum of the loads must equal the sum of the reactions.

A beam with two supports will be considered. The sum of the reactions is known from the loads, but the value of each reaction is not known. To find this, it is necessary to resort to the second condition, namely, that the resultant moment of all the forces about any point must be zero. Any point can be assumed, but it is found convenient to choose the reaction of one support as the point about which to take moments. The moment of the reaction at the point is zero, because the arm of the moment is zero. It is therefore necessary that the sum of the moments of the loads about one point of support and the moment of the other reaction about the same point shall equal zero.

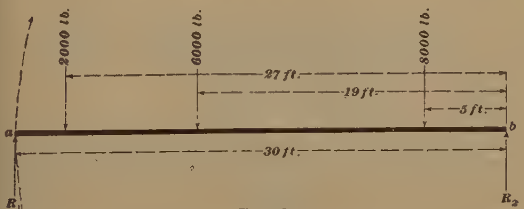


FIG. 6

A practical example will now be considered. In Fig. 6 let it be required to find the reactions R_1 and R_2 at the points of support a and b. (In all the subjoined problems, R_1 and R_2 represent the reactions.) The center of moments may be taken at either R_1 or R_2. Let the point b be taken in this case. The three loads are forces acting in a downward

direction; the sum of their moments, in foot-pounds, with respect to the assumed center, may be computed as follows:

$$\begin{aligned} 8{,}000 \times 5 &= 40000 \\ 6{,}000 \times 19 &= 114000 \\ 2{,}000 \times 27 &= 54000 \\ \text{Total,}\ & \overline{208000} \end{aligned}$$

This is the total negative moment about the point b. The positive moment about this point is of course $R_1 \times 30$. Since the resultant moment is zero, the positive moment must equal the negative moment, or $208{,}000 = 30\ R_1$ or $R_1 = 208{,}000 \div 30 = 6{,}933\frac{1}{3}$ lb. The sum of all the loads is $2{,}000 + 6{,}000 + 8{,}000 = 16{,}000$ lb. This is also the sum of the reactions. Therefore, $R_2 = 16{,}000 - 6{,}933\frac{1}{3} = 9{,}066\frac{2}{3}$ lb. In this problem, the weight of the beam itself has been neglected.

EXAMPLE 1.—What is the reaction at R_2 in Fig. 7?

SOLUTION.—In computing the moment due to a uniform, or evenly distributed, load, as at a, the lever arm is always considered as the distance from the center of moments to

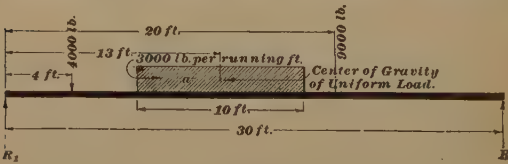


FIG. 7

the center of gravity of the load. The amount of the uniform load a is $3{,}000 \times 10 = 30{,}000$ lb., and the distance of its center of gravity from R_1 is 13 ft. Therefore, the moments of the loads on this beam about R_1, in foot-pounds, are as follows:

$$\begin{aligned} 30{,}000 \times 13 &= 390000 \\ 4{,}000 \times 4 &= 16000 \\ 9{,}000 \times 20 &= 180000 \\ \text{Total,}\ & \overline{586000} \end{aligned}$$

This is the sum of the moments of all the loads about R_1, as a center. The leverage of the reaction R_2 is 30 ft. Hence, the reaction at R_2 is $586,000 \div 30 = 19,533\frac{1}{3}$ lb.

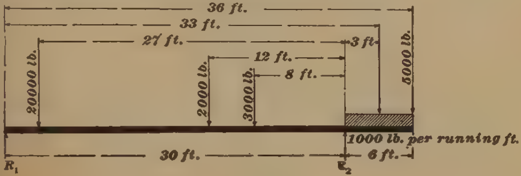


FIG. 8

EXAMPLE 2.—A beam is loaded as shown in Fig. 8. Compute the reactions R_1 and R_2.

SOLUTION.—Consider, say R_2, as the center of moments. Then, the negative moments of the loads about R_2, in foot-pounds, are

$$\begin{aligned} 20,000\times 27 &= 540000 \\ 2,000\times 12 &= 24000 \\ 3,000\times 8 &= \underline{24000} \\ \text{Total,}\ & \ 588000 \end{aligned}$$

Now, the positive moments of the loads about R_2, in foot-pounds, are

$$\begin{aligned} 1,000\times 6\times 3 &= 18000 \\ 5,000\times 6 &= \underline{30000} \\ \text{Total,}\ & \ 48000 \end{aligned}$$

The resultant moment of the loads about R_2 is negative and is $-588,000+48,000=-540,000$ ft.-lb.; $540,000 \div 30 = 18,000$ lb., which is the value of R_1.

The sum of the loads is

$$\begin{aligned} & 20000 \\ & 2000 \\ & 3000 \\ 6\times 1,000 = & 6000 \\ & \underline{5000} \\ & 36000 \end{aligned}$$

$R_2 = 36,000 - 18,000 = 18,000$ lb.

VERTICAL SHEAR

In any beam, as for instance the one shown in Fig. 9, there are forces—either loads or reactions—acting both upwards and downwards. In this figure, the left-hand reaction R_1

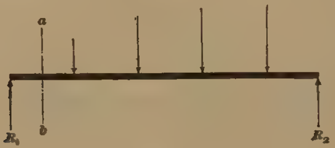


FIG. 9

acts upwards and tends to push the end of the beam up. On account of the strength of the beam, however, the end remains stationary. If the beam were suddenly cut on the line *ab*, the left-hand portion of the beam would move up in relation to the right-hand portion. This action of the forces on a beam in tending to make the surfaces at any imaginary section slide past each other, from its similarity to a shearing action, is called *shear*.

Consider now the beam shown in Fig. 10. Since the loads are symmetrically applied, each reaction is equal to 40 lb. or one-half the total load on the beam. Considering therefore,

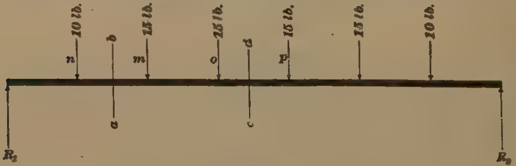


FIG. 10

any transverse section of the beam between R_1 and the point of application of the load n, it is evident that the part of the beam at the left of this transverse section is subjected to an upward thrust of 40 lb., while the part at the right is subjected to an equal downward thrust. The result is a shear-

ing action at this place, the magnitude of which is equal to the reaction R_1. This shearing action is resisted by the strength of the fibers of the beam at the section under consideration.

When the point of application of n is reached, the effect of the upward force R_1 is partly balanced by the downward force of 10 lb. due to the load n. Any section between the points of application of n and m is therefore subject to a shearing stress equal to the difference between the reaction R_1 and the load n, or $40-10=30$ lb. In the same way, it follows that the shearing stress for any section between m and o is $40-(10+15)=15$ lb. For any section, as cd, between the points of application of o and p, the shearing stress is $40-(10+15+15)=0$. In other words, it is a section in which there is no shear.

Positive and Negative Shear.—For convenience, it is customary to call the reactions, or forces, acting in an upward direction, *positive*, and the loads, or downward forces, *negative*. Since the difference between the sums of the positive and negative numbers representing a given set of values is called their algebraic sum, it follows that the shear for any section of a beam is equal to the algebraic sum of either reaction and the loads between this reaction and the transverse section under consideration. In speaking of the shear at a certain section of a beam as being positive or negative, it is simply meant that the resultant of the forces acting on the portion to the *left* of the section under consideration is either positive or negative.

If a transverse section of a simple beam is taken near the left reaction and the forces acting on the part of the beam at the left are considered, it will be seen that their resultant acts upwards. The shear at this section is therefore called *positive shear*. If, however, a section near the right reaction is taken, the resultant of the forces at the left of this section is found to act downwards, and in consequence the shear is called *negative*. It is also evident that there is a section between the two where the resultant of the forces changes from positive to negative. At such a section the shear is said to *change sign*.

It can readily be seen that in a cantilever beam the conditions are somewhat reversed; that is, in a beam having one support at the middle, the shear in the part to the left of the section taken to the left of the support is negative, while if the section is taken to the right of the support, it is positive. Like in a simple beam, however, there is an intermediate point where the shear changes sign.

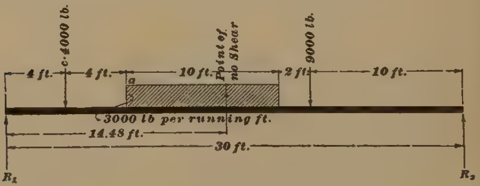


FIG. 11

EXAMPLE.—At what point in the beam loaded as shown in Fig. 11 does the shear change sign?

SOLUTION.—Compute the reaction R_1 as follows: With the center of moments at R_2, the moments of the loads, in foot-pounds, are

$$\begin{aligned} 9{,}000 \times 10 &= 90000 \\ 4{,}000 \times 26 &= 104000 \\ 3{,}000 \times 10 \times 17 &= 510000 \\ \text{Total,}\quad & \ 704000 \end{aligned}$$

The reaction at R_1 is therefore $704{,}000 \div 30 = 23{,}4662$ lb. Proceeding from R_1, the first load that occurs is c of 4,000 lb. Then, $23{,}4662 - 4{,}000 = 19{,}4662$ lb. The next load that occurs on the beam is the uniform load of 3,000 lb. per running ft. There being altogether 30,000 lb. in this load, it is evident that the load will more than counteract the remaining amount of the reaction R_1; the point where the change of sign occurs must consequently be somewhere in that part of the beam covered by the uniform load. The load being 3,000 lb. per running ft., if the remaining part

of the reaction, $19{,}466\frac{2}{3}$ lb., is divided by the 3,000 lb., the result will be the number of feet of the uniform load required to counteract the remaining part of the reaction, and this will give the distance of the section, beyond which the resultant of the forces at the left becomes negative, from the edge of the uniform load at a; thus, $19{,}466\frac{2}{3} \div 3{,}000 = 6.49$ ft. The distance from R_1 to the edge of the uniform load is 8 ft. The entire distance to the section of change of sign of the shear is, therefore, $8 + 6.48 = 14.48$ ft. from R_1.

Shear Diagram.— It is sometimes necessary to plot the shear of the forces acting on a beam in a diagram known as the *shear diagram*. In order to illustrate how this may be

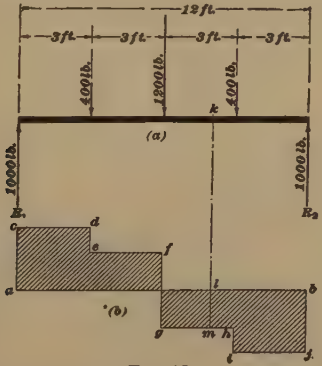


FIG. 12

done the beam shown in Fig. 12 (a), the span of which is 12 feet, will be considered. Since the beam is symmetrically loaded, each reaction is equal to one-half the sum of the loads; that is, each reaction is equal to $\frac{400 + 1{,}200 + 400}{2}$ $= 1{,}000$ lb. The shear at R_1 is equal to the reaction, or 1,000 lb. To plot the diagram, proceed as follows: Draw a line ab to any convenient scale to represent the length of

the beam. This line will also represent the *base*, or *datum*, *line* of the shearing forces, positive values being laid off above and negative values below the base line. The shear at R_1 being positive and equal to 1,000 lb., draw from a upwards a vertical line ac equal to 1,000 lb., according to any convenient scale. From c draw to the same scale as ab a horizontal line cd equal to 3 ft. Ordinates drawn from

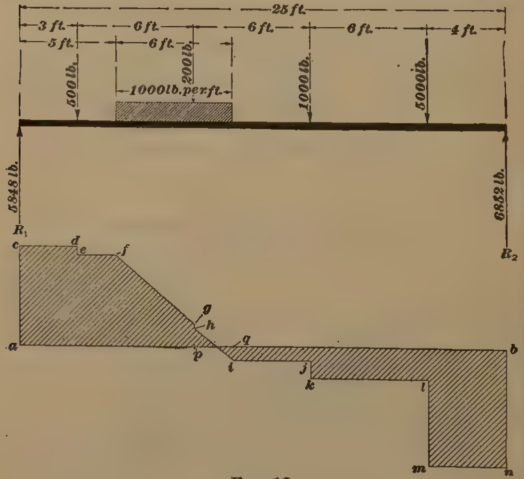


FIG. 13

any point on the line ab to cd will be of equal lengths, showing that the shear retains the value of 1,000 lb. from the reaction R_1 to the first load of 400 lb. At this point the shear is reduced by the load of 400 lb.; therefore, from d, draw downwards a vertical line de of a length corresponding to 400 lb., as shown in the diagram.

The shear is then uniform until the central load is reached, and may be represented by the distance of the line ef from

the line ab, namely, 600 lb. At this point the shear is reduced by 1,200 lb., shown on the diagram by the line fg, and becomes negative. At the third load, counting from the left, the shear is still further reduced by 400 lb., and is represented by the line hi in the diagram. From this load on to the right-hand reaction the shear is negative in sign and equal to 1,000 lb. This right-hand reaction drawn from j to b closes the diagram.

It may now be seen that to find the shear at any point in a beam it is necessary to draw an ordinate from the corresponding point on the line ab and to measure the length of the part included in the shaded diagram with the correct scale. For instance, if it is desired to ascertain the shear and its sign at the point k, draw a perpendicular line km crossing the shear diagram. The length lm measured by the scale selected for the shear will show that the shear at this point is 600 lb., and, as lm is located below the base line ab, the shear is negative.

As a general example, the shear diagram of the beam shown in Fig. 13 may be plotted. As the load here is not symmetrical, it is first necessary to calculate the reactions R_1 and R_2. The positive moments about the left-hand end of the beam, in foot-pounds, are:

$$\begin{array}{rr}
500\times 3= & 1500 \\
(1{,}000\times 6)\times 8= & 48000 \\
200\times 9= & 1800 \\
1{,}000\times 15= & 15000 \\
5{,}000\times 21= & \underline{105000} \\
\text{Total,} & 171300
\end{array}$$

The total negative moment is $R_2 \times 25$. Therefore, $R_2 = 171{,}300 \div 25 = 6{,}852$ lb. The sum of the loads is $500 + (1{,}000\times 6) + 200 + 1{,}000 + 5{,}000 = 12{,}700$ lb. R_1 is therefore equal to $12{,}700 - 6{,}852 = 5{,}848$ lb.

The plotting of the shear diagram shown in Fig. 13 may now be started. Draw the line ab equal to the length of the beam; ac equal to R_1, or 5,848 lb.; cd, horizontally, equal to 3 ft.; de, vertically downwards, equal to 500 lb.; and ef, horizontally, equal to 2 ft. From a on ab lay off 9 ft., as

at p. The shear at this point, just to the left of the concentrated load, is $+5{,}848-500-(1{,}000\times4)= +1{,}348$ lb. This value laid off above ab gives the point g. Draw fg. From g lay off vertically downwards 200 lb. to the point h. From a, on the line ab, lay off 11 ft., as at q. The shear at q is $+5{,}848-500-(1{,}000\times6)-200= -852$ lb. From q lay off, vertically, 852 lb. to i, downwards in this case because the shear is negative. Join h and i. If correctly drawn, hi should be parallel to fg. From i draw ij horizontal and equal to 4 ft.; from j draw jk vertically downwards, equal to 1,000 lb.; from k draw kl horizontally, equal to 6 ft.; from l draw lm vertically downwards, equal to 5,000 lb.; and from m draw mn horizontally, equal to 4 ft. Then, if the diagram is correctly constructed, bn should be vertical and equal to R_2, or 6,852 lb. Then $acde \ldots mnb$ is the shear diagram, and ordinates drawn from any point on ab across the diagram will, when measured by the proper scale, indicate the shear at that section, which is positive if above the line ab and negative if below it.

BENDING MOMENTS

In order to illustrate the method of calculating the *bending moment*, or the moment of a force that tends to bend a beam, the beam shown in Fig. 14 will be considered. At the point a is a joint, or hinge. It is evident that when loads

FIG. 14

are applied, as shown by the arrows, the beam will bend at the joint and take the position indicated by the dotted lines; or, to be more exact, will bend still farther until it falls off the supports entirely.

The loads and reactions (and weight of the beam itself, if this is considered) are what cause the beam to bend. Con-

sidering the portion of the beam to the left of the joint, this portion moves clockwise, and the moment that moves it is therefore positive. The magnitude of the moment tending to move the left-hand part of the beam is $R_1l - Wl_1$, since the load W acts in the opposite direction to R_1.

It will thus be seen that no matter in what part of the beam the joint a is placed, the beam will collapse. Therefore, it is evident that in any beam carrying loads, these loads and the reactions exert a moment at any transverse section that tends to bend the beam. It is only on account of its own strength that a beam does not break. A beam is therefore designed to withstand the bending moment of the forces acting on it, and for this reason the engineer must at all times be able to find this moment. With a simple beam the bending moment (considering, as usual, the part to the left of the section) is always positive, while with cantilever beams it is negative.

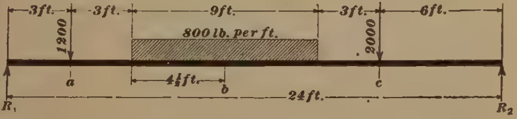


FIG. 15

As an illustration, the bending moment around various sections of the beam shown in Fig. 15 will be considered. It is of course first necessary to find the reactions, so that the moments may be calculated. To find R_2, take moments about R_1. The positive moments, in foot-pounds, are

$$
\begin{array}{rr}
1{,}200 \times 3 = & 3600 \\
(800 \times 9) \times 10\tfrac{1}{2} = & 75600 \\
2{,}000 \times 18 = & \underline{36000} \\
\text{Total,} & 115200
\end{array}
$$

The span is 24 ft. Therefore, $R_2 = 115{,}200 \div 24 = 4{,}800$ lb. The sum of the loads is $1{,}200 + (800 \times 9) + 2\,000 = 10{,}400$ lb. Therefore, $R_1 = 10{,}400 - 4{,}800 = 5{,}600$ lb

The bending moment at any section of the beam may now be found. For example, find the bending moment around

point a directly under the first load. The moment here, on the left-hand part of the beam, is $R_1 \times 3 = 5{,}600 \times 3 = 16{,}800$ ft.-lb.

Then find the moment on the left-hand portion of the beam around point b. Here the positive moment is $R_1 \times 10\frac{1}{2}$, or $5{,}600 \times 10\frac{1}{2} = 58{,}800$ ft.-lb. There are, however, two negative moments acting, and these must be subtracted from the positive moment in order to get the resultant moment. One of these moments is that due to the concentrated load of 1,200 lb., and the other is that due to the part of the distributed load to the left of the point b. The negative moment of the concentrated load is therefore $1{,}200 \times 7\frac{1}{2} = 9{,}000$ ft.-lb. The portion of the distributed load considered is $800 \times 4\frac{1}{2} = 3{,}600$ lb. Its moment arm may be considered to extend from the point b to the center of the portion under consideration, or $4\frac{1}{2} \div 2 = 2\frac{1}{4}$ ft. Its moment is therefore $3{,}600 \times 2\frac{1}{4} = -8{,}100$ ft.-lb. The resultant moment of the left-hand section about b is therefore $58{,}800 - 9{,}000 - 8{,}100 = +41{,}700$ ft.-lb.

The bending moment about c is $5{,}600 \times 18 - 1{,}200 \times 15 - (800 \times 9) \times 7\frac{1}{2} = 100{,}800 - 18{,}000 - 54{,}000 = +28{,}800$ ft.-lb.

The bending moment of the forces acting on a beam, tending to break it at any section, may be calculated from either end of the beam. It is customary to call the bending moment positive if it tends to turn the left-hand part in the direction of the hands of a clock, and negative if it tends to turn it in the opposite direction. However, the moment may be calculated from either end, remembering that if it is calculated from the right-hand end the bending moment acts in an opposite direction and will therefore receive a sign opposite to that given the left-hand end of the beam.

As has been stated, the reason a beam does not break is because its strength at any transverse section is sufficient to resist the moment of the forces about that section. It has also been shown that the bending moment is different at different points along a beam. It is therefore important to find out around what point the forces acting on a beam exert their maximum moment, and then, by the method already given, to find this maximum moment.

The point where the shear on a beam changes sign, that is, changes from positive to negative, or from negative to positive, is the point around which the maximum bending moment occurs. The point where the shear on a cantilever beam changes sign, that is, the point around which the moment is maximum, is always at the point of support.

Sometimes, the shear will change from positive to negative or vice versa two or more times on a beam, and each section, when the shear is zero, must therefore be investigated to determine around which point the maximum moment occurs. Fig. 16 shows an example of this kind. To solve this example, it is first necessary to find the reactions. The moments of the loads about R_1, in foot-pounds, are as follows:

$$\begin{aligned} 500 \times 2 &= 1000 \\ 800 \times 5 &= 4000 \\ 300 \times 14 &= 4200 \\ \text{Total,}\quad &\ 9200 \end{aligned}$$

$R_2 \times 10 = 9{,}200$ ft.-lb.; $R_2 = 9{,}200 \div 10 = 920$ lb.; and $R_1 = 500 + 800 + 300 - 920 = 680$ lb.

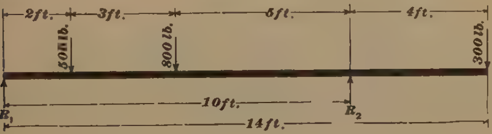


FIG. 16

At R_1, the shear is $+680$ lb., and at 2 ft. from the left-hand end the shear changes from $+680$ to $680 - 500 = +180$ lb. At 5 ft. from the left-hand end, the shear changes from $+180$ to $180 - 800 = -620$ lb. This, therefore, is one place where the shear changes sign. Under the reaction R_2, the shear changes from -620 to $920 - 620 = +300$ lb., and this is therefore another point where the shear changes sign.

There are therefore two places to be investigated for maximum bending moment, one 5 ft. from the left-hand end and the other 10 ft. from the left-hand end. The bending moment about the point 5 ft. from the left-hand end is

$680\times 5-500\times 3=+1{,}900$ ft.-lb., and that about the point 10 ft. from the left-hand end is $680\times 10-500\times 8-800\times 5=-1{,}200$ ft.-lb. It is thus seen that the greater of the two maximum bending moments occurs about the point 5 ft. from the left-hand end.

Bending-Moment Diagrams.—The bending moments that act at various points of a loaded beam may be represented graphically in the same manner in which the shearing forces were shown. For example, a beam 50 ft. long

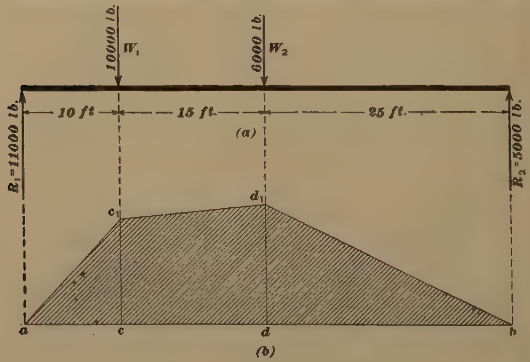


FIG. 17

supports two concentrated loads of the magnitudes and in the positions indicated in Fig. 17. Now, assuming that Fig. 17 (*a*) is drawn to scale, draw the horizontal line *ab* and produce the lines indicating the reactions and concentrated loads until they intersect line *ab* at *a*, *c*, *d*, and *b*. By calculating the moments about the left-hand reaction, the moment for the load W_1 is found to be $10{,}000\times 10=100{,}000$ ft.-lb. and that for W_2, $6{,}000\times 25=150{,}000$ ft.-lb. The reaction R_2 is therefore $\frac{100{,}000+150{,}000}{50}=5{,}000$ lb., and reaction R_1 is $10{,}000+6{,}000-5{,}000=11{,}000$ lb.

The bending moment at W_1 is $11{,}000 \times 10 = 110{,}000$ ft.-lb., and that at W_2 is $(11{,}000 \times 25) - (10{,}000 \times 15) = 125{,}000$ ft.-lb.

Lay off the line cc_1 to any convenient scale to represent the bending moment at W_1, and at dd_1 to represent that at W_2. Connect points a, c_1, d_1, and b, as shown, and the diagram is complete.

As another illustration, assume that it is desired to plot the bending-moment diagram of the beam shown in Fig. 18. This beam is 50 ft. long and has a uniform load of 1,000 lb. per running ft.

First, divide the beam into any number of smaller parts, each, for instance, 10 ft. long, and then find the bending

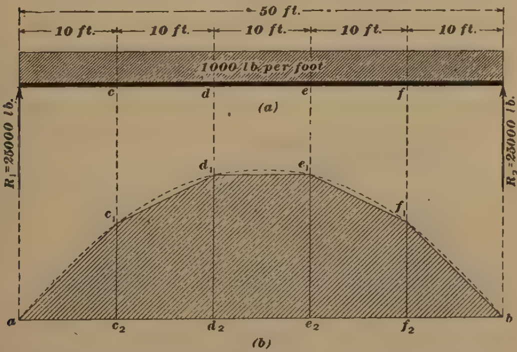
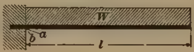
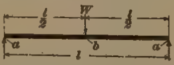
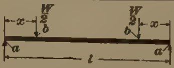


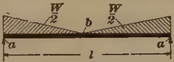
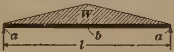
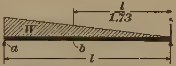
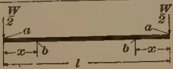
FIG. 18

moment for each of these parts. In this instance, the reactions R_1 and R_2 are each equal to one-half the total load, or $50{,}000 \div 2 = 25{,}000$ lb. The bending moment at c is $(25{,}000 \times 10) - (1{,}000 \times 10 \times 5) = 200{,}000$ ft.-lb.; at d it is $(25{,}000 \times 20) - (1{,}000 \times 20 \times 10) = 300{,}000$ ft.-lb. In a similar manner, it will be found that the bending moments at points e and f are 300,000 and 200,000 ft.-lb., respectively.

On laying off these bending moments at c_1c_2, d_1d_2, etc. and connecting points a, c_1 . . . b, the diagram is complete. The

FORMULAS FOR MAXIMUM SHEAR AND BENDING MOMENTS ON BEAMS

Case	Method of Loading	Maximum Shear	Maximum Moment
I		W	Wl
II		W	$\frac{Wl}{2}$
III		W	$\frac{Wl}{3}$
IV		W	$\frac{2Wl}{3}$
V		W	$\frac{Wl}{2}$
VI		W	$\frac{Wl}{2}$
VII		W	$\frac{Wl}{2}$
VIII		$\frac{W}{2}$	$\frac{Wl}{4}$
IX		$\frac{Wy}{l}$ or $\frac{Wx}{l}$	$\frac{Wxy}{l}$
X		$\frac{W}{2}$	$\frac{Wx}{2}$

Case	Method of Loading	Maximum Shear	Maximum Moment
XI		$\frac{W}{2}$	$\frac{Wl}{8}$
XII		$\frac{W}{2}$	$\frac{Wl}{12}$
XIII		$\frac{W}{2}$	$\frac{Wl}{6}$
XIV		$\frac{2W}{3}$	$\frac{52Wl}{405}$
XV		$\frac{11}{16}W$	$\frac{3}{16}Wl$
XVI		$\frac{5}{8}W$	$\frac{1}{8}Wl$
XVII		$\frac{W}{2}$	$\frac{Wl}{8}$
XVIII		$\frac{W}{2}$	$\frac{Wl}{12}$
XIX		$\frac{W}{2}$	$\frac{Wx}{2}$
XX		$\frac{Wx}{l}$ or $W\left(\frac{l-2x}{2l}\right)$	$\frac{Wx^2}{2l}$ or $\frac{W}{2}\left(\frac{l}{4}-x\right)$

outline of this diagram, however, is only approximately correct. To find its true form, the beam should be divided into an infinite number of parts, in which case the lines ac_1, c_1d_1, d_1e_1, e_1f_1, and f_1b would be parts of a curve, shown dotted, instead of a series of short, broken lines.

MOMENT TABLES

The methods of finding the shear and maximum moment occurring in simple and in cantilever beams have been explained. In these calculations the weight of the beam itself, if of wood or steel, is often omitted. The various unusual loadings were also considered, and the method of finding the maximum bending moment was explained. These methods will be found useful if such cases ever arise. However, in practice the loads on the beams are often more uniform, and if the weight of the beam itself is neglected, the formulas given in the accompanying table may be used.

Beams with fixed ends or even with one end fixed and one simply supported are also given in the table.

In each case in the table, the total load on the beam, in pounds, is denoted by W. If there are two separate and equal loads on the beam, each one is called $\frac{W}{2}$, so as to make the total load W. If the load is uniformly distributed over the entire length of the beam, the load per foot will be $W \div$ length of beam, in feet. The length of the beam is denoted by l. If l is taken in inches, the moment will be in inch-pounds; but if taken in feet, the moment will be in foot-pounds.

The usual method of indicating loading is adopted. A simple support under a beam is shown by the conventional arrow, while a cantilever or a beam fixed at the ends is shown as in Case I or XVII, respectively. Cases XV and XVI indicate a beam fixed at one end and supported at the other.

The maximum shear on a cantilever or on a beam fixed at the end depends somewhat on the method of holding it in the wall and the length that extends into the wall. However, the maximum shears given in the table are correct

for usual cases. In Case XX, two values are given for both shear and bending moment, and the maximum of the two values obtained must be the one used. The sign of the shears or bending moments has not been put in the table, but it can be told by inspection whether they are positive or negative, as every-day experience should enable the engineer to determine whether the loads will bend the beam up or down.

The point of maximum shear is marked on the beam at the point a, while maximum bending moment is at the

REACTIONS FOR CONTINUOUS BEAMS OVER EQUAL SPANS

(*Coefficients of W*)

Number of Spans	Number of Each Support									
	1st	2d	3d	4th	5th	6th	7th	8th	9th	10th
2	$\frac{3}{8}$	$\frac{10}{8}$	$\frac{3}{8}$							
3	$\frac{4}{10}$	$\frac{11}{10}$	$\frac{11}{10}$	$\frac{4}{10}$						
4	$\frac{11}{28}$	$\frac{32}{28}$	$\frac{26}{28}$	$\frac{32}{28}$	$\frac{11}{28}$					
5	$\frac{15}{38}$	$\frac{43}{38}$	$\frac{37}{38}$	$\frac{37}{38}$	$\frac{43}{38}$	$\frac{15}{38}$				
6	$\frac{41}{104}$	$\frac{118}{104}$	$\frac{100}{104}$	$\frac{106}{104}$	$\frac{100}{104}$	$\frac{118}{104}$	$\frac{41}{104}$			
7	$\frac{56}{142}$	$\frac{161}{142}$	$\frac{137}{142}$	$\frac{143}{142}$	$\frac{143}{142}$	$\frac{137}{142}$	$\frac{161}{142}$	$\frac{56}{142}$		
8	$\frac{153}{388}$	$\frac{440}{388}$	$\frac{374}{388}$	$\frac{392}{388}$	$\frac{386}{388}$	$\frac{392}{388}$	$\frac{374}{388}$	$\frac{440}{388}$	$\frac{153}{388}$	
9	$\frac{209}{530}$	$\frac{601}{530}$	$\frac{511}{530}$	$\frac{535}{530}$	$\frac{529}{530}$	$\frac{529}{530}$	$\frac{535}{530}$	$\frac{511}{530}$	$\frac{601}{530}$	$\frac{209}{530}$

point b. In some cases, either of these values may reach its maximum at two or more places, in which case it is so marked.

The following example will serve to show the use of the table:

Example.—A simple beam on a span of 13 ft. $2\frac{1}{2}$ in. carries a uniformly distributed load of 85 lb. per ft. What is the maximum shear and the maximum bending moment developed?

Solution.—The length of the beam, reduced to feet, is 13.2083. The total load W is therefore $85 \times 13.2083 = 1{,}122.7055$, say 1,123 lb. Referring to Case XI, the maximum shear $= \frac{W}{2} = \frac{1123}{2} = 561.5$ lb. Likewise, the maximum bending moment is

$$\frac{Wl}{8} = \frac{1{,}123 \times 13.2083}{8} = 1{,}854.115, \text{ say } 1{,}854, \text{ ft.-lb.}$$

CONTINUOUS BEAMS

When a single beam extends over three or more supports it is said to be continuous. The bending moments produced are very different from those in an ordinary beam. For this reason, the treatment of this class of beams must be considered separately.

BENDING MOMENTS FOR CONTINUOUS BEAMS OVER EQUAL SPANS

(Coefficients of Wl)

Number of Spans	Number of Each Support									
	1st	2d	3d	4th	5th	6th	7th	8th	9th	10th
2	0	$\frac{1}{8}$	0							
3	0	$\frac{1}{10}$	$\frac{1}{10}$	0						
4	0	$\frac{3}{28}$	$\frac{2}{28}$	$\frac{3}{28}$	0					
5	0	$\frac{4}{38}$	$\frac{3}{38}$	$\frac{3}{38}$	$\frac{4}{38}$	0				
6	0	$\frac{11}{104}$	$\frac{8}{104}$	$\frac{9}{104}$	$\frac{8}{104}$	$\frac{11}{104}$	0			
7	0	$\frac{15}{142}$	$\frac{11}{142}$	$\frac{12}{142}$	$\frac{12}{142}$	$\frac{11}{142}$	$\frac{15}{142}$	0		
8	0	$\frac{41}{388}$	$\frac{30}{388}$	$\frac{33}{388}$	$\frac{32}{388}$	$\frac{32}{388}$	$\frac{30}{388}$	$\frac{41}{388}$	0	
9	0	$\frac{56}{530}$	$\frac{41}{530}$	$\frac{45}{530}$	$\frac{44}{530}$	$\frac{44}{530}$	$\frac{45}{530}$	$\frac{41}{530}$	$\frac{56}{530}$	0

The first of the tables, on page 119, gives the coefficients for the reactions of the beam at its supports. The value given in the table multiplied by W, which represents the load per span, will give the reaction. By knowing the reactions

and the loads per foot, the shear at any section may be found.

The coefficient from the second table multiplied by Wl, which represents the load per span and the length of span, will give the negative bending moment over each support. As the maximum moment occurs over a support, it is seen, by referring to the table, where this moment will be. If the coefficient given in this table is multiplied by the load in pounds and the span in feet, the result will be in foot-pounds, but if the span is taken in inches, the result will be in inch-pounds.

To illustrate the method of using the tables, the following example will be assumed:

EXAMPLE.—A beam of three spans is supported by four reactions. Each span is 11 ft. The beam carries a load of 2,000 lb. per ft. (*a*) What will be the amount of the end reactions? (*b*) What is the greatest bending moment produced?

SOLUTION.—(*a*) The load W supported by each span is $2{,}000 \times 11 = 22{,}000$ lb. From the first table, the reaction at each end support is equal to $\frac{4}{10}\ W$. Substituting these values, the reaction at each end is equal to $\frac{4}{20} \times 22{,}000 = 8{,}800$ lb.

(*b*) The greatest bending moment occurs at the intermediate supports; from the second table, this moment is equal to $\frac{1}{10}\ Wl$, or $\frac{1}{10} \times 22{,}00 \times 11 = 24{,}200$ ft.-lb.

STRESSES AND STRAINS

STRESS

It is evident that the weight of the materials composing a building and its contents produces forces that must be resisted by the different members of the structure. The action of these forces has a tendency to change the relative position of the particles composing the members, and this tendency is, in turn, resisted by the cohesive force in the materials, which acts to hold the particles together.

The internal resistance with which the force of cohesion opposes the tendency of an external force to change the relative position of the particles of any body subjected to a load is called a *stress;* or, stress may be defined as the load

that produces an alteration in the form of a body, and this alteration of form is called the *strain*.

In accordance with the direction in which the forces act with reference to a body, the stress produced may be either *tensile*, *compressive*, or *shearing*.

Tensile stress is the effect produced when the external forces act in such a direction that they tend to stretch a body.

Compressive stress is the effect produced when the tendency of the forces is to compress the body.

Shearing stress is when the forces act so as to produce a tendency for the particles in one section of a body to slide over the particles of the adjacent section.

When a beam is loaded in such a manner that there is in it a tendency to bend, it is subjected to a *bending stress*. In this case, there is a combination of the three stresses already mentioned (tension, compression, and shear) in different parts of the beam.

The *unit stress* (called, also, the *intensity* of stress) is the name given to the stress per unit of area; or, it is the total stress in a tie-rod, column, or the like divided by the area of the cross-section.

Let P represent the total stress, in pounds; A, the area of cross-section, in square inches; and s, the unit stress, in pounds per square inch. Then,

$$s=\frac{P}{A}, \text{ or } P=As$$

STRAIN

When a body is stretched, shortened, or in any way deformed through the action of a force, the deformation is called a *strain*. Thus, if a rod were elongated $\frac{1}{10}$ in. by a load of 1,000 lb., the strain would be $\frac{1}{10}$ in. Within certain limits, to be given hereafter, strains are proportional to the stresses producing them.

Unit strain is the strain per unit of length or of area, but is usually taken per unit of length and called, for tension, the *elongation* per unit of length. If the unit of length is taken as 1 in., the unit strain is equal to the total strain divided by the length of the body in inches.

Let l represent the length of the body, in inches; e, the deformation, in inches; and q, the unit strain. Then,

$$q=\frac{e}{l}, \text{ or } e=lq$$

ELASTIC PROPERTIES

It can be proved by experiment that when a certain unit stress is created in a substance, a certain definite unit strain is developed. If the unit stress is doubled, it will be found that the unit strain also has doubled; that is, the alteration of shape or the strain in a body is proportional to the force applied to that body. This experimental fact is known as *Hooke's law.*

When a certain stress is created in a body a certain strain is produced. When the stress is removed, the body returns to its original shape, provided the unit stress has not been too great. For each substance, however, there is a certain maximum unit stress that the substance will stand and still return to its original shape after the external forces are removed. This unit stress is called the *elastic limit* of the material. If a body is strained beyond the elastic limit, it will maintain a permanent distortion, or *set*, even after the strain forces are removed. Hooke's law, which is almost exact for most materials below the elastic limit, does not hold good for these materials above the elastic limit, as the strain increases much more rapidly than the stress. Thus, if the unit stress is doubled beyond the elastic limit, the unit strain will be more than doubled. The unit stress that is so great that the strain increases greatly with very little increment of stress is called the *yield point.* For all practical purposes, with many materials the yield point commences at the elastic limit.

To restate Hooke's law, the ratio of the unit stress to the unit strain for any substance is constant below the elastic limit. This ratio of unit stress to unit strain is called the *modulus of elasticity*, or *coefficient of elasticity*, which will be represented by the symbol E. It is

$$E=\frac{s}{q}$$

From this equation, when s and E are known, q may be found; that is, if the modulus of elasticity of a certain substance and the unit stress are known, the unit strain can readily be found.

STRENGTH OF BUILDING MATERIALS

The *ultimate strength* of any material is that unit stress which is just sufficient to break it.

The *ultimate elongation* is the total elongation produced in a unit of length of the material by a unit stress equal to the ultimate strength of the material.

The *modulus of rupture* differs from the ultimate tensile or compressive stress, but is a quantity, something like it, that is used in calculating the strength of beams. Its use is discussed under the heading Homogeneous Beams.

FACTOR OF SAFETY

A value that is taken for the ultimate strength of any material is an average value of a number of experiments made on the material. As it is impossible to get two samples of the same material exactly alike, so is it also impossible to get two samples with the same ultimate strength. It is therefore customary in design to avoid stressing a material up to its ultimate strength or even up to its elastic limit.

The *factor of safety*, or, as it is sometimes called, the *safety factor*, is the ratio of the ultimate strength of the material to the load that, under usual conditions, the material is called on to carry. Suppose that the load required to rupture a piece of steel is 5,000 lb., and that the load it is called on to carry is 1,000 lb.; then, the factor of safety may be obtained by dividing the 5,000 lb. by the 1,000 lb. Thus, $5{,}000 \div 1{,}000 = 5$, which is the factor of safety of this material.

Another factor to be considered in selecting materials is *deterioration*, which is due to various causes. In metals, there is corrosion on account of moisture and gases in the atmosphere.

Wood is subject to decay from either dry or wet rot, caused by local conditions; it may, like iron and steel, be

subjected to *fatigue*, produced by constant stress due to the load it may have to sustain.

The preceding reasons are sufficient to require the factors now adopted by conservative constructors in all engineering work. These factors are 3 to 5 for structural steel, and 6 to 10 for cast iron.

Stone and brick are very unreliable and a high factor of safety should therefore be used. Usually, the factors employed are 10 for compressive stresses, 15 for tensile stresses, and from 10 to 20 for bending stresses. Some engineers, however, do not use such high factors. Stonework or brickwork is even more unreliable than stone or brick themselves, but the same factors are often employed. Usually, however, the strengths of stonework and brickwork, instead of being given as ultimate strength, are arranged in tables to give the *safe*, or *allowable*, *stress* per square inch or per square foot. This value embodies its own factor of safety.

Concrete is another material that sometimes requires a high factor of safety. In reinforced-concrete work a factor varying from 4 to 6 is usually employed, although some engineers use a higher one.

Ordinances in various cities throughout the country govern the allowable load that building materials should carry.

A structure withstanding shocks or containing rapidly vibrating machinery is more liable to fail than one in which the loads are quiet, even if the latter has the greater loads. Therefore, in designing a structure to withstand vibrating loads or shocks, the designer, to make the structure safe, uses a larger factor of safety. Usually, this factor averages from two to three times the ordinary factors, except those for timber, which need not as a rule be increased.

The average ultimate strength of various metals and woods are given in the accompanying tables.

AVERAGE ULTIMATE STRENGTHS OF METALS, IN POUNDS PER SQUARE INCH

	Compression	Tension	Elastic Limit	Shearing	Modulus of Rupture	Modulus of Elasticity
Aluminum:						
Aluminum, commercial	12,000	15,000	6,500	12,000		11,000,000
Aluminum, nickel		40,000	22,000			
Brass, Bronze, and Copper:						
Brass, cast	(30,000)	24,000	6,000	36,000	20,000	9,000,000
Brass wire, annealed (softened by reheating)		50,000				
Brass wire, unannealed		80,000	16,000			14,000,000
Bronze, aluminum	120,000	75,000				
Bronze, gun metal	(20,000)	32,000	10,000		53,000	10,000,000
Bronze, manganese	120,000	60,000	30,000			
Bronze, phosphor		50,000	24,000			14,000,000
Bronze, Tobin		66,000	40,000			4,500,000
Copper, bolts	30,000	30,000				
Copper, cast	(40,000)	24,000	6,000	30,000	22,000	10,000,000
Copper wire, annealed (softened by reheating)		36,000				15,000,000
Copper wire, unannealed		60,000	10,000			18,000,000

Cast and Wrought Iron:						
Iron, cast	80,000	15,000	6,000	18,000	30,000	12,000,000
Iron chains		35,000				
Iron, corrugated					40,000	
Iron wire, annealed (softened by reheating)		60,000				15,000,000
Iron wire, unannealed		80,000	27,000			25,000,000
Iron, wrought, shapes	46,000	48,000	26,000	40,000	44,000	27,000,000
Iron, wrought, rerolled bars	48,000	50,000	27,000	40,000	48,000	26,000,000
Lead:						
Lead, cast		2,000	1,000			1,000,000
Lead pipe		1,600				
Cast and Structural Steel:						
Steel, castings	70,000	70,000	40,000	60,000	70,000	30,000,000
Steel, structural, soft	56,000	56,000	30,000	48,000	54,000	29,000,000
Steel, structural, medium	64,000	64,000	33,000	50,000	60,000	29,000,000
Steel wire, annealed (softened by reheating)		80,000	40,000			29,000,000
Steel wire, unannealed		120,000	60,000			30,000,000
Steel wire, crucible		180,000	80,000			30,000,000
Steel wire, for suspension bridges		200,000	90,000			30,000,000
Steel wire, special tempered		300,000				
Tin and Zinc:						
Tin, cast	(6,000)	3,500	1,800		4,000	4,000,000
Zinc, cast	(20,000)	5,000	4,000		7,000	13,000,000

NOTE.—Compression values enclosed in parentheses indicate loads producing 10% reduction in original lengths.

AVERAGE ULTIMATE STRENGTHS OF WOODS, IN POUNDS PER SQUARE INCH

Kind of Timber	Tension		Compression			Transverse		Shearing	
	With Grain	Across Grain	With Grain		Across Grain	Extreme Fiber Stress	Modulus of Elasticity	With Grain	Across Grain
			End Bearing	Columns Under 10 or 15 Diameters					
White oak	12,000	2,000	7,000	5,000	2,000	7,000	1,500,000	800	4,000
White pine	7,000	500	5,500	3,500	700	4,000	1,000,000	400	2,000
Southern long-leaf or Georgia pine	12,000	600	7,000	5,000	1,400	7,000	1,500,000	600	5,000
Douglas fir	8,000		5,700	4,500	800	5,000	1,400,000	500	
Short-leaf yellow pine	9,000	500	6,000	4,500	1,000	6,000	1,200,000	400	4,000
Red pine (Norway pine)	8,000	500	5,000	4,000	800	5,000	1,130,000		
Spruce and Eastern fir	8,000	500	6,000	4,000	700	4,000	1,200,000	400	3,000
Hemlock	6,000			4,000	600	3,500	900,000	350	2,500
Cypress	6,000		5,000	4,000	700	5,000	900,000		
Cedar	7,000		5,500	3,500	700	4,000	700,000	400	1,500
Chestnut	8,500			4,000	900	5,000	1,000,000	600	2,000
California redwood	7,000			4,000	600	4,500	700,000	400	
California spruce				4,000		5,000	1,200,000		
Factor of safety	10	10	5	5	4	6	2	4	4

HOMOGENEOUS BEAMS

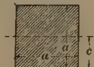
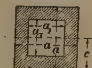
PROPERTIES OF SECTIONS

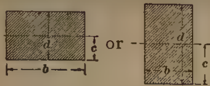
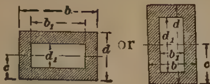
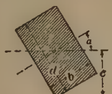
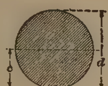
MOMENT OF INERTIA

The strength and the stiffness of a beam depend on various factors. For instance, the load that a beam can bear depends on the material of the beam, on the manner in which the load is applied, and on the length and cross-section of the beam. As to the area of the cross-section, the strength does not depend on that area itself, because, as every-day experience shows, a plank used as a beam will sustain a greater load when placed edgewise than when placed on its broad side, although it has the same area in both cases. The strength of a beam depends on the manner in which the area is distributed or disposed with respect to the neutral axis of the cross-section. The effect of the cross-section is measured by a quantity that depends on such disposition or distribution of area, and is called the *moment of inertia* of the cross-section with respect to the neutral axis. The moment of inertia of a plane figure has really nothing to do with the property of matter known as inertia, and as here used it is simply the name for a certain constant depending on the shape of the cross-section of a beam. It is not a moment as the word is generally understood, and it might have been called anything so far as the structural engineer is concerned, but it has received its name from some relations in higher mathematics where it is derived. This quantity must be considered simply as a constant that has been found suitable for use in certain formulas for the design of beams, which will be explained later.

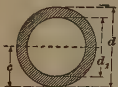
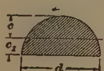
The following table, entitled Elements of Usual Sections, contains, in the column marked I, the moments of inertia of the shapes of beam sections commonly used in practice. The axis in each case passes through the center of gravity, and is designated *neutral axis* in the table for reasons that will be explained later. The square may be regarded as a

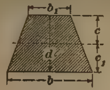
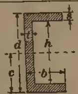
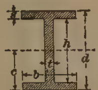
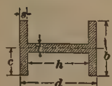
ELEMENTS OF USUAL SECTIONS

Section	Area of Section A	Distance From Neutral Axis to Extremities of Section c and c_1	Moment of Inertia About Neutral Axis I	Section Modulus $S = \frac{I}{c}$	Radius of Gyration $r = \sqrt{\frac{I}{A}}$
	a^2	$c = \frac{a}{2}$	$\frac{a^4}{12}$	$\frac{a^3}{6}$	$\frac{a}{\sqrt{12}} = .289\ a$
	$a^2 - a_1^2$	$c = \frac{a}{2}$	$\frac{a^4 - a_1^4}{12}$	$\frac{a^4 - a_1^4}{6\ a}$	$\sqrt{\frac{a^2 + a_1^2}{12}}$
	a^2	$c = \frac{a}{\sqrt{2}} = .707\ a$	$\frac{a^4}{12}$	$\frac{a^3}{6\sqrt{2}} = .118\ a^3$	$\frac{a}{\sqrt{12}} = .289\ a$
	$\frac{b\ d}{2}$	$c = \frac{2\ d}{3}$ $c = d - c_1$ $c_1 = \frac{d}{3}$ $c_1 = d - c$	$\frac{b\ d^3}{36}$	$\frac{b\ d^2}{24}$	$\frac{d}{\sqrt{18}} = .236\ d$

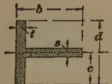
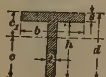
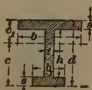
	$b\,d$	$c = \frac{d}{2}$	$\frac{b\,d^3}{12}$	$\frac{b\,d^2}{6}$	$\frac{d}{\sqrt{12}} = .289\,d$
	$b\,d - b_1\,d_1$	$c = \frac{d}{2}$	$\frac{b\,d^3 - b_1\,d_1^3}{12}$	$\frac{b\,d^3 - b_1\,d_1^3}{6\,d}$	$\sqrt{\frac{b\,d^3 - b_1\,d_1^3}{12(b\,d - b_1\,d_1)}}$
	$b\,d$	$c = \frac{b\,d}{\sqrt{b^2 + d^2}}$	$\frac{b^3\,d^2}{6\,(b^2 + d^2)}$	$\frac{b^2\,d^2}{6\sqrt{b^2 + d^2}}$	$\frac{b\,d}{\sqrt{6(b^2 + d^2)}}$
	$b\,d$	$c = \frac{d\cos\alpha + b\sin\alpha}{2}$	$\frac{b\,d}{12}(d^2\cos^2\alpha + b^2\sin^2\alpha)$	$\frac{b\,d}{6}\left(\frac{d^2\cos^2\alpha + b^2\sin^2\alpha}{d\cos\alpha + b\sin\alpha}\right)$	$\sqrt{\frac{d^2\cos^2\alpha + b^2\sin^2\alpha}{12}}$
	$\frac{\pi\,d^2}{4} = .785\,d^2$	$c = \frac{d}{2}$	$\frac{\pi\,d^4}{64} = .049\,d^4$	$\frac{\pi\,d^3}{32} = .098\,d^3$	$\frac{d}{4}$

ELEMENTS OF USUAL SECTIONS—(*Continued*)

Section	Area of Section A	Distance From Neutral Axis to Extremities of Section c and c_1	Moment of Inertia About Neutral Axis I	Section Modulus $S = \frac{I}{c}$	Radius of Gyration $r = \sqrt{\frac{I}{A}}$
	$\frac{\pi(d^2 - d_1^2)}{4} = .785(d^2 - d_1^2)$	$c = \frac{d}{2}$	$\frac{\pi(d^4 - d_1^4)}{64} = .049(d^4 - d_1^4)$	$\frac{\pi}{32}\left(\frac{d^4 - d_1^4}{d}\right) = .098\left(\frac{d^4 - d_1^4}{d}\right)$	$\frac{\sqrt{d^2 + d_1^2}}{4}$
	$\frac{\pi d^2}{8} = .393 d^2$	$c_1 = \frac{2d}{3\pi} = .212 d$ $c = \frac{(3\pi - 4)d}{6\pi} = .288 d$	$\frac{9\pi^2 - 64}{1{,}152\pi} d^4 = .007 d^4$	$\frac{9\pi^2 - 64}{192(3\pi - 4)} d^3 = .024 d^3$	$\frac{\sqrt{9\pi^2 - 64}}{12\pi} d = .132 d$
	$\frac{\pi b d}{4} = .785 b d$	$c = \frac{d}{2}$	$\frac{\pi b d^3}{64} = .049 b d^3$	$\frac{\pi b d^2}{32} = .098 b d^2$	$\frac{d}{4}$

	$\frac{b+b_1}{2} \times d$	$c_1 = \frac{b + 2b_1}{b+b_1} \times \frac{d}{3}$ $c = \frac{b_1 + 2b}{b+b_1} \times \frac{d}{3}$	$\frac{b^2 + 4bb_1 + b_1^2}{36(b+b_1)}$ $\times d^3$	$\frac{b^2 + 4bb_1 + b_1^2}{12(b_1+2b)}$ $\times d^2$	$\frac{d}{6(b+b_1)} \sqrt{2(b^2 + 4bb_1 + b_1^2)}$
 or 	bd $-h(b-t)$	$c = \frac{d}{2}$	$\frac{bd^3 - h^3}{12}$ $\times \frac{(b-t)}{12}$	$\frac{bd^3 - h^3(b-t)}{6d}$	$\sqrt{\frac{bd^3 - h^3(b-t)}{12[bd - h(b-t)]}}$
	bd $-h(b-t)$	$c - \frac{b}{2}$	$\frac{2sb^3 + ht^3}{12}$	$\frac{2sb^3 + ht^3}{6b}$	$\sqrt{\frac{2sb^3 + ht^3}{12[bd - h(b-t)]}}$
	bd $-h(b-t)$	$c_1 = \frac{2b^2s + ht^2}{2A}$ $c = b - c_1$	$\frac{2sb^3 + ht^3}{3}$ $-Ac_1^2$	$\frac{I}{b-c_1}$	$\sqrt{\frac{I}{A}}$

ELEMENTS OF USUAL SECTIONS—(*Continued*)

Section	Area of Section A	Distance From Neutral Axis to Extremities of Section c and c_1	Moment of Inertia About Neutral Axis I	Section Modulus $S = \frac{I}{c}$	Radius of Gyration $r = \sqrt{\frac{I}{A}}$
 or 	$t\,d + s(b - t)$	$c = \frac{d}{2}$	$\frac{t\,d^3 + s^3}{12} \times \frac{(b - t)}{12}$	$\frac{t\,d^3 + s^3(b - t)}{6\,d}$	$\sqrt{\frac{t\,d^3 + s^3(b - t)}{12[t\,d + s(b - t)]}}$
	$b\,s + h\,t$	$c_1 = \frac{d^2 t + s^2(b - t)}{2A}$ $c = d - c_1$	$\frac{t\,c^3 + b\,c_1^3}{3} - \frac{(b - t)(c_1 - s)^3}{3}$	$\frac{I}{d - c_1}$	$\sqrt{\frac{t\,c^3 + b\,c_1^3 - (b - t)(c_1 - s)^3}{3(b\,s + h\,t)}}$
	$b\,s + h\,t + b_1\,s$	$c_1 = \frac{t\,d^2 + s^2(b - t)}{2A} + \frac{s(b_1 - t)(2d - s)}{2A}$ $c = d - c_1$	$\left[b c_1^3 + b_1 c^3 - (b - t)(c_1 - s)^3 - (b_1 - t)(c - s)^3\right] \div 3$	$\frac{I}{d - c_1}$	$\left[\frac{b\,c_1^3 + b_1\,c^3 - (b - t)(c_1 - s)^3}{3(b\,s + h\,t + b_1\,s)} - \frac{(b_1 - t)(c - s)^3}{3(b\,s + h\,t + b_1\,s)}\right]^{\frac{1}{2}}$

particular case of a rectangle whose base b and altitude d are equal. The value of the area A is given in the second column. The distance c, given in the third column, is the distance of the most remote part of the figure from the neutral axis. The section modulus and radius of gyration, the character of which will be explained later, are given in the last two columns.

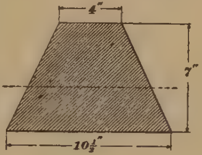


FIG. 1

EXAMPLE.—What is the area, the distance from the neutral axis to the extremities of the section, and the moment of inertia about the neutral axis of the section illustrated in Fig. 1?

SOLUTION.—Referring to the table, it will be seen that $b_1 = 4$ in., $b = 10\frac{1}{2}$ in., and $d = 7$ in. Therefore, the area is

$$\frac{b+b_1}{2} \times d = \frac{10.5+4 \times 7}{2} = 50.75 \text{ sq. in.}$$

Likewise, the distance from the neutral axis to the longer parallel side is

$$c_1 = \frac{b+2b_1}{b+b_1} \times \frac{d}{3} = \frac{10.5+2 \times 4}{10.5 \times 4} \times \frac{7}{3} = 2.977 \text{ in.}$$

The distance c, which is the distance of the neutral axis from the shorter parallel side, is

$$c = \frac{b_1+2b}{b+b_1} \times \frac{d}{3} = \frac{4+2 \times 10.5}{10.5+4} \times \frac{7}{3} = 4.023 \text{ in.}$$

As a check, $c_1 + c$ should equal d, or 7 in.; thus $2.977 + 4.023 = 7.000$ in., which indicates that the preceding solution is correct.

The moment of inertia is found by the formula

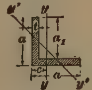
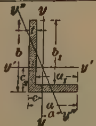
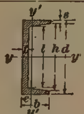
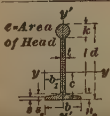
$$I = \frac{b^2+4bb_1+b_1^2}{36\ (b+b_1)} \times d^3$$

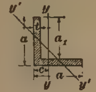
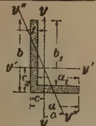
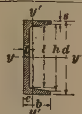
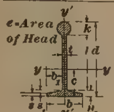
Substituting values for the letters in this formula,

$$I = \frac{10.5^2+4 \times 10.5 \times 4+4^2}{36 \times (10.5+4)} \times 7^3 = 193.348$$

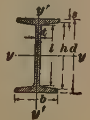
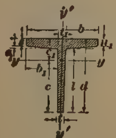
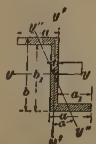
In general, if one of the values of c and c_1 is found by means of the formula, the other may be found by subtracting the

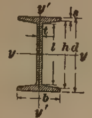
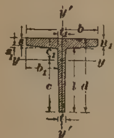
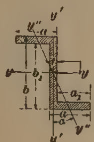
VALUES FOR STANDARD ROLLED SECTIONS

Section	Area. A	Distance From Neutral Axis to Back of Section, c and c_1	Moment of Inertia, Axis y–y I
	$t(2a-t)$	$c=\dfrac{a^2+at-t^2}{2(2a-t)}$	$\dfrac{t(a-c)^3+ac^3-(a-t)(c-t)^3}{3}$
	$t(a+b-t)$	$c=\dfrac{t(2a_1+b)+a_1^2}{2(a_1+b)}$ $c_2=\dfrac{t(2b_1+a)+b_1^2}{2(b_1+a)}$	$\dfrac{t(a-c)^3+bc^3-(b-t)(c-t)^3}{3}$
	$td+2s(b-t)+\dfrac{(b-t)^2}{6}$	$c=\left[b^2s+\dfrac{ht^2}{2}+\dfrac{(b-t)^2(b+2t)}{18}\right]\div A$	$\dfrac{bd^3}{12}-\dfrac{h^4-l^4}{16}$
	$e+t(d-k)+(b-t)\left(s+\dfrac{s_1}{2}\right)$	$c=\dfrac{e(2d-k)+t(d-k)^2}{2A}+\dfrac{(b-t)\left(s^2+ss_1+\dfrac{s^2}{3}\right)}{2A}$	$e\left[\dfrac{k^2}{16}+\left(d-\dfrac{2s+k}{2}\right)^2\right]+\dfrac{t(l+s_1)^3}{3}+\dfrac{b_1s_1^3+2bs^3}{6}-A(c-s)^2$

Section	Moment of Inertia, Axis y'–y' I'	Moment of Inertia, Axis y''–y''. I''	Tangent of Twice the Angle a
	$\dfrac{2c^4 - 2(c-t)^4 + t\left[a - \left(2c - \frac{t}{2}\right)\right]^3}{3}$		
	$\dfrac{t(b-c_2)^3 + ac_2^3 - (a-t)(c_2-t)^3}{3}$	$\dfrac{I\cos^2 a - I'\sin^2 a}{\cos 2a}$	$-\dfrac{[(2c-t)b(b-2c_2)}{2(I'-I)} + \dfrac{(2c_2-t)(a-t)(a+t-2c)]t}{2(I'-I)}$
	$\frac{1}{3}\left(2sb^3 + lt^3 + \dfrac{b^4-t^4}{12}\right) - Ac^2$		
	$\dfrac{ek^2}{16} + \dfrac{t^3(l+s_1)+sb^3}{12} + \dfrac{s_1 b_1[2b_1^2 + (2b_1+3t)^2]}{36}$		

VALUES FOR STANDARD ROLLED SECTIONS—(*Continued*)

Section	Area. A	Distance From Neutral Axis to Back of Section. c and c_1	Moment of Inertia, Axis y–y I
	$td + 2s(b - t) + \frac{(b-t)^2}{12}$		$\frac{bd^3}{12} - \frac{h^4 - l^4}{8}$
	$\frac{l(t + t_1)}{2} + n_1 t_1 + b_1(s + n_1)$	$c_1 = \frac{3s^2(b - t_1) + 2b_1 s_1(s_1 + 3s)}{6A} + \frac{3t_1 d^2 - l(t_1 - t)(3d - l)}{6A}$	$\frac{l^3(3t + t_1) + 4bn_1^3 - 2b_1 s_1^3}{12} - A(c_1 - n_1)^2$
	$[b + 2(a - t)]t$		$\frac{ab^3 - a_1(b - 2t)^3}{12}$

Section	Moment of Inertia, Axis $y'-y'$ I'	Moment of Inertia, Axis $y''-y''$. I''	Tangent of Twice the Angle α
	$\frac{b^3 s}{6} + \frac{l t^3}{12} + \frac{b^4 - t^4}{288}$		
	$\frac{s b^3 + s_1 t_1^3 + l t^3}{12} + \frac{s_1 b_1[2 b_1^2 + (2 b_1 + 3 t_1)^2]}{36} + \frac{l(t_1 - t)[(t_1 - t)^2 + 2(t_1 + 2 t)^2]}{144}$		
	$\frac{t}{12}[(a + a_1)^3 + b_1 t^2]$	$\frac{I' \cos^2 \alpha - I \sin^2 \alpha}{\cos 2 \alpha}$	$-\frac{(b t - t^2)(a^2 - a t)}{I - I'}$

known one from d. Thus, after having calculated c_1 in this example, c would be equal to $d-c_1=7-2.977=4.023$ in., which value corresponds to the one found by means of the formula.

Many of the beams used in building construction are made of steel. These steel beams are rolled with various cross-sectional shapes, from which they derive their names. These shapes are *standard;* that is, they are rolled to conform to certain sizes that have been adopted by many of the large steel companies. In the first column of the accompanying table, entitled Values for Standard Rolled Sections, are shown the cross-sections of the various structural shapes. The first section shown is known as an *angle with equal legs;* the second, as an *angle with unequal legs;* the third, as a *channel;* the fourth, as a *bulb beam;* while the fifth, sixth, and seventh sections are known, respectively, as an **I** *beam*, a **T** *bar*, and a **Z** *bar*.

In this table is given the moment of inertia for each section about at least two axes, both of which pass through the center of gravity of the section; that is, the moments of inertia are given always with respect to neutral axes. The moments of inertia with respect to different neutral axes are, in general, different, and as a rule there is one neutral axis about which the moment of inertia is less than it is about any other. With the channel, bulb beam, **I** beam, and **T** bar, the smallest moment of inertia is about the axis $y'-y'$, or the vertical axis. However, with both styles of angles and with the **Z** bar, there is another moment of inertia about a neutral axis, not horizontal nor vertical, that is smaller than the moment of inertia about any other neutral axis; that is, in certain work, more particularly in the design of columns, it is necessary to know about what neutral axis the moment of inertia will be smallest. It is also necessary to know what this moment of inertia will be. Therefore, in the section of the angle with equal legs, the section of the angle with unequal legs, and the **Z**-bar section, the moment of inertia is also given for the oblique axis giving the smallest moment of inertia. The position of this neutral axis is found by higher mathematics. With the angle with equal legs, this

oblique neutral axis $y'-y'$ makes an angle of 45° with the horizontal. With the angle section with unequal legs and with the **Z**-bar section, this oblique neutral axis $y''-y''$ makes an angle α with a vertical line. Instead of giving this angle direct, it is more convenient to give a value for the trigonometric tangent of twice the angle, and this is done in the last column of the table.

The formulas given in the table are long and difficult to use. The values of the properties of various structural-steel sections have therefore been calculated for all the standard sizes of angles, channels, etc. that are ordinarily manufactured. The tables containing these values will be found on page 147, where their use will be explained.

In the two tables just given, the moment of inertia is taken about the neutral axis. Sometimes, however, it is desirable to find the moment of inertia of the section about some axis other than the neutral axis. This may be accomplished as follows:

Let A represent the area of any section; I, the moment of inertia with respect to an axis through the center of gravity of the section; I_x, the moment of inertia with respect to any other axis parallel to the former; and p, the distance between axes. Then,

$$I_x = I + Ap^2$$

EXAMPLE.—Determine the moment of inertia of a triangle with respect to its base (see fourth section of the table entitled Elements of Usual Sections).

SOLUTION.—The distance between the base and the neutral axis c_1 is $\frac{1}{3}d$, the area is $\frac{1}{2}bd$, and $I = \frac{1}{36}bd^3$. Then, by the preceding formula,

$$I_x = \frac{1}{36}bd^3 + \frac{1}{2}bd \times \left(\frac{1}{3}d\right)^2 = \frac{bd^3}{36} + \frac{bd^3}{18} = \frac{bd^3}{12}$$

Many sections may be regarded as built up of simpler parts; they are called *compound sections*. For example, a hollow square consists of a large square and a smaller square; the **T** shown in the table entitled Elements of Usual Sections consists of two slender rectangles, one horizontal and one vertical.

The moment of inertia of a compound section with respect to any axis may be found by adding, algebraically, the moments of inertia, with respect to the same axis, of the component parts of the figure.

EXAMPLE.—Find the moment of inertia of the Z bars shown in Fig. 2, about the neutral axis $X'X$, the dimensions being as shown.

SOLUTION.—The figure may be divided into the three rectangles $efgh$, $e'f'g'h'$, and $jgig'$. The moment of inertia of $efgh$ about an axis through its center of gravity and parallel to $X'X$ is $\frac{1}{12}\,a't^3$; that of $e'f'g'h'$ about an axis through its center of gravity and parallel to $X'X$ is also $\frac{1}{12}\,a't^3$. The distance between this axis and the $X'X$ axis is $\frac{1}{2}\,(b-t)$. The moment of inertia of the rectangle $efgh$ and also of the rectangle $e'f'g'h'$ about the axis $X'X$ is then, $\frac{1}{12}\,a't^3+a't\,[\frac{1}{2}\,(b-t)]^2$. The moment of inertia of the rectangle $jgig'$ about the axis $X'X$ is $\frac{1}{12}\,tb^3$. The moment of inertia of the entire figure is, therefore,

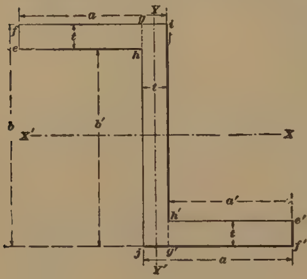


FIG. 2

$$2\left\{\frac{1}{12}\,a't^3+a't\,[\tfrac{1}{2}\,(b-t)]^2\right\}+\frac{1}{12}\,tb^3$$

Expanding and reducing this expression,

$$I=\frac{ab^3-a'\,(b-2t)^3}{12}$$

RADIUS OF GYRATION

The *radius of gyration* of a section with respect to an axis is a quantity whose square multiplied by the area of the section is equal to the moment of inertia of the section with respect to the same axis. If r and I denote, respectively,

the radius of gyration and the moment of inertia of a section, and A the area in square inches, then,

$$Ar^2 = I,$$

whence

$$r = \sqrt{\frac{I}{A}}$$

The last column of the table entitled Elements of Usual Sections gives radii of gyration corresponding to the moments of inertia given in the fourth column.

The radius of gyration of a section, or figure, may be found directly from its moment of inertia by means of the formula just given. For example, the radii of gyration for the rectangle and the hollow square in the table just referred to are found as follows:

For the rectangle,

$$r = \sqrt{\tfrac{1}{12}\, bd^3 \div bd} = \frac{d}{\sqrt{12}} = .289d$$

For the hollow square,

$$r = \sqrt{\tfrac{1}{12}\,(a^4 - a_1^4) \div (a^2 - a_1^2)} = \sqrt{\frac{a^2 + a_1^2}{12}}$$

NEUTRAL AXIS

If, in a cantilever loaded as shown in Fig. 3, any point x on the center line ab is taken as a center of moments, and a

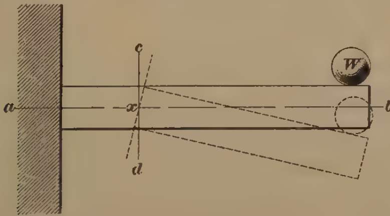


FIG. 3

section made by a vertical plane cd through this center is considered, it will be evident that the moment of the force due to the downward thrust of the load tends to turn the end of the beam to the right of cd around the center x. The

measure of this tendency is the product of the weight W multiplied by its distance from cd, and, since this tendency is the moment of a force that tends to bend the beam, it is called the *bending moment*.

A further inspection of Fig. 3 will show that through the bending action of the load the upper part of the beam is subjected to a tensile stress, while the lower part is subjected to a compressive stress.

Fig. 3 also shows that the greater the distance of the particles in the assumed section above or below the center x, the greater will be their displacement. In other words

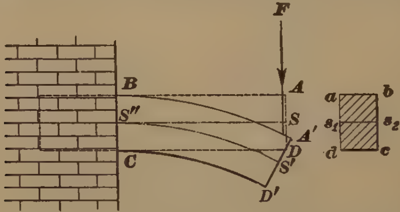


FIG. 4

since the stress in a loaded body is directly proportional to the strain, or relative displacement of the particles, it follows that the stress in a particle of any section is proportional to its distance from the center line ab, and that the greatest stress is in the particles composing the upper and lower surfaces of the beam.

In Fig. 4, let $ABCD$ represent a cantilever. A force F acts on it at its extremity A. This force will tend to bend the beam into something like the shape shown by $A'BCD'$. It is evident from what has preceded that the effect of the force F in bending the beam is to lengthen the upper fibers and to shorten the lower ones. Hence, the upper part $A'B$ is now longer than it was before the force was applied; that is, $A'B$ is longer than AB. It is also evident that $D'C$ is shorter than DC. Further consideration will show that there must be a fiber, as SS'', that is neither lengthened nor short-

ened when the beam is bent; that is, $SS'' = S'S''$. A line drawn through this fiber SS'' when the beam is straight is called the *neutral line*, and the horizontal plane in which this line lies is known as the *neutral surface* of the beam. The neutral line corresponds to the center line ab, Fig. 3, on which the center of moments x was taken.

The line s_1s_2, which passes through any beam section, as $abcd$, Fig. 4, at the neutral line, perpendicular to the direction in which the beam bends, is called the *neutral axis*. It is shown in works on mechanics that *the neutral axis always passes through the center of gravity of the cross-section of a beam made of uniform material.*

Thus, in a beam, it is evident that the neutral axis of any section is a horizontal line at which the fibers composing the beam are neither stretched nor compressed. The axis is horizontal because it is at right angles to the direction of the load, which, in structural problems, usually acts downwards. It so happens that in a beam made of one material, this line will pass through the center of gravity of the section. It is for this reason that the neutral axis has been spoken of as a line through the center of gravity of a section. It happens, however, that in many beams made of two materials, as, for example, concrete and steel, the neutral axis—that is, the horizontal line along which the particles of the beam are neither stretched nor compressed—does not pass through the center of gravity of the section.

SECTION MODULUS

The *modulus of a section* of a beam is equal to the moment of inertia of the section about its neutral axis divided by the distance from the outermost fiber in that section to the neutral axis. This fiber may be either above or below the neutral axis, and it is immaterial whether the beam is a simple beam, a cantilever, or any other kind of beam. If the moment of inertia of a section about its neutral axis is represented by I, the distance from the neutral axis to the outermost fiber by c, and the section modulus by S, then

$$S = \frac{I}{c}$$

EXAMPLE.—Prove the formula for the modulus of a section as shown in Fig. 5 to be correct as given in the table on page 133.

SOLUTION.—According to the table, the moment of inertia of the section is $\frac{b^2+4bb_1+b_1^2}{36\,(b+b_1)} \times d^3$. It is a question whether c or c_1 is the longer. However, on examining the values given for these two quantities in the table, it is seen that they are the same, with the exception of the first part of the numerator. For c_1, this part of the numerator is $b+2b_1$, while for c it is b_1+2b. In Fig. 5, b is greater than b_1; therefore, b_1+2b is greater than $b+2b_1$, and it follows that c is greater than c_1. Accordingly, the formula is

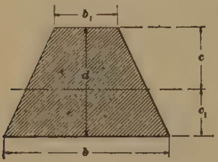


FIG. 5

$$S=\frac{I}{c}=\frac{b^2+4bb_1+b_1^2}{36\,(b+b_1)}\,d^3 \div \frac{b_1+2b}{b+b_1}\times\frac{d}{3}$$

$$=\frac{(b^2+4bb_1+b_1^2)\,d^3\,(b+b_1)\,3}{36\,(b+b_1)\,(b_1+2b)\,d}=\frac{b^2+4bb_1+b_1^2}{12\,(b_1+2b)}\times d^2,$$

which is the same as the one given in the table.

PROPERTIES OF ROLLED-STEEL SHAPES

The table of values for standard rolled sections already given shows various cross-sections of steel beams. Each one of these beams is made in various sizes and weights. On account of the confusion that ensued from using beams of different sizes from different manufacturers, the latter agreed to adopt standard sizes, which are known as *American standard*. Therefore, knowing the dimensions of the section of any beam that is rolled according to the American standard, the properties of the section may be found by using the table already given. This table, however, while quite accurate, is inconvenient to use, because the formulas it contains are long and complicated. The following tables of properties of standard sections are therefore given to diminish the labor required in beam design.

The first of the following tables gives the properties of standard **I** beams. The bevel on the inside of the flange is always made 1 to 6. The same bevel is also used for channels.

The tables dealing with **I** beams and channels give two columns dealing with the deflection, or sag, of beams. The use of these columns will be explained on page 174. The other values given in the tables will be readily understood.

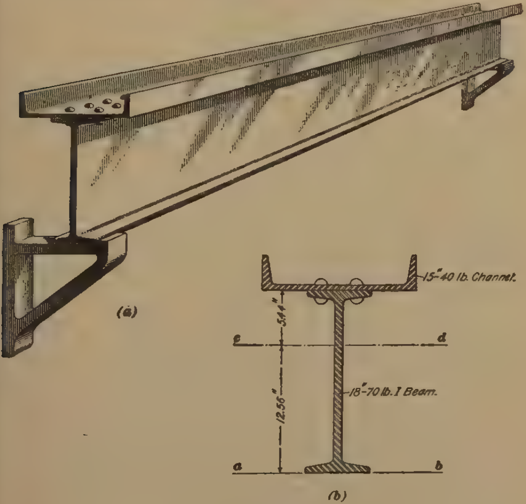


FIG. 6

Frequently, steel beams are built up of several steel sections riveted together. To find the strength of a beam of this kind it is first necessary to determine the moment of inertia of the entire section about its neutral axis. This is done by the methods previously described on page 97, but it may probably be best illustrated by an actual problem.

PROPERTIES OF STANDARD I BEAMS

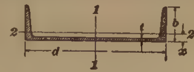
1	2	3	4	5	6	7	8	9	10	11	12	13
Depth of Beam. Inches	Weight per Foot Pounds	Area of Section Square Inches	Thickness of Web Inches	Width of Flange Inches	Moment of Inertia, Axis 1–1. Inches^4	Section Modulus, Axis 1–1. Inches^3	Radius of Gyration Axis 1–1. Inches	Moment of Inertia, Axis 2–2. Inches^4	Radius of Gyration, Axis 2–2. Inches	Increase of Thickness of Web for Each Pound Increase in Weight. Inches	Coefficient of Deflection	
											Uniform Load	Center Load
d		A	t	b	I	S	r	I'	r'	f	N	N'
3	5.50	1.63	.17	2.33	2.5	1.7	1.23	.46	.53	.098	.00031253	.00050006
3	6.50	1.91	.26	2.42	2.7	1.8	1.19	.53	.52		.00028827	.00046124
3	7.50	2.21	.36	2.52	2.9	1.9	1.15	.60	.52		.00026644	.00042630
4	7.50	2.21	.19	2.66	6.0	3.0	1.64	.77	.59	.074	.00013009	.00020815
4	8.50	2.50	.26	2.73	6.4	3.2	1.59	.85	.58		.00012209	.00019535
4	9.50	2.79	.34	2.81	6.7	3.4	1.54	.93	.58		.00011500	.00018400
4	10.50	3.09	.41	2.88	7.1	3.6	1.52	1.01	.57		.00010868	.00017389

5	9.75	2.87	.21	3.00	12.1	4.8	2.05	1.23	.65	.059	.00006417	.00010267
5	12.25	3.60	.36	3.15	13.6	5.4	1.94	1.45	.63		.00005698	.00009117
5	14.75	4.34	.50	3.29	15.1	6.1	1.87	1.70	.63		.00005122	.00008195
6	12.25	3.61	.23	3.33	21.8	7.3	2.46	1.85	.72	.049	.00003561	.00005698
6	14.75	4.34	.35	3.45	24.0	8.0	2.35	2.09	.69		.00003235	.00005177
6	17.25	5.07	.47	3.57	26.2	8.7	2.27	2.36	.68		.00002963	.00004741
7	15.00	4.42	.25	3.66	36.2	10.4	2.86	2.67	.78	.042	.00002142	.00003427
7	17.50	5.15	.35	3.76	39.2	11.2	2.76	2.94	.76		.00001980	.00003168
7	20.00	5.88	.46	3.87	42.2	12.1	2.68	3.24	.74		.00001839	.00002943
8	18.00	5.33	.27	4.00	56.9	14.2	3.27	3.78	.84	.037	.00001364	.00002183
8	20.25	5.96	.35	4.08	60.2	15.0	3.18	4.04	.82		.00001289	.00002062
8	22.75	6.69	.44	4.17	64.1	16.0	3.10	4.36	.81		.00001210	.00001936
8	25.25	7.43	.53	4.26	68.0	17.0	3.03	4.71	.80		.00001140	.00001825
9	21.50	6.31	.29	4.33	84.9	18.9	3.67	5.16	.90	.033	.00000914	.00001462
9	25.00	7.35	.41	4.45	91.9	20.4	3.54	5.65	.88		.00000844	.00001350
9	30.00	8.82	.57	4.61	101.9	22.6	3.40	6.42	.85		.00000762	.00001219
9	35.00	10.29	.73	4.77	111.8	24.8	3.30	7.31	.84		.00000694	.00001110
10	25.00	7.37	.31	4.66	122.1	24.4	4.07	6.89	.97	.029	.00000635	.00001017
10	30.00	8.82	.45	4.80	134.2	26.8	3.90	7.65	.93		.00000578	.00000925
10	35.00	1.29	.60	4.95	146.4	29.3	3.77	8.52	.91		.00000530	.00000848
10	40.00	11.76	.75	5.10	158.7	31.7	3.67	9.50	.90		.00000489	.00000782

PROPERTIES OF STANDARD I BEAMS—(*Continued*)

d		A	t	b	I	S	r	I'	r'	f	N	N'
12	31.50	9.26	.35	5.00	215.8	36.0	4.83	9.50	1.01	.025	.00000360	.00000575
12	35.00	10.29	.44	5.09	228.3	38.0	4.71	10.07	.99		.00000340	.00000544
12	40.00	11.76	.56	5.21	245.9	41.0	4.57	10.95	.96		.00000316	.00000505
15	42.00	12.48	.41	5.50	441.8	58.9	5.95	14.62	1.08	.020	.00000176	.00000281
15	45.00	13.24	.46	5.55	455.8	60.8	5.87	15.09	1.07		.00000170	.00000272
15	50.00	14.71	.56	5.65	483.4	64.5	5.73	16.04	1.04		.00000161	.00000257
15	55.00	16.18	.66	5.75	511.0	68.1	5.62	17.06	1.03		.00000152	.00000243
15	60.00	17.65	.76	5.84	538.6	71.8	5.52	18.17	1.01		.00000144	.00000231
18	55.00	15.93	.46	6.00	795.6	88.4	7.07	21.19	1.15	.016	.00000098	.00000156
18	60.00	17.65	.56	6.10	841.8	93.5	6.91	22.38	1.13		.00000092	.00000148
18	65.00	19.12	.64	6.18	881.5	97.9	6.79	23.47	1.11		.00000088	.00000141
18	70.00	20.59	.72	6.26	921.2	102.4	6.69	24.62	1.09		.00000084	.00000135
20	65.00	19.08	.50	6.25	1,169.5	117.0	7.83	27.86	1.21	.015	.00000066	.00000106
20	70.00	20.59	.58	6.33	1,219.8	122.0	7.70	29.04	1.19		.00000064	.00000102
20	75.00	22.06	.65	6.40	1,268.8	126.9	7.58	30.25	1.17		.00000061	.00000098
24	80.00	23.32	.50	7.00	2,087.2	173.9	9.46	42.86	1.36	.0123	.00000037	.00000060
24	85.00	25.00	.57	7.07	2,167.8	180.7	9.31	44.35	1.33		.00000036	.00000057
24	90.00	26.47	.63	7.13	2,238.4	186.5	9.20	45.70	1.31		.00000035	.00000056
24	95.00	27.94	.69	7.19	2,309.0	192.4	9.09	47.10	1.30		.00000034	.00000054
24	100.00	29.41	.75	7.25	2,379.6	198.3	8.99	48.55	1.28		.00000033	.00000052

PROPERTIES OF STANDARD CHANNELS



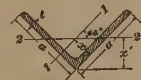
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Depth of Channel Inches	Weight per Foot Pounds	Area of Section Square Inches	Thickness of Web Inches	Width of Flange Inches	Moment of Inertia, Axis *1–1*. Inches⁴	Section Modulus, Axis *1–1*. Inches³	Radius of Gyration, Axis *1–1*. Inches	Moment of Inertia, Axis *2–2*. Inches⁴	Section Modulus, Axis *2–2*. Inches³	Radius of Gyration, Axis *2–2*. Inches	Distance of Center of Gravity From Outside of Web. Inches	Increase of Thickness of Web for Each Pound Increase in Weight. Inches	Coefficient of Deflection: Uniform Load	Coefficient of Deflection: Center Load
a		A	t	b	I	S	r	I'	S'	r'	x	f	N	N'
3	4.00	1.19	.17	1.41	1.6	1.1	1.17	.20	.21	.41	.44	.098	.0004743	.0007589
3	5.00	1.47	.26	1.50	1.8	1.2	1.12	.25	.24	.41	.44		.0004199	.0006718
3	6.00	1.76	.36	1.60	2.1	1.4	1.08	.31	.27	.42	.46		.0003751	.0006001
4	5.25	1.55	.18	1.58	3.8	1.9	1.56	.32	.29	.45	.46	.074	.0002046	.0003273
4	6.25	1.84	.25	1.65	4.2	2.1	1.51	.38	.32	.45	.46		.0001858	.0002973
4	7.25	2.13	.33	1.73	4.6	2.3	1.46	.44	.35	.46	.46		.0001698	.0002717

PROPERTIES OF STANDARD CHANNELS—(*Continued*)

a		A	t	b	I	S	r	I'	S'	r'	x	f	N	N'
5	6.50	1.93	.19	1.75	7.4	3.0	1.95	.48	.38	.50	.49	.059	.0001046	.0001674
5	9.00	2.65	.33	1.89	8.9	3.5	1.83	.64	.45	.49	.48		.0000875	.0001399
5	11.50	3.38	.48	2.04	10.4	4.2	1.75	.82	.54	.49	.51		.0000746	.0001193
6	8.00	2.38	.20	1.92	13.0	4.3	2.34	.70	.50	.54	.52	.049	.0000597	.0000855
6	10.50	3.09	.32	2.04	15.1	5.0	2.21	.88	.57	.53	.50		.0000513	.0000821
6	13.00	3.82	.44	2.16	17.3	5.8	2.13	1.07	.65	.53	.52		.0000448	.0000717
6	15.50	4.56	.56	2.28	19.5	6.5	2.07	1.28	.74	.53	.55		.0000397	.0000636
7	9.75	2.85	.21	2.09	21.1	6.0	2.72	.98	.63	.59	.55	.042	.0000368	.0000588
7	12.25	3.60	.32	2.20	24.2	6.9	2.59	1.19	.71	.57	.53		.0000321	.0000514
7	14.75	4.34	.42	2.30	27.2	7.8	2.50	1.40	.79	.57	.53		.0000286	.0000457
7	17.25	5.07	.53	2.41	30.2	8.6	2.44	1.62	.87	.56	.55		.0000257	.0000411
7	19.75	5.81	.63	2.51	33.2	9.5	2.39	1.85	.96	.56	.58		.0000234	.0000374
8	11.25	3.35	.22	2.26	32.3	8.1	3.10	1.33	.79	.63	.58	.037	.0000240	.0000384
8	13.75	4.04	.31	2.35	36.0	9.0	2.98	1.55	.87	.62	.56		.0000216	.0000345
8	16.25	4.78	.40	2.44	39.9	10.0	2.89	1.78	.95	.61	.56		.0000194	.0000311
8	18.75	5.51	.49	2.53	43.8	11.0	2.82	2.01	1.02	.60	.57		.0000177	.0000283
8	21.25	6.25	.58	2.62	47.8	11.9	2.76	2.25	1.11	.60	.59		.0000162	.0000260

9	13.25	3.89	.23	2.43	47.3	10.5	3.49	1.77	.97	.67	.61	.033	.0000164	.0000262
9	15.00	4.41	.29	2.49	50.9	11.3	3.40	1.95	1.03	.66	.59		.0000153	.0000244
9	20.00	5.88	.45	2.65	60.8	13.5	3.21	2.45	1.19	.65	.58		.0000128	.0000204
9	25.00	7.35	.61	2.81	70.7	15.7	3.10	2.98	1.36	.64	.62		.0000110	.0000176
10	15.00	4.46	.24	2.60	66.9	13.4	3.87	2.30	1.17	.72	.64	.029	.0000116	.0000186
10	20.00	5.88	.38	2.74	78.7	15.7	3.66	2.85	1.34	.70	.61		.0000099	.0000158
10	25.00	7.35	.53	2.89	91.0	18.2	3.52	3.40	1.50	.68	.62		.0000085	.0000136
10	30.00	8.82	.68	3.04	103.2	20.6	3.42	3.99	1.67	.67	.65		.0000075	.0000120
10	35.00	10.29	.82	3.18	115.5	23.1	3.35	4.66	1.87	.67	.69		.0000067	.0000107
12	20.50	6.03	.28	2.94	128.1	21.4	4.61	3.91	1.75	.81	.70	.025	.0000061	.0000097
12	25.00	7.35	.39	3.05	144.0	24.0	4.43	4.53	1.91	.78	.68		.0000054	.0000086
12	30.00	8.82	.51	3.17	161.6	26.9	4.28	5.21	2.09	.77	.68		.0000048	.0000077
12	35.00	10.29	.64	3.30	179.3	29.9	4.17	5.90	2.27	.76	.69		.0000043	.0000069
12	40.00	11.76	.76	3.42	196.9	32.8	4.09	6.63	2.46	.75	.72		.0000039	.0000063
15	33.00	9.90	.40	3.40	312.6	41.7	5.62	8.23	3.16	.91	.79	.020	.0000025	.0000040
15	35.00	10.29	.43	3.43	319.9	42.7	5.57	8.48	3.22	.91	.79		.0000024	.0000039
15	40.00	11.76	.52	3.52	347.5	46.3	5.44	9.39	3.43	.89	.78		.0000022	.0000036
15	45.00	13.24	.62	3.62	375.1	50.0	5.32	10.29	3.63	.88	.79		.0000021	.0000033
15	50.00	14.71	.72	3.72	402.7	53.7	5.23	11.22	3.85	.87	.80		.0000019	.0000031
15	55.00	16.18	.82	3.82	430.2	57.4	5.16	12.19	4.07	.87	.82		.0000018	.0000029

PROPERTIES OF STANDARD ANGLES HAVING EQUAL LEGS



1	2	3	4	5	6	7	8	9	10	11	12
Dimensions Inches	Thickness Inches	Weight per Foot Pounds	Area of Section Square Inches	Distance of Center of Gravity From Back of Leg. Inches	Moment of Inertia, Axis *1–1* Inches^4	Section Modulus, Axis *1–1* Inches^3	Radius of Gyration, Axis *1–1* Inches	Distance of Center of Gravity From External Apex. Inches	Least Moment of Inertia, Axis *2–2* Inches^4	Section Modulus, Axis *2–2* Inches^3	Least Radius of Gyration, Axis *2–2* Inches
$a \times a$	t		A	x	I	S	r	x'	I'	S'	r'
$\frac{3}{4} \times \frac{3}{4}$	$\frac{1}{8}$	.6	.18	.23	.009	.017	.22	.33	.004	.011	.14
$\frac{3}{4} \times \frac{3}{4}$	$\frac{3}{16}$	.9	.25	.25	.012	.024	.22	.36	.005	.014	.14
1×1	$\frac{1}{8}$	.8	.24	.30	.022	.031	.30	.42	.009	.021	.19
1×1	$\frac{3}{16}$	1.2	.34	.32	.030	.044	.30	.45	.013	.028	.19
1×1	$\frac{1}{4}$	1.5	.44	.34	.037	.056	.29	.48	.016	.034	.19

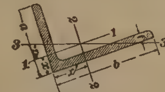
$1\frac{1}{4}\times1\frac{1}{4}$	$\frac{1}{8}$	1.1	.30	.36	.044	.049	.38	.51	.018	.035	.24
$1\frac{1}{4}\times1\frac{1}{4}$	$\frac{3}{16}$	1.5	.44	.38	.061	.071	.38	.54	.025	.047	.24
$1\frac{1}{4}\times1\frac{1}{4}$	$\frac{1}{4}$	2.0	.57	.40	.077	.091	.37	.57	.033	.057	.24
$1\frac{1}{4}\times1\frac{1}{4}$	$\frac{5}{16}$	2.4	.69	.42	.090	.109	.36	.60	.040	.066	.24
$1\frac{1}{2}\times1\frac{1}{2}$	$\frac{1}{8}$	1.3	.36	.42	.08	.072	.47	.60	.031	.053	.30
$1\frac{1}{2}\times1\frac{1}{2}$	$\frac{3}{16}$	1.8	.53	.44	.11	.104	.46	.63	.045	.072	.29
$1\frac{1}{2}\times1\frac{1}{2}$	$\frac{1}{4}$	2.4	.69	.47	.14	.134	.45	.66	.058	.088	.29
$1\frac{1}{2}\times1\frac{1}{2}$	$\frac{5}{16}$	2.9	.84	.49	.16	.162	.44	.69	.070	.101	.29
$1\frac{1}{2}\times1\frac{1}{2}$	$\frac{3}{8}$	3.4	.99	.51	.19	.188	.44	.72	.082	.114	.29
$1\frac{1}{2}\times1\frac{1}{2}$	$\frac{7}{16}$	3.9	1.13	.53	.21	.214	.43	.75	.094	.126	.29
$1\frac{3}{4}\times1\frac{3}{4}$	$\frac{3}{16}$	2.2	.63	.51	.18	.14	.54	.72	.073	.10	.34
$1\frac{3}{4}\times1\frac{3}{4}$	$\frac{1}{4}$	2.8	.82	.53	.23	.19	.53	.75	.094	.13	34
$1\frac{3}{4}\times1\frac{3}{4}$	$\frac{5}{16}$	3.4	1.00	.55	.27	.23	.52	.78	.113	.15	.34
$1\frac{3}{4}\times1\frac{3}{4}$	$\frac{3}{8}$	4.0	1.18	.57	.31	.26	.51	.81	.133	.16	.34
$1\frac{3}{4}\times1\frac{3}{4}$	$\frac{7}{16}$	4.6	1.34	.59	.35	.30	.51	.84	.152	.18	.34
$1\frac{3}{4}\times1\frac{3}{4}$	$\frac{1}{2}$	5.1	1.50	.61	.38	.33	.50	.87	.171	.20	.34
2×2	$\frac{3}{16}$	2.5	.72	.57	.27	.19	.62	.80	.11	.14	39
2×2	$\frac{1}{4}$	3.2	.94	.59	.35	.25	.61	.84	.14	.17	.39
2×2	$\frac{5}{16}$	4.0	1.16	.61	.42	.30	.60	.87	.17	.20	.39
2×2	$\frac{3}{8}$	4.7	1.36	.64	.48	.35	.59	.90	.20	.22	.39
2×2	$\frac{7}{16}$	5.3	1.56	.66	.54	.40	.59	.93	.23	.25	.38
2×2	$\frac{1}{2}$	6.0	1.75	.68	.59	.45	.58	.96	.26	.27	.38
$2\frac{1}{2}\times2\frac{1}{2}$	$\frac{3}{16}$	3.1	.91	.69	.55	.30	.78	.98	.22	.22	.49
$2\frac{1}{2}\times2\frac{1}{2}$	$\frac{1}{4}$	4.1	1.19	.72	.70	.39	.77	1.01	.29	.28	.49
$2\frac{1}{2}\times2\frac{1}{2}$	$\frac{5}{16}$	5.0	1.47	.74	.85	.48	.76	1.05	.35	.33	.49
$2\frac{1}{2}\times2\frac{1}{2}$	$\frac{3}{8}$	5.9	1.74	.76	.98	.57	.75	1.08	.41	.38	.48
$2\frac{1}{2}\times2\frac{1}{2}$	$\frac{7}{16}$	6.8	2.00	.78	1.11	.65	.75	1.11	.46	.42	.48
$2\frac{1}{2}\times2\frac{1}{2}$	$\frac{1}{2}$	7.7	2.25	.81	1.23	.72	.74	1.14	.52	.46	.48
$2\frac{1}{2}\times2\frac{1}{2}$	$\frac{9}{16}$	8.5	2.50	.83	1.34	.80	.73	1.17	.58	.49	.48

PROPERTIES OF STANDARD ANGLES HAVING EQUAL LEGS.—(*Continued*)

$a \times a$	t		A	x	I	S	r	x'	I'	S'	r'
3×3	$\frac{1}{4}$	4.9	1.44	.84	1.24	.58	.93	1.19	.50	.42	.59
3×3	$\frac{5}{16}$	6.1	1.78	.87	1.51	.71	.92	1.22	.61	.50	.59
3×3	$\frac{3}{8}$	7.2	2.11	.89	1.76	.83	.91	1.26	.72	.57	.58
3×3	$\frac{7}{16}$	8.3	2.44	.91	1.99	.95	.91	1.29	.82	.64	.58
3×3	$\frac{1}{2}$	9.4	2.75	.93	2.22	1.07	.90	1.32	.92	.70	.58
3×3	$\frac{9}{16}$	10.4	3.06	.95	2.43	1.19	.89	1.35	1.02	.76	.58
3×3	$\frac{5}{8}$	11.5	3.36	.98	2.62	1.30	.88	1.38	1.12	.81	.58
3×3	$\frac{11}{16}$	12.5	3.66	1.00	2.81	1.40	.88	1.41	1.22	.86	.58
$3\frac{1}{2}\times3\frac{1}{2}$	$\frac{5}{16}$	7.2	2.09	.99	2.45	.98	1.08	1.40	.99	.71	.69
$3\frac{1}{2}\times3\frac{1}{2}$	$\frac{3}{8}$	8.5	2.49	1.01	2.87	1.15	1.07	1.43	1.16	.81	.68
$3\frac{1}{2}\times3\frac{1}{2}$	$\frac{7}{16}$	9.8	2.88	1.04	3.26	1.32	1.07	1.46	1.33	.91	.68
$3\frac{1}{2}\times3\frac{1}{2}$	$\frac{1}{2}$	11.1	3.25	1.06	3.64	1.49	1.06	1.50	1.50	1.00	.68
$3\frac{1}{2}\times3\frac{1}{2}$	$\frac{9}{16}$	12.4	3.63	1.08	3.99	1.65	1.05	1.53	1.66	1.09	.68
$3\frac{1}{2}\times3\frac{1}{2}$	$\frac{5}{8}$	13.6	3.99	1.10	4.33	1.81	1.04	1.56	1.82	1.10	.68
$3\frac{1}{2}\times3\frac{1}{2}$	$\frac{11}{16}$	14.8	4.34	1.12	4.65	1.96	1.04	1.59	1.97	1.27	.67
$3\frac{1}{2}\times3\frac{1}{2}$	$\frac{3}{4}$	16.0	4.69	1.15	4.96	2.11	1.03	1.62	2.13	1.31	.67
$3\frac{1}{2}\times3\frac{1}{2}$	$\frac{13}{16}$	17.1	5.03	1.17	5.25	2.25	1.02	1.65	2.28	1.38	.67
$3\frac{1}{2}\times3\frac{1}{2}$	$\frac{7}{8}$	18.3	5.36	1.19	5.53	2.39	1.02	1.68	2.43	1.45	.67
4×4	$\frac{5}{16}$	8.2	2.41	1.12	3.71	1.29	1.24	1.58	1.50	.95	.79
4×4	$\frac{3}{8}$	9.8	2.86	1.14	4.36	1.52	1.23	1.61	1.77	1.10	.79
4×4	$\frac{7}{16}$	11.3	3.31	1.16	4.97	1.75	1.23	1.64	2.02	1.23	.78
4×4	$\frac{1}{2}$	12.8	3.75	1.18	5.56	1.97	1.22	1.67	2.28	1.36	.78
4×4	$\frac{9}{16}$	14.3	4.19	1.21	6.12	2.19	1.21	1.71	2.52	1.48	.78
4×4	$\frac{5}{8}$	15.7	4.62	1.23	6.66	2.40	1.20	1.74	2.76	1.59	.77

4×4	$\frac{11}{16}$	17.1	5.03	1.25	7.17	2.61	1.19	1.77	3.00	1.70	.77
4×4	$\frac{3}{4}$	18.5	5.44	1.27	7.66	2.81	1.19	1.80	3.23	1.80	.77
4×4	$\frac{13}{16}$	19.9	5.84	1.29	8.14	3.01	1.18	1.83	3.46	1.89	.77
4×4	$\frac{7}{8}$	21.2	6.24	1.31	8.59	3.20	1.17	1.86	3.69	1.99	.77
6×6	$\frac{3}{8}$	14.9	4.36	1.64	15.39	3.53	1.88	2.32	6.19	2.67	1.19
6×6	$\frac{7}{16}$	17.2	5.06	1.66	17.68	4.07	1.87	2.34	7.13	3.04	1.19
6×6	$\frac{1}{2}$	19.6	5.75	1.68	19.91	4.61	1.86	2.38	8.04	3.37	1.18
6×6	$\frac{9}{16}$	21.9	6.44	1.71	22.07	5.14	1.85	2.41	8.94	3.70	1.18
6×6	$\frac{5}{8}$	24.2	7.11	1.73	24.16	5.66	1.84	2.45	9.81	4.01	1.17
6×6	$\frac{11}{16}$	26.5	7.78	1.75	26.19	6.17	1.83	2.48	10.67	4.31	1.17
6×6	$\frac{3}{4}$	28.7	8.44	1.78	28.15	6.66	1.83	2.51	11.52	4.59	1.17
6×6	$\frac{13}{16}$	31.0	9.09	1.80	30.06	7.15	1.82	2.54	12.35	4.86	1.17
6×6	$\frac{7}{8}$	33.1	9.74	1.82	31.92	7.63	1.81	2.57	13.17	5.12	1.16
6×6	$\frac{15}{16}$	35.3	10.38	1.84	33.72	8.11	1.80	2.60	13.98	5.37	1.16
6×6	1	37.4	11.00	1.86	35.46	8.57	1.80	2.64	14.78	5.61	1.16
8×8	$\frac{1}{2}$	26.4	7.75	2.19	48.65	8.37	2.51	3.09	19.56	6.33	1.59
8×8	$\frac{9}{16}$	29.6	8.69	2.21	54.09	9.34	2.50	3.12	21.79	6.98	1.58
8×8	$\frac{5}{8}$	32.7	9.61	2.23	59.43	10.30	2.49	3.16	23.97	7.60	1.58
8×8	$\frac{11}{16}$	35.8	10.53	2.25	64.64	11.25	2.48	3.19	26.13	8.20	1.58
8×8	$\frac{3}{4}$	38.9	11.44	2.28	69.74	12.18	2.47	3.22	28.24	8.77	1.57
8×8	$\frac{13}{16}$	42.0	12.34	2.30	74.72	13.11	2.46	3.25	30.33	9.33	1.57
8×8	$\frac{7}{8}$	45.0	13.24	2.32	79.58	14.02	2.45	3.28	32.38	9.86	1.56
8×8	$\frac{15}{16}$	48.1	14.13	2.34	84.34	14.91	2.44	3.32	34.40	10.38	1.56
8×8	1	51.0	15.00	2.37	88.98	15.80	2.44	3.35	36.40	10.88	1.56
8×8	$1\frac{1}{16}$	54.0	15.88	2.39	93.53	16.67	2.43	3.38	38.38	11.36	1.56
8×8	$1\frac{1}{8}$	56.9	16.74	2.41	97.97	17.53	2.42	3.41	40 33	11.83	1.55

PROPERTIES OF STANDARD ANGLES HAVING UNEQUAL LEGS



1	2	3	4	5	6	7	8	9	10	11	12	13	14
Dimensions Inches	Thickness Inches	Weight per Foot Pounds	Area of Section Square Inches	Distance of Center of Gravity From Back of Longer Leg. Inches	Moment of Inertia, Axis *1–1*. Inches^4	Section Modulus Axis *1–1*. Inches^3.	Radius of Gyration Axis *1–1*. Inches	Distance of Center of Gravity From Back of Shorter Leg. Inches	Moment of Inertia, Axis *2–2*. Inches^4	Section Modulus, Axis *2–2*. Inches^3	Radius of Gyration, Axis *2–2*. Inches	Tangent of Angle	Least Radius of Gyration, Axis *3–3*. Inches
$b \times a$	t	W	A	x	I	S	r	x'	I'	S'	r'	a	r''
$2\frac{1}{2} \times 2$	$\frac{3}{16}$	2.8	.81	.51	.29	.20	.60	.76	.51	.29	.79	.632	.43
$2\frac{1}{2} \times 2$	$\frac{1}{4}$	3.7	1.07	.54	.37	.25	.59	.79	.65	.38	.78	.626	.42
$2\frac{1}{2} \times 2$	$\frac{5}{16}$	4.5	1.31	.56	.45	.31	.58	.81	.79	.47	.78	.620	.42
$2\frac{1}{2} \times 2$	$\frac{3}{8}$	5.3	1.55	.58	.51	.36	.58	.83	.91	.55	.77	.614	.42
$2\frac{1}{2} \times 2$	$\frac{7}{16}$	6.1	1.78	.60	.58	.41	.57	.85	1.03	.62	.76	.607	.42
$2\frac{1}{2} \times 2$	$\frac{1}{2}$	6.8	2.00	.63	.64	.46	.56	.88	1.14	.70	.75	.600	.42
$2\frac{1}{2} \times 2$	$\frac{9}{16}$	7.6	2.22	.65	.69	.51	.56	.90	1.24	.77	.75	.592	.42

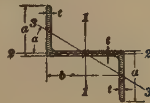
$3 \times 2\frac{1}{2}$	$\frac{1}{4}$	4.5	1.32	.66	.74	.40	.75	.91	1.17	.56	.95	.684	.53
$3 \times 2\frac{1}{2}$	$\frac{5}{16}$	5.6	1.63	.68	.90	.49	.74	.93	1.42	.69	.94	.680	.53
$3 \times 2\frac{1}{2}$	$\frac{3}{8}$	6.6	1.93	.71	1.04	.58	.74	.96	1.66	.81	.93	.676	.52
$3 \times 2\frac{1}{2}$	$\frac{7}{16}$	7.6	2.22	.73	1.18	.66	.73	.98	1.88	.93	.92	.672	.52
$3 \times 2\frac{1}{2}$	$\frac{1}{2}$	8.5	2.50	.75	1.30	.74	.72	1.00	2.08	1.04	.91	.666	.52
$3 \times 2\frac{1}{2}$	$\frac{9}{16}$	9.5	2.78	.77	1.42	.82	.72	1.02	2.28	1.15	.91	.661	.52
$3 \times 2\frac{1}{2}$	$\frac{5}{8}$	10.4	3.05	.79	1.53	.90	.71	1.04	2.46	1.26	.90	.655	.52
$3\frac{1}{2} \times 2\frac{1}{2}$	$\frac{1}{4}$	4.9	1.44	.61	.78	.41	.74	1.11	1.80	.75	1.12	.506	.54
$3\frac{1}{2} \times 2\frac{1}{2}$	$\frac{5}{16}$	6.1	1.78	.64	.94	.50	.73	1.14	2.19	.93	1.11	.501	.54
$3\frac{1}{2} \times 2\frac{1}{2}$	$\frac{3}{8}$	7.2	2.11	.66	1.09	.59	.72	1.16	2.56	1.09	1.10	.496	.54
$3\frac{1}{2} \times 2\frac{1}{2}$	$\frac{7}{16}$	8.3	2.44	.68	1.23	.68	.71	1.18	2.91	1.26	1.09	.491	.54
$3\frac{1}{2} \times 2\frac{1}{2}$	$\frac{1}{2}$	9.4	2.75	.70	1.36	.76	.70	1.20	3.24	1.41	1.09	.486	.53
$3\frac{1}{2} \times 2\frac{1}{2}$	$\frac{9}{16}$	10.4	3.06	.73	1.49	.84	.70	1.23	3.55	1.56	1.08	.480	.53
$3\frac{1}{2} \times 2\frac{1}{2}$	$\frac{5}{8}$	11.5	3.36	.75	1.61	.92	.69	1.25	3.85	1.71	1.07	.472	.53
$3\frac{1}{2} \times 2\frac{1}{2}$	$\frac{11}{16}$	12.5	3.66	.77	1.72	.99	.69	1.27	4.13	1.85	1.06	.468	.53
$3\frac{1}{2} \times 2\frac{1}{2}$	$\frac{3}{4}$	13.4	3.94	.79	1.83	1.07	.68	1.29	4.40	1.99	1.06	.461	.54
$3\frac{1}{2} \times 3$	$\frac{5}{16}$	6.6	1.94	.81	1.58	.72	.90	1.06	2.33	.95	1.10	.724	.63
$3\frac{1}{2} \times 3$	$\frac{3}{8}$	7.9	2.30	.83	1.85	.85	.90	1.08	2.72	1.13	1.09	.721	.62
$3\frac{1}{2} \times 3$	$\frac{7}{16}$	9.1	2.66	.85	2.09	.98	.89	1.10	3.10	1.29	1.08	.718	.62
$3\frac{1}{2} \times 3$	$\frac{1}{2}$	10.2	3.00	.88	2.33	1.10	.88	1.13	3.45	1.45	1.07	.714	.62
$3\frac{1}{2} \times 3$	$\frac{9}{16}$	11.4	3.34	.90	2.55	1.21	.87	1.15	3.79	1.61	1.07	.711	.62
$3\frac{1}{2} \times 3$	$\frac{5}{8}$	12.5	3.68	.92	2.76	1.33	.87	1.17	4.11	1.76	1.06	.707	.62
$3\frac{1}{2} \times 3$	$\frac{11}{16}$	13.6	4.00	.94	2.96	1.44	.86	1.19	4.41	1.91	1.05	.703	.62
$3\frac{1}{2} \times 3$	$\frac{3}{4}$	14.7	4.32	.96	3.15	1.54	.85	1.21	4.70	2.05	1.04	.698	.62
$3\frac{1}{2} \times 3$	$\frac{13}{16}$	15.8	4.63	.98	3.33	1.65	.85	1.23	4.98	2.20	1.04	.694	.62
$3\frac{1}{2} \times 3$	$\frac{7}{8}$	16.8	4.93	1.00	3.50	1.75	.84	1.25	5.24	2.33	1.03	.689	.63

PROPERTIES OF STANDARD ANGLES HAVING UNEQUAL LEGS.—(*Continued*)

$b \times a$	t	W	A	x	I	S	r	x'	I'	S'	r'	a	r''
4×3	$\frac{5}{16}$	7.2	2.09	.76	1.65	.73	.89	1.26	3.38	1.23	1.27	.554	.65
4×3	$\frac{3}{8}$	8.5	2.49	.78	1.92	.87	.88	1.28	3.96	1.46	1.26	.551	.64
4×3	$\frac{7}{16}$	9.8	2.88	.80	2.18	.99	.87	1.30	4.52	1.68	1.25	.547	.64
4×3	$\frac{1}{2}$	11.1	3.25	.83	2.42	1.12	.86	1.33	5.05	1.89	1.25	.543	.64
4×3	$\frac{9}{16}$	12.4	3.63	.85	2.66	1.23	.86	1.35	5.55	2.09	1.24	.538	.64
4×3	$\frac{5}{8}$	13.6	3.99	.87	2.87	1.35	.85	1.37	6.03	2.30	1.23	.534	.64
4×3	$\frac{11}{16}$	14.8	4.34	.89	3.08	1.46	.84	1.39	6.49	2.49	1.22	.529	.64
4×3	$\frac{3}{4}$	16.0	4.69	.92	3.28	1.57	.84	1.42	6.93	2.68	1.22	.524	.64
4×3	$\frac{13}{16}$	17.1	5.03	.94	3.47	1.68	.83	1.44	7.35	2.87	1.21	.518	.64
4×3	$\frac{7}{8}$	18.3	5.36	.96	3.66	1.79	.83	1.46	7.75	3.05	1.20	.512	.64
5×3	$\frac{5}{16}$	8.2	2.41	.68	1.75	.75	.85	1.68	6.26	1.89	1.61	.368	.66
5×3	$\frac{3}{8}$	9.8	2.86	.70	2.04	.89	.84	1.70	7.37	2.24	1.61	.364	.65
5×3	$\frac{7}{16}$	11.3	3.31	.73	2.32	1.02	.84	1.73	8.43	2.58	1.60	.361	.65
5×3	$\frac{1}{2}$	12.8	3.75	.75	2.58	1.15	.83	1.75	9.45	2.91	1.59	.357	.65
5×3	$\frac{9}{16}$	14.3	4.19	.77	2.83	1.27	.82	1.77	10.43	3.23	1.58	.353	.65
5×3	$\frac{5}{8}$	15.7	4.61	.80	3.06	1.39	.82	1.80	11.37	3.55	1.57	.349	.64
5×3	$\frac{11}{16}$	17.1	5.03	.82	3.29	1.51	.81	1.82	12.28	3.86	1.56	.345	.64
5×3	$\frac{3}{4}$	18.5	5.44	.84	3.51	1.62	.80	1.84	13.15	4.16	1.55	.340	.64
5×3	$\frac{13}{16}$	19.9	5.84	.86	3.71	1.74	.80	1.86	13.98	4.46	1.55	.336	.64
5×3	$\frac{7}{8}$	21.2	6.24	.88	3.91	1.85	.79	1.88	14.78	4.75	1.54	.331	.64
$5 \times 3\frac{1}{2}$	$\frac{5}{16}$	8.7	2.56	.84	2.72	1.02	1.03	1.59	6.60	1.94	1.61	.489	.77
$5 \times 3\frac{1}{2}$	$\frac{3}{8}$	10.4	3.05	.86	3.18	1.21	1.02	1.61	7.78	2.29	1.60	.485	.76
$5 \times 3\frac{1}{2}$	$\frac{7}{16}$	12.0	3.53	.88	3.63	1.39	1.01	1.63	8.90	2.64	1.59	.482	.76

5×3½	½	13.6	4.00	.91	4.05	1.56	1.01	1.66	9.99	2.99	1.58	.479	.75
5×3½	9/16	15.2	4.47	.93	4.45	1.73	1.00	1.68	11.03	3.32	1.57	.476	.75
5×3½	⅝	16.8	4.93	.95	4.83	1.90	.99	1.70	12.03	3.65	1.56	.472	.75
5×3½	11/16	18.3	5.38	.97	5.20	2.06	.98	1.72	12.99	3.97	1.56	.468	.75
5×3½	¾	19.8	5.82	1.00	5.55	2.22	.98	1.75	13.92	4.28	1.55	.464	.75
5×3½	13/16	21.3	6.25	1.02	5.89	2.37	.97	1.77	14.81	4.58	1.54	.460	.75
5×3½	⅞	22.7	6.68	1.04	6.21	2.52	.96	1.79	15.67	4.88	1.53	.455	.75
5×3½	15/16	24.2	7.09	1.06	6.52	2.67	.96	1.81	16.49	5.17	1.53	.451	.75
6×3½	⅜	11.7	3.43	.79	3.34	1.23	.99	2.04	12.86	3.24	1.94	.350	.77
6×3½	7/16	13.5	3.97	.81	3.81	1.41	.98	2.06	14.76	3.75	1.93	.347	.76
6×3½	½	15.3	4.50	.83	4.25	1.59	.97	2.08	16.59	4.24	1.92	.344	.76
6×3½	9/16	17.1	5.03	.86	4.67	1.77	.96	2.11	18.37	4.72	1.91	.341	.75
6×3½	⅝	18.9	5.55	.88	5.08	1.94	.96	2.13	20.08	5.19	1.90	.338	.75
6×3½	11/16	20.6	6.06	.90	5.47	2.11	.95	2.15	21.74	5.65	1.89	.334	.75
6×3½	¾	22.4	6.57	.93	5.84	2.27	.94	2.18	23.34	6.10	1.89	.331	.75
6×3½	13/16	24.0	7.06	.95	6.20	2.43	.94	2.20	24.89	6.55	1.88	.327	.75
6×3½	⅞	25.7	7.55	.97	6.55	2.59	.93	2.22	26.39	6.98	1.87	.323	.75
6×3½	15/16	27.3	8.03	.99	6.88	2.74	.93	2.24	27.84	7.41	1.86	.320	.75
6×3½	1	28.9	8.50	1.01	7.21	2.90	.92	2.26	29.15	7.80	1.85	.317	.75
6×4	⅜	12.3	3.61	.94	4.90	1.60	1.17	1.94	13.47	3.32	1.93	.446	.88
6×4	7/16	14.3	4.19	.96	5.60	1.85	1.16	1.96	15.46	3.83	1.92	.443	.87
6×4	½	16.2	4.75	.99	6.27	2.08	1.15	1.99	17.40	4.33	1.91	.440	.87
6×4	9/16	18.1	5.31	1.01	6.91	2.31	1.14	2.01	19.26	4.83	1.90	.438	.87
6×4	⅝	20.0	5.86	1.03	7.52	2.54	1.13	2.03	21.07	5.31	1.90	.434	.86
6×4	11/16	21.8	6.41	1.06	8.11	2.76	1.13	2.06	22.82	5.78	1.89	.431	.86
6×4	¾	23.6	6.94	1.08	8.68	2.97	1.12	2.08	24.51	6.25	1.88	.428	.86
6×4	13/16	25.4	7.47	1.10	9.23	3.18	1.11	2.10	26.15	6.70	1.87	.425	.86
6×4	⅞	27.2	7.99	1.12	9.75	3.39	1.11	2.12	27.73	7.15	1.86	.421	.86
6×4	15/16	28.9	8.50	1.14	10.26	3.59	1.10	2.14	29.26	7.59	1.86	.418	.86
6×4	1	30.6	9.00	1.17	10.75	3.79	1.09	2.17	30.75	8.02	1.85	.414	.86

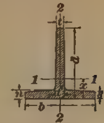
PROPERTIES OF Z BARS



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Depth of Bar Inches	Length of Legs Inches	Thickness of Web and Legs. Inches	Weight per Foot Pounds	Area of Section Square Inches	Moment of Inertia, Axis *1–1*. Inches^4	Section Modulus Axis *1–1*. Inches^3	Radius of Gyration, Axis *1–1*. Inches	Moment of Inertia, Axis *2–2*. Inches^4	Section Modulus, Axis *2–2*. Inches^3	Radius of Gyration, Axis *2–2*. Inches	Tangent of Angle	Least Radius of Gyration, Axis *3–3*. Inches	Coefficient of Deflection: Uniform Load	Coefficient of Deflection: Center Load
b	a	t		A	I	S	r	I'	S'	r'	a	r''	N	N'
3	$2\frac{11}{16}$	$\frac{1}{4}$	6.7	1.97	2.87	1.92	1.21	2.81	1.10	1.19	.986	.55	.000270	.000432
$3\frac{1}{16}$	$2\frac{3}{4}$	$\frac{5}{16}$	8.4	2.48	3.64	2.38	1.21	3.64	1.40	1.21	1.000	.55	.000213	.000341
3	$2\frac{11}{16}$	$\frac{3}{8}$	9.7	2.86	3.85	2.57	1.16	3.92	1.57	1.17	.990	.54	.000201	.000322
$3\frac{1}{16}$	$2\frac{3}{4}$	$\frac{7}{16}$	11.4	3.36	4.57	2.98	1.17	4.75	1.88	1.19	.975	.55	.000170	.000272
3	$2\frac{11}{16}$	$\frac{1}{2}$	12.5	3.69	4.59	3.06	1.12	4.85	1.99	1.15	.965	.53	.000169	.000271
$3\frac{1}{16}$	$2\frac{3}{4}$	$\frac{9}{16}$	14.2	4.18	5.26	3.43	1.12	5.68	2.30	1.17	.951	.54	.000148	.000236

4	$3\frac{1}{16}$	$\frac{1}{4}$	8.2	2.41	6.28	3.14	1.62	4.23	1.44	1.33	.778	.67	.000124	.000198
$4\frac{1}{16}$	$3\frac{1}{8}$	$\frac{5}{16}$	10.3	3.03	7.94	3.91	1.62	5.46	1.84	1.34	.788	.68	.000098	.000156
$4\frac{1}{8}$	$3\frac{3}{16}$	$\frac{3}{8}$	12.4	3.66	9.63	4.67	1.62	6.77	2.26	1.36	.798	.69	.000081	.000129
4	$3\frac{1}{16}$	$\frac{7}{16}$	13.8	4.05	9.66	4.83	1.54	6.73	2.37	1.29	.794	.66	.000080	.000129
$4\frac{1}{16}$	$3\frac{1}{8}$	$\frac{1}{2}$	15.8	4.66	11.18	5.50	1.55	7.96	2.77	1.31	.804	.67	.000069	.000111
$4\frac{1}{8}$	$3\frac{3}{16}$	$\frac{9}{16}$	17.9	5.27	12.74	6.18	1.55	9.26	3.19	1.32	.814	.68	.000061	.000098
4	$3\frac{1}{16}$	$\frac{5}{8}$	18.9	5.55	12.11	6.05	1.48	8.73	3.18	1.25	.808	.65	.000064	.000103
$4\frac{1}{16}$	$3\frac{1}{8}$	$\frac{11}{16}$	20.9	6.14	13.52	6.65	1.48	9.95	3.58	1.27	.818	.67	.000057	.000092
$4\frac{1}{8}$	$3\frac{3}{16}$	$\frac{3}{4}$	23.0	6.75	14.97	7.26	1.49	11.24	4.00	1.29	.828	.68	.000052	.000083
5	$3\frac{1}{4}$	$\frac{5}{16}$	11.6	3.40	13.36	5.34	1.98	6.18	2.00	1.35	.611	.75	.000058	.000093
$5\frac{1}{16}$	$3\frac{5}{16}$	$\frac{3}{8}$	13.9	4.10	16.18	6.39	1.99	7.65	2.45	1.37	.619	.76	.000048	.000077
$5\frac{1}{8}$	$3\frac{3}{8}$	$\frac{7}{16}$	16.4	4.81	19.07	7.44	1.99	9.20	2.92	1.38	.628	.76	.000041	.000065
5	$3\frac{1}{4}$	$\frac{1}{2}$	17.9	5.25	19.19	7.68	1.91	9.05	3.02	1.31	.616	.74	.000040	.000065
$5\frac{1}{16}$	$3\frac{5}{16}$	$\frac{9}{16}$	20.2	5.94	21.83	8.62	1.92	10.51	3.47	1.33	.623	.75	.000036	.000057
$5\frac{1}{8}$	$3\frac{3}{8}$	$\frac{5}{8}$	22.6	6.64	24.53	9.57	1.92	12.06	3.94	1.35	.631	.76	.000032	.000051
5	$3\frac{1}{4}$	$\frac{11}{16}$	23.7	6.96	23.68	9.47	1.84	11.37	3.91	1.28	.619	.73	.000033	.000052
$5\frac{1}{16}$	$3\frac{5}{16}$	$\frac{3}{4}$	26.0	7.64	26.16	10.34	1.85	12.83	4.37	1.30	.626	.74	.000030	.000048
$5\frac{1}{8}$	$3\frac{3}{8}$	$\frac{13}{16}$	28.3	8.33	28.70	11.20	1.86	14.37	4.84	1.31	.633	.76	.000027	.000043
6	$3\frac{1}{2}$	$\frac{3}{8}$	15.6	4.59	25.32	8.44	2.35	9.11	2.75	1.41	.519	.83	.000031	.000049
$6\frac{1}{16}$	$3\frac{9}{16}$	$\frac{7}{16}$	18.3	5.39	29.80	9.83	2.35	10.94	3.27	1.43	.526	.83	.000026	.000042
$6\frac{1}{8}$	$3\frac{5}{8}$	$\frac{1}{2}$	21.0	6.19	34.36	11.22	2.36	12.87	3.81	1.44	.532	.84	.000023	.000036
6	$3\frac{1}{2}$	$\frac{9}{16}$	22.7	6.68	34.64	11.55	2.28	12.59	3.91	1.37	.520	.81	.000022	.000036
$6\frac{1}{16}$	$3\frac{9}{16}$	$\frac{5}{8}$	25.4	7.46	38.87	12.82	2.28	14.41	4.44	1.39	.526	.82	.000020	.000032
$6\frac{1}{8}$	$3\frac{5}{8}$	$\frac{11}{16}$	28.1	8.25	43.18	14.10	2.29	16.34	4.98	1.41	.532	.84	.000018	.000029
6	$3\frac{1}{2}$	$\frac{3}{4}$	29.3	8.63	42.12	14.04	2.21	15.44	4.94	1.34	.519	.81	.000018	.000030
$6\frac{1}{16}$	$3\frac{9}{16}$	$\frac{13}{16}$	31.9	9.39	46.13	15.22	2.22	17.27	5.47	1.36	.525	.82	.000017	.000027
$6\frac{1}{8}$	$3\frac{5}{8}$	$\frac{7}{8}$	34.6	10.17	50.22	16.40	2.22	19.18	6.02	1.37	.530	.83	.000015	.000025
$7\frac{1}{2}$	3	$\frac{3}{8}$	16.3	4.78	38.19	10.18	2.83	5.59	1.99	1.08	.29	.72	.000020	.000033
8	3	$\frac{1}{2}$	22.1	6.50	56.54	14.14	2.95	7.01	2.55	1.04	.26	.71	.000014	.000022

PROPERTIES OF T BARS



EQUAL LEGS

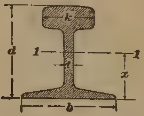
1	2	3	4	5	6	7	8	9	10	11	12	13
Dimensions												
Width of Flange Inches	Depth of Bar Inches	Thickness of Flange Inches	Thickness of Stem Inches	Weight per Foot Pounds	Area of Section Square Inches	Distance of Center of Gravity From Outside of Flange. Inches	Moment of Inertia, Axis 1–1. Inches⁴	Section Modulus Axis 1–1. Inches³	Radius of Gyration, Axis 1–1. Inches	Moment of Inertia, Axis 2–2. Inches⁴	Section Modulus, Axis 2–2. Inches³	Radius of Gyration, Axis 2–2. Inches
b	d	s to n'	t to t'		A	x	I	S	r	I'	S'	r'
1	1	$\frac{1}{8}$ to $\frac{5}{32}$	$\frac{1}{8}$ to $\frac{5}{32}$	1.0	.27	.29	.02	.03	.30	.01	.02	.21
$1\frac{1}{8}$	$1\frac{1}{8}$	$\frac{3}{16}$ to $\frac{7}{32}$	$\frac{5}{32}$ to $\frac{7}{32}$	1.4	.41	.33	.04	.05	.32	.02	.04	.25
$1\frac{3}{16}$	$1\frac{3}{16}$	$\frac{3}{16}$ to $\frac{1}{4}$	$\frac{5}{32}$ to $\frac{7}{32}$	1.6	.45	.34	.05	.06	.33	.03	.05	.26
$1\frac{1}{4}$	$1\frac{1}{4}$	$\frac{3}{16}$ to $\frac{1}{4}$	$\frac{5}{32}$ to $\frac{1}{4}$	1.7	.48	.36	.06	.07	.35	.03	.05	.27
$1\frac{3}{8}$	$1\frac{3}{8}$	$\frac{3}{16}$ to $\frac{1}{4}$	$\frac{5}{32}$ to $\frac{1}{4}$	1.9	.55	.39	.08	.08	.39	.05	.07	.29
2	2	$\frac{1}{4}$ to $\frac{5}{16}$	$\frac{1}{4}$ to $\frac{5}{16}$	3.7	1.07	.59	.37	.26	.59	.18	.18	.42
2	2	$\frac{5}{16}$ to $\frac{3}{8}$	$\frac{5}{16}$ to $\frac{3}{8}$	4.4	1.28	.61	.43	.31	.59	.23	.23	.42

$2\frac{1}{4}$	$2\frac{1}{4}$	$\frac{1}{4}$ to $\frac{5}{16}$	$\frac{1}{4}$ to $\frac{5}{16}$	4.2	1.21	.68	.51	.32	.65	.24	.21	.45
$2\frac{1}{4}$	$2\frac{1}{4}$	$\frac{5}{16}$ to $\frac{3}{8}$	$\frac{5}{16}$ to $\frac{3}{8}$	5.0	1.46	.67	.64	.40	.66	.32	.29	.47
$2\frac{1}{2}$	$2\frac{1}{2}$	$\frac{5}{16}$ to $\frac{3}{8}$	$\frac{5}{16}$ to $\frac{3}{8}$	5.6	1.63	.73	.87	.49	.74	.44	.35	.52
3	3	$\frac{5}{16}$ to $\frac{3}{8}$	$\frac{5}{16}$ to $\frac{3}{8}$	6.8	1.99	.86	1.58	.74	.90	.75	.50	.62
3	3	$\frac{3}{8}$ to $\frac{7}{16}$	$\frac{3}{8}$ to $\frac{7}{16}$	7.9	2.31	.88	1.82	.86	.90	.92	.61	.64
3	3	$\frac{1}{2}$ to $\frac{9}{16}$	$\frac{1}{2}$ to $\frac{9}{16}$	10.1	2.96	.93	2.27	1.10	.88	1.20	.80	.64
$3\frac{1}{2}$	$3\frac{1}{2}$	$\frac{3}{8}$ to $\frac{7}{16}$	$\frac{3}{8}$ to $\frac{7}{16}$	9.3	2.74	.99	3.10	1.23	1.08	1.42	.81	.73
4	4	$\frac{3}{8}$ to $\frac{7}{16}$	$\frac{3}{8}$ to $\frac{7}{16}$	10.9	3.19	1.12	4.54	1.58	1.21	2.11	1.06	.83

UNEQUAL LEGS

$1\frac{1}{4}$	$1\frac{1}{16}$	$\frac{3}{16}$ to $\frac{1}{4}$	$\frac{5}{32}$ to $\frac{7}{32}$	1.5	.44	.29	.04	.05	.29	.03	.01	.28
$2\frac{1}{2}$	$1\frac{1}{4}$	$\frac{3}{16}$ to $\frac{9}{32}$	$\frac{3}{16}$ to $\frac{5}{16}$	3.0	.86	.30	.08	.09	.31	.28	.22	.58
$2\frac{1}{2}$	$1\frac{3}{4}$	$\frac{11}{32}$ to $\frac{13}{32}$	$\frac{1}{4}$ to $\frac{17}{64}$	4.5	1.31	.43	.21	.16	.40	.47	.38	.60
$2\frac{1}{2}$	3	$\frac{3}{8}$ to $\frac{7}{16}$	$\frac{3}{8}$ to $\frac{7}{16}$	7.2	2.10	.95	1.72	.84	.91	.53	.42	.50
$2\frac{3}{4}$	2	$\frac{5}{16}$ to $\frac{3}{8}$	$\frac{3}{4}$	7.5	2.21	.74	.83	.66	.62	.64	.46	.54
3	$2\frac{1}{2}$	$\frac{3}{8}$ to $\frac{7}{16}$	$\frac{3}{8}$ to $\frac{7}{16}$	7.2	2.11	.71	1.08	.60	.64	.90	.60	.66
3	4	$\frac{3}{8}$ to $\frac{7}{16}$	$\frac{3}{8}$ to $\frac{7}{16}$	9.3	2.74	1.27	4.12	1.51	1.24	.90	.60	.58
$3\frac{1}{2}$	4	$\frac{3}{8}$ to $\frac{7}{16}$	$\frac{3}{8}$ to $\frac{7}{16}$	10.0	2.94	1.20	4.33	1.54	1.23	1.42	.81	.70
$4\frac{1}{2}$	$2\frac{1}{2}$	$\frac{5}{16}$ to $\frac{3}{8}$	$\frac{5}{16}$ to $\frac{3}{8}$	8.0	2.29	.57	1.04	.54	.68	2.51	1.12	1.05
$4\frac{1}{2}$	$3\frac{1}{2}$	$\frac{7}{16}$ to $\frac{9}{16}$	$\frac{11}{16}$	14.9	4.37	1.09	4.89	2.03	1.06	3.68	1.64	.92
$4\frac{1}{2}$	$3\frac{1}{2}$	$\frac{7}{16}$ to $\frac{9}{16}$	$\frac{11}{16}$ to $\frac{7}{8}$	15.9	4.65	1.11	5.08	2.13	1.05	3.73	1.66	.90
5	3	$\frac{1}{2}$ to $\frac{9}{16}$	$\frac{13}{32}$ to $\frac{5}{8}$	13.6	3.99	.72	2.42	1.06	.78	5.42	2.17	1.17

PROPERTIES AND PRINCIPAL DIMENSIONS OF STANDARD T RAILS



1	2	3	4	5	6	7	8	9
							Axis *1–1*	
Weight per Yard Pounds	Area Square Inches	*b* Inches	*d* Inches	*k* Inches	*t* Inches	*x* Inches	Moment of Inertia *I*	Section Modulus *S*
8	.78	$1\frac{1}{2}$	$1\frac{1}{2}$	$\frac{13}{16}$	$\frac{5}{32}$	.75	.23	.31
12	1.18	$1\frac{7}{8}$	$1\frac{7}{8}$	$1\frac{1}{16}$	$\frac{3}{16}$	.92	.55	.58
16	1.57	$2\frac{1}{4}$	$2\frac{1}{4}$	$1\frac{1}{4}$	$\frac{15}{64}$	1.10	1.1	.95
20	2.00	$2\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{3}{8}$	$\frac{17}{64}$	1.2	1.7	1.3
25	2.5	$2\frac{3}{4}$	$2\frac{3}{4}$	$1\frac{1}{2}$	$\frac{19}{64}$	1.3	2.6	1.8
30	2.9	3	3	$1\frac{5}{8}$	$\frac{21}{64}$	1.4	3.6	2.3
35	3.4	$3\frac{1}{4}$	$3\frac{1}{4}$	$1\frac{3}{4}$	$\frac{23}{64}$	1.6	4.9	2.9
40	3.9	$3\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{7}{8}$	$\frac{25}{64}$	1.7	6.6	3.6
45	4.4	$3\frac{11}{16}$	$3\frac{11}{16}$	2	$\frac{27}{64}$	1.8	8.1	4.2
50	4.9	$3\frac{7}{8}$	$3\frac{7}{8}$	$2\frac{1}{8}$	$\frac{7}{16}$	1.9	9.8	4.9
55	5.4	$4\frac{1}{16}$	$4\frac{1}{16}$	$2\frac{1}{4}$	$\frac{15}{32}$	2.0	12.2	5.9
60	5.9	$4\frac{1}{4}$	$4\frac{1}{4}$	$2\frac{3}{8}$	$\frac{31}{64}$	2.1	14.7	6.7
65	6.4	$4\frac{7}{16}$	$4\frac{7}{16}$	$2\frac{13}{32}$	$\frac{1}{2}$	2.2	17.0	7.4
70	6.9	$4\frac{5}{8}$	$4\frac{5}{8}$	$2\frac{7}{16}$	$\frac{33}{64}$	2.2	20.0	8.4
75	7.4	$4\frac{13}{16}$	$4\frac{13}{16}$	$2\frac{15}{32}$	$\frac{17}{32}$	2.3	23.0	9.1
80	7.8	5	5	$2\frac{1}{2}$	$\frac{35}{64}$	2.4	26.7	10.1
85	8.3	$5\frac{3}{16}$	$5\frac{3}{16}$	$2\frac{9}{16}$	$\frac{9}{16}$	2.5	30.5	11.2
90	8.8	$5\frac{3}{8}$	$5\frac{3}{8}$	$2\frac{5}{8}$	$\frac{9}{16}$	2.6	34.4	12.3
95	9.3	$5\frac{9}{16}$	$5\frac{9}{16}$	$2\frac{11}{16}$	$\frac{9}{16}$	2.7	38.6	13.3
100	9.8	$5\frac{3}{4}$	$5\frac{3}{4}$	$2\frac{3}{4}$	$\frac{9}{16}$	2.8	43.4	14.7
150	14.7	6	6	$4\frac{1}{4}$	1	3.0	69.3	23.1

Sometimes, the sections are not symmetrical and the neutral axis itself must be found. As an example, consider the beam shown in Fig. 6 (*a*). It is composed of an **I** beam and a channel riveted together. Such a beam is often used to carry the track of a traveling crane.

The first problem is to locate the neutral axis *cd*, Fig. 6 (*b*). Assume any axis, as *ab*, about which to take ordinary moments of the areas. The moment of each area about *ab* is equal in each case to the product of the area and the distance from its center of gravity to *ab*. The areas and location of the centers of gravity of the beam and column sections can be obtained from the tables. Adding these moments, the work is as follows:

Moment of **I** beam $= 20.59 \times 9$ .... $= 185.31$
Moment of channel $= 11.76 \times (18 + .78) = 220.85$
Total.......................... 406.16

The total area of the section is $20.59 + 11.76 = 32.35$ sq. in. Therefore, the distance from the line *ab* to the neutral axis *cd* is $\frac{406.16}{32.35} = 12.56$ in.

It now remains, by the method given, to find the moments of inertia of the channel and the **I** beam about the axis *cd* and to add them together. This can be done with the help of the tables, as follows: The moment of inertia of the channel about *cd* is $9.39 + 11.76 \times (5.44 + .78)^2 = 464.36$. Likewise, the moment of inertia of the **I** beam about the axis *cd* is $921.2 + 20.59 \times (12.56 - 9)^2 = 1{,}182.15$; therefore, the total moment of inertia is $464.36 + 1{,}182.15 = 1{,}646.51$ in^4.

FORMULAS FOR DESIGN

BENDING-MOMENT FORMULAS

In any homogeneous beam the maximum stress at any section of the beam developed by the loads may be found by the following formula:

$$M = \frac{s}{c} I,$$

in which M is the bending moment at that point, in inch-

pounds; I, the moment of inertia of the section referred to the inch; c, the distance, in inches, from the neutral axis to the most remote fiber; and s, the maximum stress, in pounds per square inch. This stress, of course, occurs in the most remote fiber.

This is the general equation for beams. It expresses the bending moment about any section, caused by the loads and reactions, in terms of the maximum unit stress at the section and the dimensions of the beam at that section $\left(\text{that is, } \frac{I}{c}\right)$. The stress s may be either tension or compression, but it is always the maximum stress in the section.

It is evident that if a beam is going to break, the break will occur at that section about which the external bending moment is maximum. If this section holds, every other part of the beam will be strong enough. Therefore, it is the section of a beam about which the external bending moment is maximum that is always investigated.

The quantity $\frac{I}{c}$ is equal to the section modulus; therefore, the formula is often written

$$M = sS,$$

in which S is the section modulus.

In the preceding tables the moments of inertia, etc. are given with the inch as a base, and s is usually measured in pounds per square inch. Therefore, in the preceding formulas M must be given in inch-pounds and not in foot-pounds.

Example.—What is the maximum stress in a simple beam 12 ft. long and uniformly loaded with 1,000 lb. per ft. of length? The beam is rectangular in section, 10 in. broad and 14 in. deep.

Solution.—The total load on the beam is $1{,}000 \times 12 = 12{,}000$ lb. The maximum bending moment, in foot-pounds, is $\frac{Wl}{8}$. In this case, $W = 12{,}000$ lb. and $l = 12$ ft. Therefore, the maximum moment, which shall be called M,

equals $\frac{12{,}000 \times 12}{8} = 18{,}000$ ft.-lb. To change this to inch-pounds, multiply by 12. Thus, $12 \times 18{,}000 = 216{,}000$ in.-lb. The section modulus of the beam is $\frac{bd^2}{6} = \frac{10 \times 14 \times 14}{6} = 326.67$. Substituting these values in the formula, $216{,}000 = s \times 326.67$, and $s = 216{,}000 \div 326.67 = 661.22$ lb. per sq. in. Ans.

MODULUS OF RUPTURE

Assume that the load on a beam is increased until it breaks. From the loading that causes failure, M can be found, while S can be found from the shape of the beam section. Substituting these values in the formula $M = Ss$, a value for s will be obtained. This value of s, which corresponds to the moment that causes the beam to break, is called the *modulus of rupture*, or *ultimate unit bending stress*, of the material. Since the formula $M = Ss$ is strictly correct only for values of s below the elastic limit, the modulus of rupture will not agree exactly with either the ultimate unit tensile or the ultimate unit compressive strength of the material. The modulus of rupture is a valuable constant, however, because by substituting it in the formula $M = Ss$ the breaking moment of a beam can be ascertained.

In designing a beam, of course the beam is not intended to develop the ultimate unit bending stress, but only a fraction of it, depending on the factor of safety used. For timber, a factor of 6 is usually employed; for wrought iron, one of 4 is sufficient; while for cast iron, from 6 to 10 is employed. For structural steel, the factor of safety generally used is about 4. With structural steel, however, instead of dividing the modulus of rupture by the factor of safety it is customary to use certain approved unit working bending stresses. For light roof construction, this value is often taken as high as 18,000 to 20,000 lb. per sq. in In ordinary building construction, 16,000 lb. is usually employed and in bridge work, 12,500 lb. per sq. in. is often used.

Example.—Design a rectangular white-oak beam to carry a safe load of 2,000 lb. located at the center, the span being 11 ft. 5 in.

SOLUTION.—The maximum external bending moment is $\frac{Wl}{4}=\frac{2{,}000\times 137}{4}=68{,}500$ in.-lb. The modulus of rupture for white oak is 7,000, and a factor of safety of 6 will be used. Therefore, $s=7{,}000\div 6=1{,}167$ lb. per sq. in. Substituting these values in the formula, $68{,}500=S\times 1{,}167$, and $S=68{,}500\div 1{,}167=58.71$. $S=\frac{bd^2}{6}$; therefore, either the breadth or the depth of the beam may be assumed and the other dimensions found. It will also be noted that in the value of S the breadth is involved only as a first power, while the depth is squared. Therefore, to design an economical beam, the better plan is to make the beam narrow and as deep as possible. Of course, there are practical considerations that govern this matter, such as obtaining commercial sizes of material and the like.

In the problem at hand, let is be assumed that the beam will be 10 in. deep. Then, $d=10$ in., $S=58.71=\frac{bd^2}{6}=\frac{b\times 10^2}{6}$, and $b=\frac{6\times 58.71}{100}=3.523$ in. The next larger size of commercial timber is 4 in. $\times$ 10 in., and is therefore the size to be used.

The modulus of rupture of metals and timbers will be found in the two tables on pages 126 and 128.

WEIGHT OF BEAMS

Every beam, whether it is made of wood, steel, or stone, has a certain weight, and the question is whether it should be considered or not. Neglecting the weight of the beam itself in beam design does not make so much difference on a short span with heavy loads as it does on a long span with comparatively light loads. Just when the weight of the beam itself should be considered and just when it should not, is a matter of experience, and no set rule can be laid down. Usually, however, if the weight of the beam is less than 5% of the load it is intended to carry, its weight may be neglected.

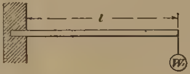
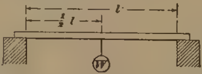
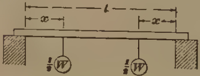
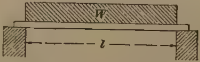
Since, in some cases, it has been decided that the weight of the beam itself must be taken into account, the methods of attaining these results will be considered. As the weight of the beam cannot be obtained until its size is known, and as the size of the beam cannot be found until the total bending moment is known, this problem can be solved only by trial. The following example will serve to illustrate the method to be pursued:

EXAMPLE.—Calculate the size of I beam required to carry, besides its own load, a uniformly distributed load of 960 lb. per ft. over a span of 20 ft.

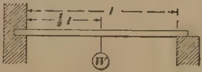
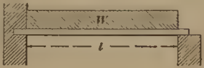
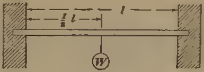
SOLUTION.—The total load on the beam, exclusive of its own weight, is $960 \times 20 = 19{,}200$ lb. The maximum bending moment is $\frac{Wl}{8} = \frac{19{,}200 \times 20 \times 12}{8} = 576{,}000$ in.-lb. $M = Ss = \frac{Wl}{8} = 576{,}000$. Giving s a value of 16,000 and neglecting the weight of the beam, $16{,}000 \times S = 576{,}000$, or $S = 576{,}000 \div 16{,}000 = 36$. On consulting the table on page 148, it will be seen that the value of S here found corresponds to that of a 12-in. 31.5-lb. I beam. This beam would satisfy the requirements if the weight of the beam itself were left out of consideration, but as it is necessary to provide for this additional load, the next larger size may be chosen and a trial calculation made to see whether it will support the combined load. This beam is a 12-in. one, weighing 35 lb. per ft.; hence, the weight of the beam is $35 \times 20 = 700$ lb. From the preceding formula $\frac{Wl}{8}$, the maximum bending moment due to the weight of the beam alone is $\frac{700 \times 20 \times 12}{8} = 21{,}000$ in.-lb. The sum of this moment and that of the external load is $576{,}000 + 21{,}000 = 597{,}000$ in.-lb. $= M$. $M = Ss$, or $597{,}000 = 16{,}000 \times S$; therefore, $S = 597{,}000 \div 16{,}000 = 37.31$. As the value given for S in the table is greater than this, the beam selected is of ample strength.

As was stated, it is usually considered safe to neglect the weight of the beam itself in calculations of beam design.

FORMULAS FOR DEFLECTION OF BEAMS

Case	Method of Loading	Deflection Inches
I		$\frac{Wl^3}{3\,EI}$
II		$\frac{Wl^3}{8\,EI}$
III		$\frac{Wl^3}{15\,EI}$
IV		$\frac{Wl^3}{48\,EI}$
V		$\frac{Wxy(2l-x)\sqrt{3x(2l-x)}}{27\,lEI}$
VI		$\frac{Wx}{48\,EI}(3l^2-4x^2)$
VII		$\frac{5\,Wl^3}{384\,EI}$
VIII		$\frac{3\,Wl^3}{320\,EI}$

TABLE—(*Continued*)

Case	Method of Loading	Deflection Inches
IX		$\dfrac{Wl^3}{60\,EI}$
X		$\dfrac{47\,Wl^3}{3{,}600\,EI}$
XI		$\dfrac{3\,Wl^3}{322\,EI}$
XII		$\dfrac{5\,Wl^3}{926\,EI}$
XIII		$\dfrac{Wl^3}{192\,EI}$
XIV		$\dfrac{Wl^3}{384\,EI}$
XV		For overhang: $\dfrac{Wx}{12\,EI}(3\,xl-4\,x)^2$ For part between supports: $\dfrac{Wx}{16\,EI}(l-2\,x)^2$

As the amount of load a beam must carry, particularly the live load, is very uncertain at best, the addition of a slight weight due to the weight of the beam itself is seldom considered to be a factor of great importance. When beams carry floors, it is customary to find the weight of the floor per square foot and then multiply this value by the distance between beams and by the span to get the total load on the beam. Many engineers assume that the weight of the beams themselves add a certain weight per square foot to the weight of the floor. This added weight is assumed to be 8 lb. for wooden beams and 6 lb. for steel beams. Thus, if all the materials composing a floor, exclusive of beams, were calculated to weigh 12 lb. per sq. ft., a weight of 20 lb. per sq. ft. would be taken to constitute the total load of a floor supported by wooden beams, while 18 lb. per sq. ft. would be taken for the total load of one supported by steel beams. This method, while not absolutely accurate, is one way of estimating the weight of the beams in a floor. *In concrete beams and stone beams, however, the weight of the beam itself must almost always be considered.*

DEFLECTION

Deflection is the name applied to the distortion or bending produced in a beam when subjected to bending stresses. The measure of the deflection at any point on a beam is the vertical displacement of the point from its original position.

Stiffness is a measure of the ability of a body to resist bending; this property is very different from the strength of the material or its power to resist rupture.

The stiffness of a beam does not depend so much on the elasticity of the material of which it is composed as on its length of span. This property of stiffness is as important in building construction as mere strength, and the two should be considered together; thus, the floor joists of a building may be strong enough to resist breaking, but they may also be so long as to lack stiffness, in which case the floor will be springy and will vibrate from persons walking on it. If there is a plastered ceiling on the under side of the joists of such a floor, the deflection of the joists may cause the

plaster to crack and fall into the room below. The allowable deflection of a plastered ceiling is usually placed at $\frac{1}{360}$ of the span, or $\frac{1}{30}$ in. for each foot of span. Where stiffness is lacking in the rafters of a roof, they will be liable to sag, thereby causing unsightly hollows in the surface, in which moisture and snow may lodge.

The amount of deflection that exists in beams loaded and supported in different ways may be calculated by the formulas given in the accompanying table. In using these formulas, all the loads should be expressed in pounds and the lengths in inches. The modulus of elasticity is denoted by E, and the moment of inertia of the section by I.

EXAMPLE.—A 10-in. 35-lb. steel **I** beam supported at the ends must sustain a uniformly distributed load of 10,000 lb. The span of the beam is 20 ft., and its moment of inertia is 146.4. There is to be a plastered ceiling on its under side, the allowable deflection of which is $\frac{1}{30}$ in. for each foot of span. Will the deflection of the beam be excessive?

SOLUTION.—The formula of the deflection of a beam of this character, from the table, is $\frac{5\,Wl^3}{384\,EI}$. The modulus of elasticity of structural steel is 29,000,000. Substituting the values of the example in the formula, the deflection equals

$$\frac{5\times 10{,}000\times 240^3}{384\times 29{,}000{,}000\times 146.4}=.42, \text{ or about } \tfrac{7}{16} \text{ in.}$$

Since the allowable deflection is $\frac{1}{360}$ of the span, the total allowable deflection is $\frac{1}{360}\times 240=\frac{2}{3}$ in. This is greater than the calculated deflection, and the beam therefore satisfies the required conditions.

The values for N and N', the coefficients of deflection for uniform and center loads, respectively, given in the tables containing the properties of sections of **I** beams, channels, and **Z** bars, were obtained from the formulas $N=\frac{5\,Wl^3}{384\,EI}$ and $N'=\frac{Wl^3}{48\,EI}$, in which W equals 1,000 lb.; l, 12 in.; E, 29,000,000; and I, the moment of inertia about the axis $1-1$. Therefore, these coefficients represent the deflection, in

inches, of a beam 1 ft. long having a load of 1,000 lb. Multiplying the proper coefficient by the cube of the span, in feet, and by the number of 1,000-lb. units in the given load, will give the deflection of a beam for any load and span.

Example.—What is the deflection of a 20-in. 65-lb. I beam that carries a center load of 28,000 lb. and has a span of 20 ft.?

Solution.—The amount of deflection is obtained by multiplying the coefficient of deflection for beams with center loads (column 13 in the table of properties of I beams) by the cube of the span, in feet, and the number of 1,000-lb. by units in the load. Hence, the deflection equals $.00000106 \times 20^3 \times \frac{28000}{1000} = .237$ in.

SUDDENLY APPLIED LOADS

In the formulas and investigations so far discussed, it has been assumed that the loads on the beams were laid gently in place. This, however, is not always the case, for the load may be suddenly or almost instantaneously applied, or it may even be dropped on the beam. Of course, in designing such beams, a large factor of safety may be employed, but if the load is dropped or very suddenly applied, this method is at best a matter of guesswork and experience.

The investigation of sudden loads divides itself naturally into two classes. The first includes loads that are not raised above a beam and whose weight is suddenly applied to the beam. The second class of loads includes those that fall vertically on a beam, as when heavy boxes or crates are dropped on the beams of a floor.

As the problems of the first class are the simplest to solve, they will be taken up first. When a load is placed suddenly on a beam, the stress produced is twice as great as if the same load had been at rest; that is, a beam to sustain a suddenly applied load should have twice the transverse strength required to sustain the same load at rest.

Often, a problem occurs concerning suddenly applied loads in which the beam has a quiet load, which is the dead load, and a suddenly applied load, which is the live load. Such a problem should be solved as follows:

EXAMPLE.—Design an I beam to carry a uniformly distributed load of 140 lb. per ft. on a span of 12 ft., and also a centrally concentrated, suddenly applied load of 3,700 lb.

SOLUTION.—The bending moment due to the uniformly distributed load is $\frac{Wl}{8}=\frac{(140\times 12)\times 12}{8}=2{,}520$ ft.-lb., or $2{,}520\times 12=30{,}240$ in.-lb. The concentrated load, if gently applied, would cause a bending moment of $\frac{3{,}700\times 12}{4}$ $=11{,}100$ ft.-lb., or $11{,}100\times 12=133{,}200$ in.-lb. Since, however, the load is suddenly applied, it will produce stresses equivalent to twice this bending moment, or $133{,}200\times 2$ $=266{,}400$ in.-lb. The total moment that the beam must be designed to withstand is therefore $266{,}400+30{,}240$ $=296{,}640=Ss$. Since $s=16{,}000$, then, $S=296{,}640\div 16{,}000$ $=18.54$. Referring to the table on page 148, it will be found that a 9-in., 21.5-lb. I beam is required.

The other class of loads referred to are those which drop on a beam from a distance above it. It is customary in considering the effect of a falling concentrated load to determine the statical or quiet load concentrated at the center that would produce the same stress, and then to design the beam for this statical load. The formula used to accomplish this is

$$W_1=W\left(1+\sqrt{\frac{2\,ah}{d}+1}\right),$$

in which W_1 is the static load, in pounds, concentrated at the center, that would produce the same stress in the beam as the falling load; W, the falling load, in pounds, that strikes the beam in the center of the span; h, the distance, in inches, that the load falls; d, the deflection of beam, in inches, produced by load W statically applied; and a, the constant.

The value of d is found as previously explained, while a is found by the formula

$$a=\frac{1}{1+.489\,\frac{W_2}{W}},$$

in which W_2 is the combined weight, in pounds, of beam and dead load that it supports; and W is the falling load.

From the construction of the formulas, it will be noted that the size of a beam required to sustain a certain falling load cannot be found direct. The size of beam must be assumed; then the formulas are used to ascertain whether the beam will meet the requirements.

Example.—A 12-in., 40-lb. I beam carries, besides its own weight, a uniform load of 260 lb. per ft. The span is 10 ft. If a load of 400 lb. drops on the beam from a distance of 18 in., will it develop a unit stress beyond the safe unit stress of 12,500 lb.?

Solution.—The total static load per foot on the beam is 260 + weight of beam per foot = 260 + 40 = 300 lb. per ft. The total static load on the beam, therefore, is $300 \times$ span $= 300 \times 10 = 3{,}000$ lb. The deflection due to the falling load of 400 lb., according to column 13, of the table on page 000 is $.00000505 \times 10^3 \times .4 = .00202$ in. The constant a thus equals

$$\frac{1}{1+.489\,\dfrac{W_2}{W}} = \frac{1}{1+.489\times\dfrac{3{,}000}{400}} = .2142$$

Therefore,

$$W_1 = W\left(1+\sqrt{\frac{2\,ah}{d}+1}\right) = 400\left(1+\sqrt{\frac{2\times .2142\times 18}{.00202}+1}\right)$$
$$= 25{,}117 \text{ lb.}$$

The maximum bending moment due to this load is $\dfrac{25{,}117\times 10}{4} = 62{,}792.5$ ft.-lb., or $62{,}792.5 \times 12 = 753{,}510$ in.-lb. The maximum bending moment due to the static or dead load is $\dfrac{3{,}000\times 10}{8} = 3{,}750$ ft.-lb., or $3{,}750 \times 12 = 45{,}000$ in.-lb. The total bending moment of both the static and sudden load is therefore $753{,}510 + 45{,}000 = 798{,}510$ in.-lb. $= Ss$. From the table on page 148, $S = 41$. Therefore, $798{,}510 = 41s$, and $s = 798{,}510 \div 41 = 19{,}476$ lb. per sq. in.

This is greater than 12,500, which was assumed as the allowable unit stress. Even if 16,000 lb. were taken as the allowable unit stress, the actual stress would still be too large and a beam of larger size would have to be assumed.

WOOD AND CAST-IRON COLUMNS

WOODEN POSTS

MATERIALS

The kinds of timber usually employed for columns are the long-leaf and short-leaf yellow pines, red pine, white and red oak, spruce, hemlock, cypress, fir, and redwood. The timber generally preferred is the yellow pine, the long-leaf variety being stronger and more durable than the short-leaf. The disadvantage of this wood is that the resinous sap makes it very inflammable.

The *compressive strength* of timber varies greatly, according to the amount of moisture it contains, a decrease in moisture resulting in increased strength. The percentage of moisture in wood is usually reckoned from the dry weight. Thus, if a certain piece of timber that weighed 165 lb. when green weighs 100 lb. when kiln dried, it would be said that in this instance the wood when green contained 65% of moisture.

The drying of green wood does not effect an increase in strength until the moisture is decreased to a value amounting to about 20 to 30% of the weight of the dry material; that is, the strength of a piece of green wood when being dried remains constant until the moisture remaining in the piece is reduced to from about 20 to 30% of the weight of the dry wood, and from this stage the strength starts to increase.

Experiments made by Tieman, the results of which are given in the Proceedings of the American Society for Testing Materials, show, for instance, the following variations in strength of long-leaf pine: If the strength of the green wood is taken as 1, then the strength of air-dried wood containing 12% moisture is 2.4, and that of kiln-dried wood containing $3\frac{1}{2}$% moisture is 2.9.

Thus, it is important when consulting tables of strength of timber to know the percentage of moisture contained in

the samples tested. Generally, this percentage amounts to about 18% in ordinary commercial stock.

The table on page 128 gives the ultimate compressive strengths of the more common kinds of timber, together with their moduli of elasticity.

SHORT POSTS

A post, or column, may in its elementary form be considered as a cubical or rectangular block, as shown in Fig. 1. If the post does not exceed in length from six to ten times the smallest dimension of its cross-sectional area, it is designated as a *short post*, or *column*.

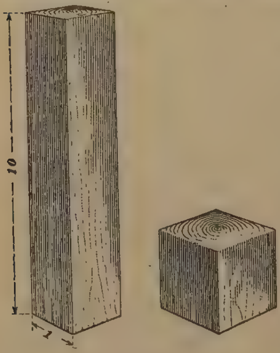


Fig. 1

The load that a short post may safely carry may be estimated by multiplying its sectional area, in square inches, by the safe unit compression of the material parallel to the grain. The ultimate unit values for compression are given in the table on page 128. A factor of safety of 5 is generally used, but in some instances it may be good practice to use 6. The proper factor of safety to choose is usually governed by the conditions to be met.

Example.—A short post of Georgia yellow pine is 12 in. square and 6 ft. long. What safe load will it support while standing on end? The factor of safety is 5.

Solution.—The ultimate strength of Georgia yellow pine is 5,000 lb.; hence, the safe unit compressive stress is $5{,}000 \div 5 = 1{,}000$ lb. The area of the post is 12 in. $\times$ 12 in. $=$ 144 sq. in. Therefore, the safe load is $144 \times 1{,}000 = 144{,}000$ lb.

Posts under compression develop more strength if the end surfaces are true and level. The tendency then is to resist

compression equally, and not to crush at one place before the remainder of the section can be brought under compression, as would be the case if the bearing surfaces were uneven and rough.

LONG POSTS

If the length of a post is over ten times its diameter or the width of its narrowest side, it is termed a *long post*. A post of this length, if not secured against yielding sidewise, is liable to bend before breaking, as shown in Fig. 2 (*a*). In this case, the compressive stress is not uniformly distributed over the cross-sectional area of the post, but will decrease from a maximum value at the concave side of the post to a minimum value at the convex side. Or, if the bending proceed far enough, the compressive stress at the convex side may change into one of tension. In some cases, the post will split, as shown in Fig. 2 (*b*), the two halves bending independently.

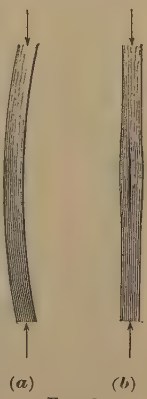


FIG. 2

The formula generally used for long square or rectangular wooden columns with flat ends, deducted from elaborate tests made on full-sized specimens at the Watertown Arsenal, is:

$$u = s - \frac{sl}{100\,d}$$

in which u is the ultimate strength of post per square inch of sectional area; s, the ultimate compressive strength of material, in pounds per square inch; l, the length of post, in inches; and d, the dimensions of least side of post, in inches.

EXAMPLE.—A white-pine post with flat ends is 10 in. square and 20 ft. long. Using a factor of safety of 6, what safe load will the post support?

SOLUTION.—The ultimate compressive strength of white pine parallel to the grain is 3,500 lb. per sq. in. Inserting the several values in the formula,

$$u = 3{,}500 - \left(\frac{3{,}500 \times 240}{100 \times 10}\right) = 2{,}660 \text{ lb.}$$

Since the factor of safety is 6, the safe bearing value per square inch of sectional area is $2{,}660 \div 6 = 443\frac{1}{3}$ lb. The area of the post being 100 sq. in., the safe load is $100 \times 443\frac{1}{3} = 44{,}333$ lb.

The column formulas in general use do not give a direct method of calculating the dimensions of a post that will safely support a given load. The usual method of obtaining this information is to assume values for the dimensions of the post, substitute these values in the formula, and then solve for u, the ultimate average compression per square inch of sectional area of post. If the assumed dimensions give a value of u that is satisfactory for the given conditions, they are accepted as correct. If, however, the resulting value of u is smaller than desirable, it shows that the sectional area is too small. Larger dimensions must then be chosen and the solution repeated until a satisfactory result is obtained.

If, on the contrary, the value of u is much greater than the required ultimate strength per square inch of the post section, a smaller cross-sectional area is chosen and the corresponding value of u is found. After a few trials, a size that gives a satisfactory stress for the given conditions is found.

Example.—Design to the nearest inch a white-oak post that is to be 15 ft. long and that is to carry a load of 40,000 lb. with a factor of safety of 5. The post is to be square in cross-section.

Solution.—Since a factor of safety of 5 is to be used, the required post must crush under a load of $5 \times 40{,}000 = 200{,}000$ lb. The ultimate compressive strength of white oak, from the table on page 128, is 5,000 lb. As a trial, first try a $7'' \times 7''$ post.

Substituting the correct values in the preceding formula,

$$u = 5{,}000 - \frac{5{,}000 \times 15 \times 12}{100 \times 7} = 3{,}714 \text{ lb. per sq. in.}$$

The ultimate strength of the post is therefore $3{,}714 \times 7 \times 7 = 181{,}986$ lb. But the required ultimate strength is

200,000 lb. Therefore, a 7″×7″ post is not strong enough. Next, try a 10″×10″ post. Thus,

$$u = 5{,}000 - \frac{5{,}000 \times 15 \times 12}{100 \times 10} = 4{,}100 \text{ lb. per sq. in.}$$

The ultimate strength of this post is therefore 4,100×10 ×10=410,000 lb. Since this value is greater than 200,000, the post is safe; but perhaps a smaller post would also fill the requirements. Therefore, try an 8″×8″ post. Thus,

$$u = 5{,}000 - \frac{5{,}000 \times 15 \times 12}{100 \times 8} = 3{,}875 \text{ lb. per sq. in.}$$

The ultimate strength of this post is therefore 3,875×8×8 =248,000 lb. Since this result is greater than the required ultimate strength, the post is strong enough. Moreover, it has been found that a 7″×7″ post is not strong enough. Therefore, unless the timber be cut to fractions of an inch, the 8″×8″ post is the smallest post that will fulfil the requirements.

If the formula just given is transposed so as to read $u = s\left(1 - \frac{l}{100\,d}\right)$, the factor $\left(1 - \frac{l}{100\,d}\right)$ may be calculated for various values of l and d and arranged in the form of a table, as shown on pages 184–186, that will prove of great assistance when using the formula.

In this table, the first column contains various lengths of posts, in inches, and in the top horizontal row are values of the least thickness of a post, in inches. In order to find the factor for a given post, first find the nearest length in the first column; then proceed from this value to the right until a point is reached that is below the least dimension in width. The value found at this point is the factor desired.

EXAMPLE.—A 12″×14″ Georgia-pine post is 18 ft. long. What safe load will it carry?

SOLUTION.—According to the table, the ultimate strength per square inch for Georgia pine is 5,000 lb. The length of the post, in inches, is 18×12=216, and the area of the post is 12×14=168 sq. in. According to the table, the factor is .821. Inserting this value in the formula, $u = s\left(1 - \frac{l}{100\,d}\right)$

CONSTANTS FOR RECTANGULAR WOODEN COLUMNS

Length Inches	Least Thickness, in Inches															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
10	.900															
15	.850															
20	.800	.900														
25	.750	.875														
30	.700	.850	.900													
35	.650	.825	.883													
40	.600	.800	.867	.900												
45		.775	.850	.888												
50		.750	.833	.875	.900											
55		.725	.817	.863	.890											
60		.700	.800	.850	.880	.900										
65		.675	.783	.838	.870	.892										
70		.650	.767	.825	.860	.883	.900									
75		.625	.750	.813	.850	.875	.893									
80		.600	.733	.800	.840	.867	.886	.900								
85			.717	.788	.830	.858	.879	.894								
90			.700	.775	.820	.850	.871	.887	.900							
95			.683	.763	.810	.842	.864	.881	.894							
100			.667	.750	.800	.833	.857	.875	.889	.900						
105			.650	.738	.790	.825	.850	.869	.883	.895						
110			.633	.725	.780	.817	.843	.863	.878	.890	.900					

115			.617	.713	.770	.808	.836	.856	.872	.885	.895					
120			.600	.700	.760	.800	.829	.850	.867	.880	.891	.900				
125				.688	.750	.792	.821	.844	.861	.875	.886	.896				
130				.675	.740	.783	.814	.838	.856	.870	.882	.892	.900			
135				.663	.730	.775	.807	.831	.850	.865	.877	.888	.896			
140				.650	.720	.767	.800	.825	.844	.860	.873	.883	.892	.900		
145				.638	.710	.758	.793	.819	.839	.855	.868	.879	.888	.896		
150				.625	.700	.750	.786	.813	.833	.850	.864	.875	.885	.893	.900	
155				.613	.690	.742	.779	.806	.828	.845	.859	.871	.881	.889	.897	
160				.600	.680	.733	.771	.800	.822	.840	.855	.867	.877	.886	.893	.900
165					.670	.725	.764	.794	.817	.835	.850	.863	.873	.882	.890	.897
170					.660	.717	.757	.787	.811	.830	.845	.858	.869	.879	.887	.894
175					.650	.708	.750	.781	.806	.825	.841	.854	.865	.875	.883	.891
180					.640	.700	.743	.775	.800	.820	.836	.850	.862	.871	.880	.888
185					.630	.692	.736	.769	.794	.815	.832	.846	.858	.868	.877	.884
190					.620	.683	.729	.763	.789	.810	.827	.842	.854	.864	.873	.881
195					.610	.675	.721	.756	.783	.805	.823	.838	.850	.861	.870	.878
200					.600	.667	.714	.750	.778	.800	.818	.833	.846	.857	.867	.875
205						.658	.707	.744	.772	.795	.814	.829	.842	.854	.863	.872
210						.650	.700	.738	.767	.790	.809	.825	.838	.850	.860	.869
215						.642	.693	.731	.761	.785	.805	.821	.835	.846	.857	.866
220						.633	.686	.725	.756	.780	.800	.817	.831	.843	.853	.863
225						.625	.679	.719	.750	.775	.795	.813	.827	.839	.850	.859
230						.617	.671	.713	.744	.770	.791	.808	.823	.836	.847	.856
235						.608	.664	.706	.739	.765	.786	.804	.819	.832	.843	.853
240						.600	.657	.700	.733	.760	.782	.800	.815	.829	.840	.850
245							.650	.694	.728	.755	.777	.796	.811	.825	.837	.847
250							.643	.687	.722	.750	.773	.792	.808	.821	.833	.844

CONSTANTS FOR RECTANGULAR WOODEN COLUMNS—(*Continued*)

Length Inches	Least Thickness, in Inches															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
255							.636	.681	.717	.745	.768	.788	.804	.818	.830	.841
260							.629	.675	.711	.740	.764	.783	.800	.814	.827	.838
265							.621	.669	.706	.735	.759	.779	.796	.811	.823	.834
270							.614	.663	.700	.730	.755	.775	.792	.807	.820	.831
275							.607	.656	.694	.725	.750	.771	.788	.804	.817	.828
280							.600	.650	.689	.720	.745	.767	.785	.800	.813	.825
285								.644	.683	.715	.741	.763	.781	.796	.810	.822
290								.638	.678	.710	.736	.758	.777	.793	.807	.819
295								.631	.672	.705	.732	.754	.773	.789	.803	.816
300								.625	.667	.700	.727	.750	.769	.786	.800	.813
305								.619	.661	.695	.723	.746	.765	.782	.797	.809
310								.613	.656	.690	.718	.742	.762	.779	.793	.806
315								.606	.650	.685	.714	.738	.758	.775	.790	.803
320								.600	.644	.680	.709	.733	.754	.771	.787	.800
325									.639	.675	.705	.729	.750	.768	.783	.797
330									.633	.670	.700	.725	.746	.764	.780	.794
335									.628	.665	.695	.721	.742	.761	.777	.791
340									.622	.660	.691	.717	.738	.757	.773	.788
345									.617	.655	.686	.713	.735	.754	.770	.784
350									.611	.650	.682	.708	.731	.750	.767	.781
355									.606	.645	.677	.704	.727	.746	.763	.778
360									.600	.640	.673	.700	.723	.743	.760	.775

$=5{,}000 \times .821 = 4{,}105$. The ultimate strength of the post is therefore $168 \times 4{,}105 = 689{,}640$ lb. With 5 as a factor of safety, the safe strength of the post is $689{,}640 \div 5 = 137{,}928$ lb.

CAST-IRON COLUMNS

COLUMN FORMULAS

Cast-iron columns are most frequently used in buildings of moderate height, but in some cases they have been used in buildings of sixteen, and even more, stories. The uncertain strength of cast iron has compelled the adoption of a low unit stress per square inch, or, in other words, a high factor of safety.

The safe loads that cast-iron columns will carry can be obtained by the use of the table on pages 188–189. The first column of this table gives the external diameters of various columns, and the second column, the several thicknesses that the column of a given diameter is likely to possess.

EXAMPLE.—A hollow, cylindrical column is 18 ft. long, and has an external diameter of 14 in. and an internal one of $11\frac{1}{2}$ in. (*a*) What safe load will it support? (*b*) What load will it support if the internal diameter is 11 in.?

SOLUTION.—(*a*) The thickness of the metal is $\dfrac{14-11.5}{2} = 1.25$ in. Proceeding in the table from 14 in the first column to the value $1\frac{1}{4}$ in the second column, and then continuing to the right until the column headed 18 is reached, the value 386 is found. This shows that the column will support a safe load of 386,000 lb.

(*b*) In a column having an internal diameter of 11 in., the thickness of the metal is $\dfrac{14-11}{2} = 1\frac{1}{2}$ in. Proceeding to the right from the value $1\frac{1}{2}$ in the second column of the table, the value 454 is found in the column headed 18; hence, the safe load for this column is 454,000 lb.

It is thus evident that a difference in the thickness of the

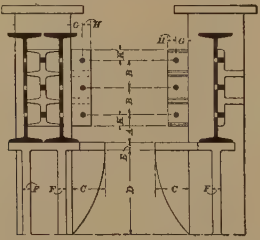
SAFE LOADS, IN THOUSANDS OF POUNDS, FOR HOLLOW, CYLINDRICAL CAST-IRON COLUMNS

(*Flat Ends. Factor of Safety = 8*)

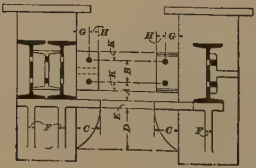
Outside Diameter. Inches	Thickness of Metal. Inches	Length of Column, in Feet: 6	8	10	12	14	16	18	20	22	24	Area of Section Square Inches	Weight per Foot of Column Pounds
6	$\frac{3}{4}$	105	94	82	72							12.37	38.65
	$\frac{7}{8}$	119	107	94	82							14.09	44.03
7	$\frac{3}{4}$	130	119	108	96	86						14.73	46.03
	$\frac{7}{8}$	149	136	123	110	98						16.84	52.63
8	$\frac{3}{4}$	155	145	133	122	110	99					17.08	53.38
	$\frac{7}{8}$	178	166	153	139	126	114					19.59	61.22
	1	200	186	172	157	142	128					21.99	68.72
9	$\frac{3}{4}$	180	170	160	151	135	124	113				19.44	60.75
	$\frac{7}{8}$	207	196	183	169	156	142	130				22.34	69.81
	1	233	220	206	190	175	160	146				25.13	78.53
	$1\frac{1}{8}$	258	244	228	211	194	177	162				27.83	86.97
10	$\frac{3}{4}$	205	195	185	173	161	149	138	117			21.80	68.13
	$\frac{7}{8}$	236	225	213	199	185	172	158	146			25.08	78.38
	1	266	253	240	225	209	194	179	164			28.28	88.38
	$1\frac{1}{8}$	295	281	266	249	232	215	198	182			31.37	98.03
	$1\frac{1}{4}$	323	308	291	273	254	235	217	200			34.36	107.38
11	$\frac{7}{8}$	268	254	242	230	216	202	188	175	162		27.83	86.97
	1	298	287	274	259	243	228	212	197	183		31.42	98.19
	$1\frac{1}{8}$	331	319	304	287	270	253	236	219	203		34.90	109.06
	$1\frac{1}{4}$	363	350	333	315	297	277	258	240	223		38.29	119.66
	$1\frac{3}{8}$	394	380	362	342	322	301	281	261	242		41.58	129.94

12	1	331	320	307	293	278	262	246	230	215	201	34.56	108.00
	1⅛	368	356	342	326	309	291	274	256	240	224	38.44	120.13
	1¼	404	391	375	358	339	320	301	281	263	245	42.22	131.94
	1⅜	439	425	408	389	369	348	327	306	286	267	45.90	143.44
	1½	474	458	440	419	397	375	352	330	308	288	49.48	154.63
13	1	363	353	341	327	312	296	280	264	249	234	37.70	117.81
	1⅛	404	393	379	364	347	330	312	294	277	260	41.97	131.16
	1¼	444	432	417	400	382	363	343	324	305	286	46.14	144.19
	1⅜	484	470	454	435	415	395	373	352	331	311	50.21	156.91
	1½	522	507	490	470	448	426	403	380	358	336	54.19	169.34
14	1⅛		430	417	402	386	369	352	333	315	298	45.51	142.22
	1¼		473	459	442	424	405	386	366	347	328	50.07	156.47
	1⅜		515	500	482	462	442	420	399	378	357	54.54	170.44
	1½		556	540	520	499	448	454	431	408	385	58.91	184.09
	1⅝		597	579	558	535	512	487	462	437	413	63.18	197.44
15	1¼			500	484	467	448	429	409	389	370	54.00	168.71
	1⅜			545	528	509	489	467	446	424	403	58.85	183.91
	1½			589	570	550	528	505	482	459	436	63.61	198.78
	1⅝			632	612	590	567	542	517	492	467	68.28	213.38
	1¾			674	653	630	605	579	552	525	499	72.84	227.63
16	1⅜				574	555	535	516	493	471	450	63.17	197.40
	1½				621	601	579	557	533	510	486	68.33	213.53
	1⅝				666	645	622	598	573	548	522	73.38	229.31
	1¾				711	689	644	638	611	585	558	78.34	244.81
	1⅞				756	731	705	678	649	621	592	83.20	260.00
18	1½					700	680	658	635	612	588	77.76	243.00
	1⅝					754	732	709	684	659	633	83.60	261.25
	1¾					806	782	757	731	704	677	89.34	279.19
	1⅞					856	832	805	777	749	720	94.99	296.84
	2					907	880	852	823	792	762	100.53	314.16

DIMENSIONS, IN INCHES, OF STANDARD CONNECTIONS TO CAST-IRON COLUMNS



Depth of Beam	A	B	C	D	E	F	G	H	K	Thickness of Lugs	Holes Cored for $\frac{3}{4}$-Inch Bolts
20	5	5	6	$10\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	2	$1\frac{1}{2}$	2	1	
18	4	5	6	$10\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	2	$1\frac{1}{2}$	2	1	
15	4	$3\frac{1}{2}$	$5\frac{1}{2}$	$9\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{4}$	2	$1\frac{1}{2}$	$1\frac{3}{4}$	1	
12	3	3	$4\frac{1}{2}$	$7\frac{3}{4}$	$1\frac{1}{4}$	$1\frac{1}{4}$	2	$1\frac{1}{2}$	$1\frac{1}{2}$	1	



Depth of Beam	A	B	C	D	E	F	G	H	K	Thickness of Lugs	Holes Cored for $\frac{3}{4}$-Inch Bolts
10	$3\frac{1}{4}$	$3\frac{1}{2}$	4	7	$1\frac{1}{4}$	1	2	$1\frac{1}{2}$	$1\frac{1}{2}$	1	
9	3	3	4	7	1	1	2	$1\frac{1}{2}$	$1\frac{1}{2}$	1	
8	$2\frac{1}{2}$	3	4	7	1	1	2	$1\frac{1}{2}$	$1\frac{1}{2}$	$\frac{3}{4}$	
7	$2\frac{1}{4}$	$2\frac{1}{2}$	4	7	1	1	2	$1\frac{1}{2}$	$1\frac{1}{2}$	$\frac{3}{4}$	

metal of $1.5-1.25=.25$ in. gives an increase in strength of $454{,}000-386{,}000=68{,}000$ lb.

The table may also be used for calculating the strength of columns when a factor of safety other than 8 is to be used.

Let W be safe load given in the table: W_1 the safe load corresponding to another factor of safety; and f, the new factor of safety. Then,

$$W : W_1 = f : 8,$$

and

$$W_1 = \frac{8\,W}{f}$$

$$W = \frac{W_1 f}{8}$$

EXAMPLE.—A hollow, cast-iron column of 13-in. external diameter and $1\frac{1}{4}$-in. metal is 18 ft. long. Assuming that the factor of safety is 6, find the safe load.

SOLUTION.—From the table $W=343{,}000$; hence, according to the formula,

$$W_1 = \frac{8 \times 343{,}000}{6} = 457{,}000 \text{ lb., nearly.}$$

Many concerns have their own standard designs for column connections and brackets. These are usually embodied in tables that give the required dimensions. The accompanying table gives the standard dimensions of brackets on cast-iron columns for I-beam connections, both for double and for single beams. The top surface of the shelf should have a pitch away from the column of $\frac{1}{8}$ in. to the foot to allow for the deflection of the beam. The top surface of the shelf should have a pitch away from the column of $\frac{1}{8}$ in. to the foot to allow for the deflection of the beam. As the holes in the column are cored, it will usually be necessary to have the beams drilled in the field in order to insure alinement.

In this table the values given in the columns marked A, B, C, etc. are the various dimensions for brackets, these dimensions being represented by corresponding letters in the figures accompanying the table. Thus, for a 12-in. I-beam connection, the distance from the bottom of the beam flange

to the center of the outside bolt of the vertical lug should be 3 in.; the pitch of the bolts, 3 in.; the projection of the bracket beyond the column, $4\frac{1}{2}$ in.; the depth of the vertical leg of the bracket, $7\frac{3}{4}$ in.; etc.

SAND AND CEMENT

CEMENTING MATERIALS

Any substance that becomes plastic under certain treatment and subsequently reverts to a tenacious and inelastic condition may, in a broad sense, be termed a *cement*. However, nearly all the cementing materials employed in building construction are obtained by the heating, or *calcination*, as it is called, of minerals composed wholly or in part of lime. The different composition of these minerals, as well as the properties of the calcined products, enables the various resulting substances to be classified as *limes*, *hydraulic cements*, *plasters*, and *miscellaneous cements*. Although all these materials have cementing properties, the term *cement* is commonly used to apply only to the group made up of hydraulic cements, *hydraulic* meaning that these substances possess the ability to *set*, or become hard, under water.

Limes and hydraulic cements (commonly called simply *cements*) are composed essentially of oxide of calcium, or lime, generally called *quicklime*, with which may be combined certain argillaceous, or clayey, elements, notably silica and alumina, it being to these elements that the hydraulic properties of certain of these materials are due. The quantity of silica and alumina present in these substances enables them to be classified as *common limes*, *hydraulic limes*, and *cements*.

The ratio of the quantity of silica and alumina present in these materials to the quantity of lime is called the *hydraulic index*. In common limes, this index is less than $\frac{10}{100}$; in hydraulic limes it lies between $\frac{10}{100}$ and $\frac{40}{100}$; and in cements, it exceeds $\frac{40}{100}$. These limes merge into each other so gradually, however, that it is often difficult to distinguish the dividing line between them.

LIMES

The commercial varieties of lime may be classified as common, hydrated, and hydraulic. The common limes, also called quicklimes, may be subdivided into rich, or fat, lime, and meager, or poor, lime.

Common Limes.—The grade of common lime known as *fat*, or *rich*, *lime* is almost pure oxide of calcium, CaO, and contains only about 5% of impurities. It has a specific gravity of about 2.3 and a great affinity for water, of which it absorbs about one-quarter of its weight. This absorption is accompanied by a great rise in temperature, by the lime bursting, and by the giving off of vapor. The lime finally crumbles into a powder. This powder occupies from two and one-half to three and one-half times as much volume as the original lime, the exact amount depending on its initial purity. When the lime is in this plastic state, it is said to be *slaked*. It is then unctuous and soft to the touch, and from this peculiarity it derives the name of *fat* or *rich*.

Meager, or *poor*, *lime* consists of from 60 to 90% of pure lime, the remainder being impurities, such as sand or other foreign matter. These impurities have no chemical action on the lime, but simply act as adulterants. Compared with fat lime, poor lime slakes more slowly and evolves less heat. The resulting paste is also thinner and not so smooth, greatly resembling fat slaked lime mixed with sand. Poor lime is not so good for building purposes as fat lime, nor has it such extensive use.

Hydrated Lime.—The class of lime called *hydrated lime* (calcium hydrate) is merely thoroughly slaked fat lime dried in the form of a fine powder, $Ca(OH)_2$. It is used extensively in conjunction with cement for making mortar, and also in the sand-lime brick industry.

Hydraulic Limes.—Limes that contain enough quicklime to slake when water is added, and enough clay or sand to form a chemical combination when wet, thus giving them the property of setting under water, are called *hydraulic limes*.

Limes of this class are made by burning limestones containing from 5 to 30% of clay or sand. They are often

considered as divided into three classes, namely, *feebly hydraulic*, *ordinarily hydraulic*, and *eminently hydraulic*, in proportion to the quantity of argillaceous materials present. The slaking qualities vary from slaking in a few minutes with considerable heat after water is added, in the feebly hydraulic to slaking only after many hours, with practically no evolution of heat and without cracking or powdering, in the eminently hydraulic. The time of setting under water also varies from setting as hard as soap in 2 years, with the feebly hydraulic, to becoming as hard as stone in 3 or 4 days, with the eminently hydraulic. If carbonate of magnesia is present in the lime, it reduces the energy of the slaking, but increases the rapidity of the setting and the ultimate strength when set.

CEMENTS

Cement may be divided into four general classes: Portland, natural, puzzolan (also called pozzuolana), and mixed. The relative importance of each cement is indicated by the order in which it is named.

Portland cement may be defined as the product resulting from the process of grinding an intimate mixture of calcareous (containing lime) and argillaceous (containing clay) materials, calcining (heating) the mixture until it starts to fuse, or melt, and grinding the resulting clinker to a fine powder. It must contain not less than 1.7 times as much lime by weight as it does of those materials which give the lime its hydraulic properties, and must contain no materials added after calcination, except small quantities of certain substances used to regulate the activity or the time of setting.

Natural cement is the product resulting from the burning and subsequent pulverization of an argillaceous limestone or other suitable rock in its natural condition, the heat of burning being insufficient to cause the material to start to melt.

Puzzolan cement is a material resulting from grinding together, without subsequent calcination, an intimate mixture of slaked lime and a puzzolanic substance, such as

blast-furnace slag or volcanic scoria. The only variety of puzzolan cement employed at all extensively in American practice is *slag cement*. This cement is made by grinding together a mixture of blast-furnace slag and slaked lime. The slag used for this purpose is granulated, or quenched, in water as soon as it leaves the furnace, which operation drives off most of the dangerous sulphides and renders the slag puzzolanic.

Mixed cements cover a wide range of products obtained by mixing, or blending, the foregoing cements with each other or with other inert substances. *Sand cements*, *improved cements*, and many second-grade cements belong to this class. Mixed cements. however, are of comparatively little importance.

Properties of Cements.—The hydraulic cements differ from the limes in that they do not slake after calcination, and that they set, or harden, under water. They can be formed into a paste with water without any sensible increase in volume and with little, if any, disengagement of heat. They do not shrink appreciably in hardening, so that the sand and broken stone with which they are mixed are employed merely through motives of economy and not, as with limes, of necessity.

The color of the different grades of cement is variable, but in certain cases it is distinctive. Portland cement is a dark-bluish or greenish gray; if it is a light yellow, it may indicate underburning. Natural cement ranges in color from a light straw, through the grays, to a chocolate brown. Slag cement is gray with usually a tinge of lilac. In general, however, the color of cement is no criterion of its quality

Cement is packed either in wooden barrels or in cloth or paper bags, the latter being the form of package most commonly employed. A barrel of Portland or of slag cement contains the equivalent of 4 bags, while but 3 bags of natural cement equals a barrel. The average weights of the various cements are given in the table on page 196.

In proportioning mortar or concrete by volume, the common assumption is that a bag of Portland cement occupies .9 cu. ft.

AVERAGE WEIGHTS OF HYDRAULIC CEMENTS

Kind of Cement	Net Weight of Bag Pounds	Net Weight of Barrel Pounds	Weight per Cubic Foot Pounds	
			Packed	Loose
Portland ...	94	376	100–120	70–90
Natural....	94	282	75–95	45–65
Slag........	82½	330	80–100	55–75

Portland cement may be distinguished by its dark color, heavy weight, slow rate of setting, and greater strength. Natural cement is characterized by lighter color, lighter weight, quicker set, and lower strength. Slag cement is somewhat similar to Portland, but may be distinguished from it by its lilac-bluish color, by its lighter weight, and by the greater fineness to which it is ground.

Portland cement is adaptable to any class of mortar or concrete construction, and is unquestionably the best material for all such purposes. Natural and slag cements, however are cheaper, and, under certain conditions, may be substituted for the more expensive Portland cement. All heavy construction, especially if exposed, all reinforced-concrete work, sidewalks, concrete blocks, foundations of buildings, piers, walls abutments, etc. should be made with Portland cement. In second-class work, as in rubble masonry, brick sewers, unimportant work in damp or wet situations, or in heavy work in which the working loads will not be applied until long after completion, natural cement may be employed to advantage. Slag cement is best adapted to heavy foundation work that is immersed in water or at least continually damp. This kind of cement should never be exposed directly to dry air, nor should it be subjected either to attrition or impact.

SAND AND ITS MIXTURES

SAND

Sand is an aggregation of loose grains of crystalline structure, derived from the disintegration of rocks. It is called *silicious*, *argillaceous*, or *calcareous*, according to the character of the rock from which it is derived. Sand is obtained from the seashore, from the banks and beds of rivers, and from land deposits. The first class, called *sea sand*, contains alkaline salts that attract and retain moisture and cause efflorescence in brick masonry. This efflorescence is not at first apparent but becomes more marked as time goes on. It can be removed temporarily at least by washing the stonework in very dilute hydrochloric acid. The second, termed *river sand*, is generally composed of rounded particles, and may or may not contain clay or other impurities. The third, called *pit sand*, is usually composed of grains that are more angular; it often contains clay and organic matter. When washed and screened, it is a good sand for general purposes.

Sand is used in making mortar because it prevents excessive shrinkage and reduces the quantity of lime or cement required. Lime adheres better to the particles of sand than it does to its own particles; hence, it is considered that sand adds strength to lime mortar. On cement mortar, on the contrary, sand has a weakening effect.

Properties of Sand.—The weight of sand is determined by merely filling a cubic-foot measure with dried sand and obtaining its weight. Dry sand weighs from 80 to 120 lb. per cu. ft.; moist sand, however, occupies more space and weighs less per cubic foot. The weight of sand is more or less dependent on its specific gravity and on the size and shape of the sand grains, but, other things being equal, the heaviest sand makes the best mortar.

The specific gravity of sand ranges from 2.55 to 2.80. For all practical purposes the specific gravity may be assumed to be 2.65 with little danger of error.

By *percentage of voids* is meant the amount of air space in the sand. Structurally, it is one of the most important

properties of sand. The greater these voids, the more cement paste will be required to fill them in order to give a dense mortar. The percentage of voids may be determined by observing the quantity of water that can be introduced into a vessel filled with sand, but it is best computed as follows:

$$\text{percentage of voids} = 100 - \frac{100 \times \text{weight per cubic foot}}{62.5 \times \text{specific gravity}}$$

EXAMPLE.—What is the percentage of voids in a sand having a specific gravity of 2.65 and weighing 105 lb. per cu. ft.?

SOLUTION.—Substituting in the formula, the percentage of voids is

$$100 - \frac{100 \times 105}{62.5 \times 2.65} = 100 - 63.4 = 36.6$$

The percentage of voids depends principally on the size and shape of the sand grains and the gradation of its fineness, and hence will vary from 25 to 50%. Sand containing over 45% of voids should not be used to make mortars.

The shape of the grains of sand is of chief importance in the influence that the sand exerts on the percentage of voids. Obviously a sand with rounded grains will compact into a more dense mass than one whose grains are angular or flat like particles of mica. Therefore the more nearly the grains approach the spherical in shape, the more dense and strong will be the mortar. This fact is contrary to the common opinion on the subject.

The fineness of sand is determined by passing a dried sample through a series of sieves having 10, 20, 30, 40, 50, 74, 100, and 200 meshes, respectively, to the linear inch. The result of this test, expressed in the amount of sand passing each sieve, is known as the *granulometric composition* of the sand. Material that does not pass a ¼-in. screen is not considered to be sand, and should be separated by screening. Sand that is practically all retained on a No. 30 sieve is called *coarse*, while 80 or 90% of sand known as *fine* will pass through this sieve. Fine sand produces a weaker mortar than coarse sand, but a mixture of fine and coarse sand will surpass either one.

The purity, or cleanness, of sand may be roughly ascertained by rubbing it between the fingers and observing how much dirt remains. To determine the percentage of the impurities more accurately, a small dried and weighed sample is placed in a vessel and stirred up with water. The sand is allowed to settle, the dirty water poured off, and the process repeated until the water pours off clear. The sand is then dried and weighed. The loss in weight gives the quantity of impurities contained in the sand. The presence of dirt, organic loam, mica, etc. is decidedly injurious and tends to weaken the resulting mortar. Clay or fine mineral matter in small proportions may actually result in increased strength, but excessive quantities of these materials may be a possible source of weakness. The best modern practice limits the quantity of impurities found by this washing test to 5%.

It is advisable, prior to the selection of a sand, to determine what its strength will be when made into mortar.

Preparation of Sand.—Sand is prepared for use by (1) screening to remove the pebbles and coarser grains the fineness of the meshes of the screen depending on the kind of work in which the sand is to be used; (2) washing, to remove salt, clay, and other foreign matter; and (3) drying if necessary. When dry sand is required, it is obtained by evaporating the moisture either in a machine, called a *sand dryer*, or in large, shallow, iron pans supported on stones, with a wood fire placed underneath.

LIME AND CEMENT MORTARS

Mortars are composed of lime or cement and sand mixed to the proper consistency with water. The proportions of the ingredients depend on the character of the work in which the mortar is to be used.

In proportioning mortar, the quantities of the separate ingredients are usually designated by a ratio, such as 1–1, 1–2, 1–3, etc. Thus, 1–2 signifies that 1 part of lime or cement is used to 2 parts of sand; etc. For great accuracy these measurements should be made by weight, but they are usually made by volume, which is almost the same thing.

Lime Mortars.—In *lime mortar*, besides effecting an economy, the presence of sand is necessary to prevent the shrinkage that would otherwise occur during the hardening of the paste.

When a mortar is made of lime and sand, enough lime should be present to just cover completely each grain of sand. An excess of lime over this quantity will cause the mortar to shrink excessively on drying, while a deficiency of lime will produce a weak and crumbly mortar. The correct quantity of lime depends on the character of the ingredients, the method of treatment, and, to some extent on the judgment of the builder. The mixtures employed vary from 1–2½ to 1–5. Building laws in many municipalities require the use of a 1–3 mixture, and for most materials this proportion will be found satisfactory, although for rich, fat limes a 1–3½ or a 1–4 mixture is sometimes preferable.

In mixing lime mortar, a bed of sand is made in a mortar box, and the lime distributed as evenly as possible over it, first measuring both the lime and the sand in order that the proportions specified may be obtained. The lime is then slaked by pouring on water, after which it should be covered with a layer of sand, or, preferably, a tarpaulin, to retain the vapor given off while the lime is undergoing the chemical reaction of slaking. Additional sand is then used, if necessary, until the mortar attains the proper proportions.

Care should be taken to add just the proper quantity of water to slake the lime completely to a paste. If too much water is used, the mortar will never attain its proper strength, while if too little is used at first, and more is added during the process of slaking, the lime will have a tendency to chill, thereby injuring its setting and hardening properties. Rather than make up small batches, it is considered better practice to make lime mortar in large quantities and to keep it standing in bulk so that it can be used as needed.

Lime mortar is employed chiefly for brickwork of the second class and its use is continually decreasing as that of cement increases. It is absolutely unsuitable for any important construction, because it possesses neither strength

nor the property of resisting water. It cannot be used in damp or wet situations, nor should it ever be laid in cold weather, as it is very susceptible to the action of frost, being much injured thereby. Moreover, since it hardens by the action of dry air, only the exterior of lime mortar ever becomes fully hardened, so that anything like a concrete with lime as a matrix is impossible. However, for second-class brickwork, such as is commonly used in the walls of smaller buildings, lime mortars are economical and sufficiently good.

The strength of lime mortars is extremely variable, depending on the ingredients themselves and on their treatment, environment, etc. Moreover, it is unsafe to figure a lime-mortar joint as possessing much strength, since only a part of the joint is hardened and capable of developing any strength at all. The tensile strength of thoroughly hardened 1–3 lime mortars averages from 40 to 70 lb. per sq. in., and the compressive strength from 150 to 300 lb.

Cement Mortars.—The sand for all mortars should be clean, of suitable size and granulometric composition. For structures designed to withstand heavy unit stresses, or for those intended to resist either the penetration of moisture or the actual pressure of water, the selection of the sand should be most carefully made. Generally, it is not advisable to use a sand containing over 5% of loam by the washing test, nor one that soils the fingers when it is rubbed between them. Very fine sand, such as is found on the seashore, should not be employed in mortar unless it is intended simply for pointing or for grouting.

A simple method of determining the best sand for cement mortar is to prepare mixtures of the cement, sand, and water, using the same quantities in each case, and then to place each mixture in a measure; that mixture giving the least volume of mortar may be considered to contain the most desirable sand for use.

Limestone screenings, brick dust, crushed cinders, etc., are sometimes substituted for sand in making mortars, and, if care is taken in their selection, they may prove economical and entirely suitable for certain purposes.

The theory of the composition of a correctly proportioned mortar is that the cement paste will just a little more than fill all the voids between the particles of sand, thus giving an absolutely dense mortar at the least expense. The correct proportion of cement to sand, therefore, is more or less variable, depending on the granulometric composition of the sand. Since, however, Portland-cement paste that has set weighs nearly as much as sand, and since the average sand contains about 30 to 40% of voids, it is evident that 1–3 mixtures most nearly approach the best and most economical proportion.

Mortars, however, are made in proportions varying from 1–1 to 1–8. The richer mixtures are used for facing, pointing, waterproofing, granolithic mixtures, etc., the 1–2 mixture being usually made for such purposes. The leaner mixtures are used for rough work, filling, backing, etc., but should never be employed where either much strength or much density is desired. Natural-cement mortars are commonly made 1 part of sand less than Portland-cement mortars intended for the same purpose; that is, where a 1–3 Portland-cement mortar would be used, a 1–2 natural mortar would be required, although natural-cement mortars should be decreased by about 2 parts of sand to equal the strength of Portland. In other words, a 1–4 Portland mortar closely equals the strength of a 1–2 natural mortar. Puzzolan cements are usually proportioned the same as Portlands.

Cements are commonly proportioned by volume, the unit volume of the cement barrel being assumed. If a 1–3 mortar is desired, a box having a capacity of 10.8 cu. ft. is filled with sand and mixed with 4 bags or 1 bbl. of cement. A box 3 ft. $3\frac{7}{16}$ in. square and 1 ft. deep will have a capacity of very nearly 10.8 cu. ft. and, besides, makes a convenient size of box for actual work.

For general purposes, the mortar should be of a plastic consistency—firm enough to stand at a considerable angle yet soft enough to work easily. Wet mortars are easiest to work and are the strongest. However, they are subject to greater shrinkage, are slower setting, and are more easily

attacked by frost. Dry mortars, on the other hand, are often friable and porous.

In the following table are given the quantities of materials required to produce 1 cu. yd. of compacted mortar. The proportions are by volume, a cement barrel being assumed to contain 3.6 cu. ft.

MATERIALS REQUIRED PER CUBIC YARD OF MORTAR

Kind of Mixture	Portland Cement Barrels	Loose Sand Cubic Yards
1–1	4.95	.65
1–2	3.28	.88
1–3	2.42	1.01
1–4	1.99	1.06
1–5	1.62	1.11
1–6	1.34	1.15
1–7	1.18	1.17
1–8	1.05	1.18

Example.—How much cement and sand will be required to obtain 8.5 cu. yd. of 1–3 Portland-cement mortar?

Solution.—According to the table, 1 cu. yd. of a 1–3 Portland-cement mortar requires 2.42 bbl. of cement; therefore, 8.5 cu. yd. will require $8.5 \times 2.42 = 20.57$ bbl. of cement. Also, since 1 cu. yd. of a mixture of this kind requires 1.01 cu. yd. of sand, the quantity of sand required will be $8.5 \times 1.01 = 8.59$ cu. yd.

Mortar that is to be mixed by hand is prepared on a platform or in a mortar box. The sand is first measured by means of a bottomless box with handles on the sides. After filling the box, the sand is struck off level, the box lifted up, and the sand spread in a low, flat pile. The required number of bags of cement are then emptied on the sand and spread evenly over it. The pile is then mixed with shovels, working through it not less than four times. After this operation, the dry mixture is formed into a ring, or crater, and the water intended to be used is poured into the center. The material from the sides of the basin is then shoveled

into the center until the water is entirely absorbed, after which the pile is worked again with shovels and hoes until the mixture is uniform and in a plastic condition.

Another method of mixing, where a mortar box is used, is to gather the mixed dry materials at one end of the box and pour in the water at the other end drawing the mixture into the water with a hoe, a little at a time, and hoeing until a plastic consistency is obtained.

Properties and Uses of Cement Mortars.—The strength of a mortar is measured by its resistance to tensile, compressive,

TENSILE STRENGTH OF CEMENT MORTARS

Proportions		Tensile Strength, in Pounds per Square Inch					
		Portland Cement			Natural Cement		
Cement Parts	Sand Parts	7 da.	28 da.	3 mo.	7 da.	28 da.	3 mo.
1	1	450	600	610	160	245	280
1	2	280	380	395	115	175	215
1	3	170	245	280	85	130	165
1	4	125	180	220	60	100	135
1	5	80	140	175	40	75	110
1	6	50	115	145	25	60	90
1	7	30	95	120	15	50	75
1	8	20	70	100	10	45	65

cross-breaking, and shearing stresses, and also by determinations of its adhesion to inert surfaces, its resistance to impact, abrasion, etc. There is no definitely fixed ratio between the strength of mortar subjected to these different stresses, but there is nevertheless a close relation between them, so that, practically, it may be assumed that if a mortar shows either abnormally high or low values in any one test, the same relation will develop when tested under other stresses. In practice, therefore, the strength of mortar is

commonly determined through its resistance to tensile stresses, and its resistance to other forms of stress is computed from these results.

The tensile strength of mortar has been shown to vary with the character of its ingredients, with its consistency, its age, and with many other factors. In the accompanying table is given a fair average of the tensile strength that may be expected from mortars of Portland and natural cements that are made in the field and with a sand of fair quality but not especially prepared.

The strength of Portland-cement mortar increases up to about 3 mo.; after that period, it remains practically constant for an indefinite time. Natural-cement mortar, on the other hand, continues to increase in strength for 2 or 3 yr., its ultimate strength being about 25% in excess of that attained in 3 mo. The strength of slag-cement mortar averages about three-quarters of that of Portland-cement mortar.

The compressive strength of cement mortars is usually given in textbooks as being from eight to ten times the tensile strength. This value is rather high for the average mortar, a ratio of from 6 to 8 being one more nearly realized in practice. The ratio increases with the age and richness of the mortar, and varies considerably with the quality of the sand. Portland-cement mortars of 1–3 mixture that are 3 mo. old develop, on an average, a compressive strength of about 1,800 lb. per sq. in., while 1–2 natural-cement mortars average about 1,600 lb.

The strength of mortars in cross-breaking and shear may be taken at about one and one-half to two times the tensile strength, with a fair amount of accuracy.

The adhesion of mortars to inert materials varies both with the character of the mortar and with the roughness and porosity of the surfaces with which they are in contact. The adhesion of 1–2 Portland-cement mortar, 28 da. old to sandstone averages about 100 lb. per sq. in.; to limestone, 75 lb.; to brick, 60 lb.; to glass, 50 lb.; and to iron or steel, 75 to 125 lb. Natural-cement mortars have nearly the same adhesive strength as those made of Portland cement.

In bricklaying and in other places in which mortar is employed it is frequently desired to use a material that is more plastic or smoother than pure cement mortar. This quality is usually obtained by adding from 10 to 25% of lime to the mortar. This addition of lime not only renders the mortar more plastic, and hence easier to work, but also increases both its adhesive strength and its density, which assists in making the mortar waterproof. Hydrated lime is to be preferred for use in cement mortar, because its complete slaking is assured. Hydrated lime may also be readily handled and measured on the work.

Occasionally, small quantities of cement are added to lime mortars so as to make them set quicker and to increase their strength. Such mixtures, however, are not especially economical nor are they convenient in practice.

Retempering of Mortar.—Mortar composed of cement, sand, and water soon begins to set and finally becomes hard. When it is desired to use this material, more water has to be added and the mixture worked until it again becomes plastic. This process is called *retempering*. Laboratory tests generally show that retempering slightly increases the strength of mortar, but the reworking is more thorough as a rule in the laboratory than would be the case in actual work. Any part of the hardened mortar that is not retempered is a source of weakness when incorporated in the building. The adhesive strength of cement, moreover, is greatly diminished. by this process For these reasons, it is generally inadvisable to permit the use of retempered mortars; but if they are allowed, great care should be taken to see that the second working is thorough and complete.

Laying Mortar in Freezing Weather.—Frost or even cold has a tendency to retard greatly the set of cement mortars. When the temperature, moreover, is so low that the water with which the mortar is mixed freezes before it combines with the cement, it may, if care is not exercised, result in complete destruction of the work. A single freezing is not particularly harmful because when thawing occurs, the arrested chemical action continues. A succession of alternate freezings and thawings, however, is extremely injurious.

Nevertheless, Portland-cement mortars may be laid even under the worst conditions if certain precautions are observed, but mortars of natural cement should never be used in extremely cold weather, as they are generally completely ruined by freezing.

The bad results that arise during mild frosts may be successfully guarded against by heating the sand and water and by using a quick-setting cement mixed rich and as dry as possible. In extremely cold weather, salt must be added to the water, so as to convert it into a brine that requires a temperature lower than 32° F. to freeze it. The common rule for adding salt is to use a quantity equal to 1% of the weight of the water for each degree of temperature that is expected below 33° F. Thus, at 32° F., a 1% solution would be used, while at 25°, an 8% solution would be required. Solutions greater than 12% should not be employed, and if a temperature below 20° F. is expected, heat must be used in addition to the salt. The finished work should also be protected with canvas or straw. Manure should not be used for this purpose, because the acids it contains tend to rot the cement. Unless the conditions are such as to make it imperative, it is not advisable to lay mortars during freezing weather.

Shrinkage of Mortars.—Cement mixtures exposed to the air shrink during the process of hardening, while those immersed in water tend to expand. The shrinkage of ordinary cement mortars is slight, and when they are used as a bonding material it need not be considered. When used as a monolith, as in sidewalks, shrinkage is guarded against by keeping the mortar wet during setting. This can be done by covering with moist straw or by sprinkling the mixture with water.

Grouting.—By *grouting* is meant the process of filling spaces in masonry with a thin, semifluid mixture known as *grout*. This mixture consists of cement, 1 or 2 parts of sand, and an excess of water. Grout can be used for filling the voids in walls of rubble masonry for backing arches and tunnels, and for filling the joints between paving brick. In fact, it can be used in all places where mortar cannot be laid in the ordinary

manner. When hardened, grout is weak, friable, and porous.

Coloring of Mortars.—Colors are often used in mortars to effect contrasts, or to subdue the glaring tone of cement in sidewalks or in similar situations. Red lead weakens mortar and should not be used. The color of hardened mortar is quite different in appearance from one that is still wet, so that where it is important to secure the correct tints, preliminary trials should be made until the proportions desired have been determined.

The various materials employed to produce different colors in mortar, together with the quantity required per barrel of cement, are as follows: For gray, 2 lb. of lampblack; for black, 45 lb. of manganese dioxide; for blue, 19 lb. of ultramarine; for red 22 lb. of iron oxide; for bright red, 22 lb. of Pompeian or English red; and for violet, 22 lb. of violet oxide of iron.

TESTS ON CEMENT

FIELD INSPECTION

In order to determine correctly the structural value of a shipment of cement, an examination in the field is very necessary. A number of packages of cement should be weighed at intervals, and the average weight should never be permitted to fall below 94 lb. per bag, since mortar and concrete are usually proportioned on the assumption of this weight. Each package should also be plainly marked with the brand and name of the manufacturer; those not branded should be discarded, and, if possible, a mixture of different brands should be avoided.

A possible indication of inferiority is the presence of lumps throughout the bulk of the material. On standing, cement gradually absorbs moisture from the air. At first this moisture is present in merely a minute and harmless state, but eventually it combines chemically with the cement; that is, in the same manner as when cement and water are actually mixed together in practice. In the first condition, lumps usually appear, but they are so soft that they may be readily

crushed with the fingers, and of course would be entirely broken up when mixed into mortar. When, however, the cement contains lumps that are hard and pebble-like and that can be crushed only with considerable effort, it indicates that chemical action has begun. Cement containing any appreciable amount of these hardened lumps is generally of decidedly inferior quality, and it should never be permitted to enter any important part of a structure.

Storing cement too long will tend to weaken it. Cement from 2 to 6 mo. old is usually the safest and will produce the best results.

The color of Portland cement, ranging from bluish to yellowish gray, affords no indication of quality except in cases where different shipments or different parts of the same shipment show a variation in color, thus pointing to a lack of uniformity.

SAMPLING

In securing a sample for testing, the essential point is to get one that will fairly represent the entire shipment whose qualities are to be determined. The common practice is to take a small portion of material from every tenth barrel, or, what is the same thing, from every fortieth bag. When tests are to be made, however, on a shipment of only a few barrels, more packages than one in ten should be opened; and when the shipment is large, say over 150 bbl., it should be subdivided and each portion tested separately. The bags selected should be taken at random and from different layers and not all from one part of the pile.

The cement, moreover, should be taken not only from the top of the packages, but from the center and sides as well. When the cement is contained in barrels, a sampling auger is used to extract the sample, a hole being bored in the staves midway between the heads.

After the samples of cement have been taken from the packages they are thoroughly mixed in a can or basin, and this mixed sample is used for the various tests. To make a complete series of tests, the sample should contain from 6 to 8 lb. The cement, after sampling and before testing,

must be well protected, as exposure to heat, cold, dampness, or any other abnormal condition may seriously affect the results.

PURPOSE AND CLASSIFICATION OF TESTS

In order that a mortar or a concrete made with cement shall give good results in actual construction it must possess two important properties, namely, *strength* and *durability*. The primary purpose of cement testing, therefore, is to determine whether any particular shipment of cement possesses sufficient strength and durability to admit of its use in construction.

A determination of the quality of cement necessitates the employment of several tests, which may be classified as *primary tests* and *secondary tests*. The former tests, which include tests for *soundness* and tensile strength, are made to give directly a measure of the essential qualities of strength and durability. Unfortunately, neither of these tests is capable of being made with precision. Therefore, the secondary tests, which include tests to determine the time of setting, the fineness, the specific gravity, and the chemical analysis, are made to obtain additional information in regard to the character of the material. However, with the possible exception of the test of time of setting, the secondary tests have but little importance and only indicate by their results indirectly the properties of the material.

PRIMARY TESTS

TESTS FOR SOUNDNESS

Soundness may be defined as the property of cement that tends to withstand any forces that may operate to destroy or disintegrate it. This property of soundness, or, as it is sometimes called, *constancy of volume*, is the most important requisite of a good cement.

The most common cause of unsoundness in Portland cement is an excess of free or uncombined lime, which crystallizes with great increase of volume, and thus breaks up and destroys the bond of the cement. This excess of lime

may be due to incorrect proportioning or to insufficient grinding of the raw materials, to underburning, or to lack of sufficient storing before use, called *seasoning*. A certain amount of seasoning is usually necessary, because almost every cement, no matter how well proportioned or burned it may be, will contain a small amount of this excess of lime, which, on standing, will absorb moisture from the air, slake, and become inert.

Excess of magnesia or the alkalies may also cause unsoundness, but the ordinary cement rarely contains a sufficient amount of these ingredients to be harmful. Sulphate of lime is occasionally responsible for unsoundness, but this ingredient usually acts in the opposite direction, tending to make sound a cement that otherwise might disintegrate.

The property of soundness is determined in one or more of three ways: by measurements of expansion, by normal tests, and by accelerated tests.

Measurements of expansion are made by forming specimens of cement, usually in the shape of prisms, and measuring the change in volume by means of a micrometer screw. At the present time, however, it is believed that expansion is not a sure index of unsoundness, so that this test is seldom employed.

Normal tests consist in making specimens of cement mixed with water, preserving them in air or in water under normal conditions, and observing their behavior. The common practice is to make from a paste of neat, or pure, cement on glass plates about 4 in. square, two circular pats, about 3 in. in diameter, $\frac{1}{2}$ in. thick at the center, and tapering to a thin edge. These pats are kept in moist air for 24 hr.; then one of them is placed in fresh water of ordinary temperature and the other is preserved in air. The condition of the pats is observed 7 da. and 28 da. from the date of making, and thereafter at such times as may be desired.

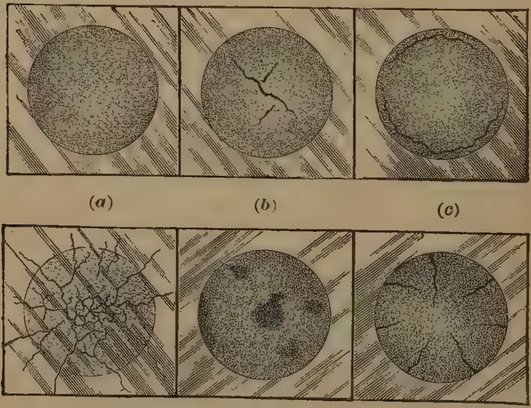
The most characteristic forms of failure are illustrated in Figs. 1 and 2.

Fig. 1 (*a*) shows a pat in good condition.

Fig. 1 (*b*) illustrates shrinkage cracks that are due, not to inferior cement, but to the fact that the pat has been allowed

to dry out too quickly after being made. Pats must be kept in a moist atmosphere while hardening, or these cracks, indicative merely of careless manipulation will develop.

Fig. 1 (*c*) shows cracks that are due to the expansion of the cement. This condition is common in the air pats, and is not indicative of injurious properties. Pats kept in water, however, should not show these cracks.



(*a*) (*b*) (*c*)

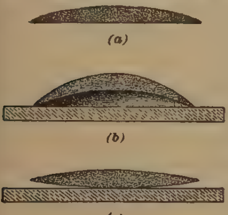
(*d*) (*e*) (*f*)

FIG. 1

Fig. 1 (*d*) shows cracking of the glass plate to which the pat is attached. This cracking is caused by expansion or contraction of the cement, combined with strong adhesion to the glass. It rarely indicates injurious properties.

Fig. 1 (*e*) illustrates blotching of the pats, the cause of which should always be investigated by chemical analysis or otherwise, which may or may not warrant the rejection of the material. Slag cements or cements adulterated with slag invariably show this blotching.

Fig. 1 (*f*) shows the radial cracks that mark the first stages of disintegration. Such cracks should never occur with good material. They are signs of real failure, and cement showing them should never be used.



(*a*) (*b*) (*c*)

FIG. 2

Fig. 2 shows three pats that, for different reasons, have left the glass plate on which they were made. The disk shown in (*a*) left the plate because of lack of adhesion; the one in (*b*), through contraction; and the one in (*c*), through expansion. The condition illustrated in (*a*) is never dangerous in either air or water; that in (*c*) is only dangerous when existing in a marked degree; and that in (*b*) hardly ever occurs in water, but in air it often indicates dangercus properties. Air pats that develop the curvature shown in (*b*) generally disintegrate later. A curvature of about $\frac{3}{8}$ in. in a 3-in. pat can be considered to be about the limit of safety.

The normal pat tests are the only absolutely fair and accurate methods of testing cements for soundness, but the serious objection to them lies in the fact that frequently several months or even years elapse before failure in the cement so tested becomes apparent. To overcome this difficulty the *accelerated tests* have been devised. These tests are intended to produce in a few hours results that require months in the normal tests.

Many forms of accelerated tests have been devised. At present, however, the only tests employed commercially are the boiling test and the steam test.

The *boiling test* is made by forming specimens of neat-cement paste into pats, such as are employed for the normal tests, or preferably into balls about $1\frac{1}{2}$ in. in diameter. The specimens are allowed to remain in moist air for 24 hr. and are then tested.

The form of apparatus used for the boiling test is shown in Fig. 3. It consists of a copper tank that is heated by a

Bunsen burner and is filled with water. The water in the tank is kept at a uniform height by means of a constant-level bottle. A wire screen placed an inch from the bottom of the tank prevents the specimens from coming into contact with the heated bottom. The test pieces, which are 24 hr. old, are placed in the apparatus, which is filled with water of a normal temperature, and heat is applied at a rate such that the water will come to boiling in about ½ hr. Quiet boiling is continued for 3 hr., after which the specimens are removed and examined. Care must be taken that the water employed

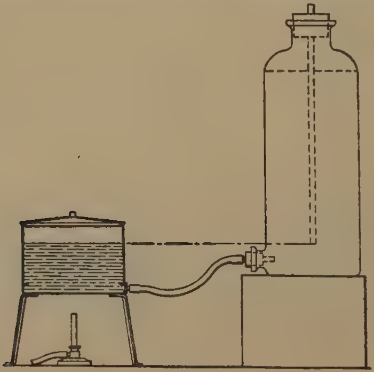


FIG. 3

is clean and fresh, because impure water may seriously affect the results. The same water, also, should never be used for more than one test. A good cement will not be affected by this treatment, and the ball will remain firm and hard. Inferior cement will fail by checking, cracking, or entirely disintegrating.

The *steam test* is made in the same way as the boiling test, except that instead of immersing the specimens in water, they are kept in the steam above the water. The apparatus employed is the same as that used for the boiling test. The

wire screen, however, is raised so that it is an inch above the surface of the water; also, there must be provided a cover that is close enough to retain the steam without creating pressure. The steam test is less severe than the boiling test and is somewhat less accurate.

Results of Tests for Soundness.—The result of the normal tests, if properly made and interpreted, may be considered reliable guides to the soundness of the material, and cement failing in these tests should always be rejected. The accelerated tests, on the other hand, furnish merely indications, and are by no means infallible. A cement passing the boiling test can generally be assumed sound and safe for use, but, if failure occurs, it simply means that other tests should be performed with greater care and watchfulness. It often is advisable to hold for a few weeks cement that fails in boiling, so that the expansive elements may have an opportunity to hydrate and become inert; but if the material fulfils all the conditions except the boiling test, and is sound in the normal tests up to 28 da., it is generally safe for use. All things being equal, however, a cement that will pass the boiling test is to be preferred.

TESTS FOR TENSILE STRENGTH

The *tensile-strength test* is for the purpose of ascertaining a measure of the ability of the material to withstand the loads that the structure must carry. This test is made by forming specimens, called *briquets*, of cement and cement mortar, and determining the force necessary to rupture them in tension at the expiration of fixed intervals of time. Cement constructions are rarely called on to withstand tensile stresses, but if the tensile strength is known, the resistance to other forms of stress may be computed with a fair degree of accuracy. The tensile-strength test is the most convenient for laboratory determinations, on account of the small size of the specimens and the comparatively low stress required to cause rupture.

Cement is tested both neat or pure and in a mortar commonly composed of 1 part of cement and 3 parts of sand. The period at which the briquets are broken have been fixed

by usage at 7 da. and 28 da. after making, although tests covering much longer periods of time are necessary in research or in investigative work.

Normal Consistency.—The strength of cement and cement mortars varies considerably with the amount of water employed in making the briquets. Dry mixtures ordinarily give the higher results for short-time tests, and wet mixtures show stronger with a greater lapse of time. For testing purposes, therefore, it is essential that all cements be mixed, not with the same amount of water, but with the amount that will bring all the cements to the same physical condition, or to what is called *normal consistency*. Different cements require different percentages of water because of their varying chemical composition, degree of burning, age, fineness, etc.

The normal consistency of neat-cement pastes may be determined by the method that follows.

This method is to form of the paste a ball about 2 in. in diameter and to drop this ball on a table from a height of about 2 ft. If the cement is of the correct consistency, the ball will not crack nor will it flatten to less than half its original thickness. The percentage of water required will vary from 16 to 25, depending on the characteristics of the material, the average cement taking about 20%.

Consistency of Sand Mortars.—The consistency of sand mortars, however, cannot be obtained by the foregoing method, because the mixture is too incoherent. For mortars, therefore, it is necessary to employ a formula by means of which the sand consistency can be computed when that of the neat paste is known. Several such formulas have been devised, of which the following is adaptable to the greatest variety of conditions.

Let x be the per cent. of water required for the sand mixture; N, the per cent. of water required to bring the neat cement to normal consistency; n, the parts of sand to one of cement; and S, a constant depending on the character of the sand. Then,

$$x=\frac{3N+Sn+1}{4(n+1)}$$

For crushed-quartz sand, the constant S is 30, for Ottawa sand, it becomes 25; and for the bar and bank sands used in

construction, it varies from 25 to 35, and must be determined for each particular sand.

EXAMPLE.—How much water is required in a mixture of 1 part of cement and 3 parts of crushed-quartz sand? The neat cement requires 19% of water to give normal consistency.

SOLUTION.—Here, $N=19$, $S=30$, and $n=3$. Substituting these values in the formula,

$$x=\frac{3\times 19+30\times 3+1}{4\times(3+1)}=9.3\%$$

Sand for Mortar Tests.—The size, gradation, and shape of the particles of sand with which cement mortars are made have great influence on the resulting strength. There are two varieties of standard sand for cement testing, one an artificial sand of crushed quartz, the particles of which are angular in shape, and the other a natural sand from Ottawa, Illinois, the particles of which are almost spherical. Both sands are sifted to a size that will pass a sieve of 20 meshes to the inch and be retained on a sieve of 30 meshes, the diameters of the sieve wires being .0112 and .0165 in., respectively. The Ottawa sand will develop strengths in 1-3 mortars about 20 to 30% greater than those obtained with crushed quartz, and it is theoretically the better sand for testing, but, at present, crushed quartz is more extensively employed. On most important works, tests for purposes of comparison are also made of the actual sand entering the construction.

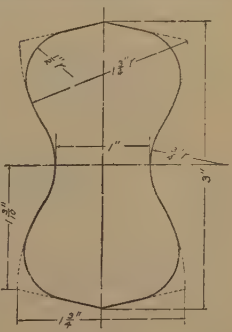


FIG. 4

Form of Briquet.—The form of tensile briquet, adopted as standard in the United States, is shown in Fig. 4. Its cross-section is exactly 1 sq. in.

Molds.—Cement briquets are made in molds that come either single or in gangs of three, four, or five. The gang molds are preferable, as they tend to produce greater uniformity in the results. Molds should be made of brass or of some other non-corrodible material; those made of cast iron soon rust and become unfit for use.

Method of Making Briquets.—First, 1,000 g. of cement is carefully weighed and placed on the mixing table in the form of a crater, and into the center of this is poured the amount of water that has previously been determined to give the correct normal consistency. Cement from the sides of the crater is then turned into the center, by means of a trowel, until all the water is absorbed, after which the mass is vigorously worked with the hands, as dough is kneaded, for 1½ min. When sand mixtures are being tested, 250 g. of cement and 750 g. of sand are first weighed and thoroughly mixed dry until the color of the pile is uniform; then the water is added and the operation is completed by vigorous kneading.

After kneading, the material is immediately placed into the molds, which should first have been wiped with oil to prevent the cement from sticking to them. The entire mold is filled with material at once—not compacted in layers—and pressed in firmly with the fingers without any ramming or pounding. An excess of material is then placed on the mold and a trowel drawn over it under moderate pressure, at each stroke cutting off more and more of the excess material, until the surface of the briquets is smooth and even. The mold is then turned over, and more material placed in it and smoothed, as before. The mixing and molding should be performed on a surface of slate, glass, or some other smooth, non-absorbent material. During the mixing the operator should wear rubber gloves, so as to protect his hands from the action of the lime in the cement.

Storage of Briquets.—For 24 hr. after making, the briquets are stored in a damp closet so that the cement can harden in a moist atmosphere. The damp closet is simply a tight box of soapstone with doors of wood lined with zinc, or some similar arrangement, with a receptacle for water at the

bottom and racks for holding the briquets. The briquets remain in the molds while in the damp closet, but at the expiration of 24 hr. they are removed, marked, and placed in clean water near 70° F. until broken.

Testing Machines.—There are many styles of testing machines on the market. In Fig. 5 is shown what is called a *shot machine*, made by the Fairbanks Company. It is

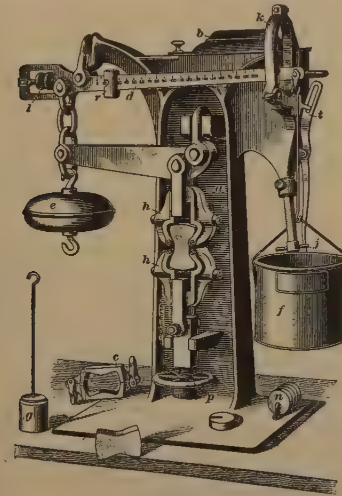


FIG. 5

constructed on the cast-iron frame *a*, and is operated as follows: The cup *f* is hung on the end of the beam *d*, the poise *r* placed at the zero mark, and the beam balanced by turning the weight *l*. The hopper *b* is then filled with fine shot, and the briquet to be tested is placed in the clips *h*. The hand-wheel *p* is now tightened sufficiently to cause the graduated beam *d* to rise to the stop *k*, and the automatic valve *j*

opened so as to allow the shot to run into the cup *f*. The flow of the shot can be regulated by means of a small valve located where the spout joins the reservoir. When the briquet breaks, the beam *d* drops and by means of the lever *t* automatically closes the valve *j*. After the specimen has broken, the cup with its contents is removed, and the counterpoise *g* is hung in its place. The cup *f* is then hung on the hook under the large ball *e*, and the shot weighed. The weighing is done by using the poise *r* on the graduated beam *d* and the weights *n* on the counterpoise *g*. The result will show the number of pounds required to break the specimen. A mold for a single briquet is shown at *c*.

Rate of Loading Testing Machine.—The load should be applied in all tests at the uniform rate of 600 lb. per min. The briquets should be broken as soon as they are removed from the storage tanks and while they are still wet, because drying out tends to lower their strength. The average of from three to five briquets should be taken as the result of a test.

Results of Tensile-Strength Tests.—The tensile strength of cement tested in the preceding manner should increase with age up to about 3 mo. and should then remain practically stationary for longer periods. The average results of tests of Portland cement made in the Philadelphia laboratories, covering a period of several years and based on over 200,000 briquets, are given in the accompanying table.

Specifications for strength commonly stipulate minimum values for the 7- and 28-da. tests, the customary requirements for Portland cement being 500 lb. at 7 da. and 600 lb. at 28 da., when tested neat, and 170 lb. at 7 da. and 240 lb. at 28 da., when tested in a mortar consisting of 1 part of cement and 3 parts of crushed-quartz sand. When Ottawa sand is used, the requirements for mortar should be raised to 200 and 280 lb., respectively. Retrogression in strength of the neat briquets between 7 and 28 da. is not necessarily indicative of undesirable properties, but if the mortar briquets show retrogression, the cement should be condemned. Abnormally high strength in the 7-da. test of neat cement, say over 900 lb., may generally be taken as an indication of

TENSILE STRENGTH OF CEMENT BRIQUETS

(Pounds per Square Inch)

Mixture	1 Hour in Air 23 Hours in Water	1 Day in Air 6 Days in Water	1 Day in Air 27 Days in Water	1 Day in Air 89 Days in Water
Neat	420	710	770	775
1 cement, 1 crushed-quartz sand	360	590	695	700
1 cement, 2 crushed-quartz sand	210	370	455	465
1 cement, 3 crushed-quartz sand	105	210	300	310
1 cement, 4 crushed-quartz sand	60	130	210	230
1 cement, 5 crushed-quartz sand	35	80	155	195

weakness rather than of superiority, because such a condition is usually created by an excess of lime or of sulphates, either of which may be injurious. Neat cement testing from 600 to 800 lb. at 7 da. is generally the most desirable.

SECONDARY TESTS

TESTS FOR TIME OF SETTING

The *time-of-setting test* is made to determine whether or not the cement will become hard at the time most desirable in actual construction. If it begins to set too soon, the crystallization of the particles will have begun before the mortar or concrete is thoroughly tamped into place. If, on the other hand, the cement sets too slowly the material is more likely to suffer from exposure to heat, cold, dampness, and frost; also, the progress of the work will be much delayed on account of the greater interval required between different sections, and the longer time the forms must be left up.

In the setting of cements, two stages are recognized: (1) When the paste begins to harden or to offer resistance to

change of form, called *initial set*, and (2) when the setting is complete, or when the mass cannot be appreciably distorted without rupture, called *hard set*. The time-of-setting test consists, therefore, in determining the time required for the cement to reach these two critical points.

The test is made by mixing cement with the amount of water required to produce normal consistency, in the same manner as for neat tensile briquets, forming specimens, placing them under one of the forms of apparatus, and observing the time that elapses between the moment the mixing water is added and the moments when the paste acquires initial set and hard set.

FIG. 6

The *Vicat needle*, shown in Fig. 6, consists of a frame *k*, holding a movable rod *l*, which carries a cap *d* at the upper end and a needle *h* at the lower. A screw *f* holds the rod in any desired place. The position of the needle is shown by a pointer moving over a graduated scale. The rod with needle and cap weighs exactly 300 gr. and the needle is 1 mm. in diameter with the end cut off square. When making tests of normal consistency, the plunger *b* is substituted for the needle *h*, and the cap *a* for the cap *d*, the difference in weight between the needle and plunger being compensated by the difference in the weight of the caps. The mold *i* for holding the cement paste is in the form of a truncated cone. It has an upper diameter of 6 cm., a lower diameter of 7 cm., and a height of 4 cm., and rests on a $4'' \times 4'' \times \frac{1}{8}''$ glass plate *j*.

After the cement paste is mixed, the mold is filled by forcing the cement through the large end; then, after turning it over and smoothing the top, it is placed on the glass plate

under the needle. The needle is lowered until it is exactly in contact with the surface of the paste, then quickly released and the depth to which it penetrates read from the graduated scale. Initial set is said to have taken place when the needle ceases to penetrate to within 5 mm. of the bottom of the specimen; and hard set takes place when the same needle ceases to make an impression on the surface. Trials of penetration are made every 5 or 10 min. until these points are reached.

Time of setting varies considerably with the amount of mixing water employed, so that it is essential that every sample tested be brought exactly to normal consistency; otherwise, the results may be in decided error. Variations in temperature in both environment and in the mixing water, also influence the results. Standard practice requires that both the materials and the room in which the tests are made be at a temperature of as nearly 70° F. as practicable.

Results of Time-of-Setting Tests.—In specifying results to be obtained in testing the time of setting, it is obvious that a minimum value should be stipulated for initial set and a maximum, as well as a minimum, for hard set. It must also be remembered that a cement mixed with an aggregate and with an excess of water in the field, will require from two to four times as long to set as the neat-cement paste mixed with little water in the laboratory. Cement, therefore, showing an initial set at the expiration of 20 min. with the Vicat needle, will rarely begin to set on the actual work in less than $\frac{3}{4}$ hr. which gives ample time for mixing and placing the materials, and cement setting in less than 10 hr., will usually have hardened completely in the work in 24 or at least in 36 hr. Specifications usually stipulate that Portland cement shall show initial set in not less than 20 minutes and shall develop hard set in not less than 1 hr. nor more than 10 hr. Cement reaching initial set in less than 12 or 15 min. should never be used for any work.

TESTS FOR FINENESS

Apparatus for Fineness Test.—The fineness of cement is important, because it affects both the strength and the sound-

ness of the product. The fineness of cement is determined by passing it through a series of sieves of different mesh and then measuring the amount retained on each. Three sieves are commonly employed, namely, those having 50, 100, and 200 wires to the linear inch. Sieves for cement testing should never be used until they have been carefully examined and found to conform to the following standard specification

1. Cloth for cement sieves shall be of woven brass wire of the following diameters: No. 50, .0090 in.; No. 100, .0045 in.; and No. 200, .00235 in.

2. Mesh to count on any part of the sieve as follows: No. 50, not less than 48 nor more than 50 per lin. in.; No. 100, not less than 96 nor more than 100 per lin. in.; and No. 200, not less than 188 nor more than 200 per lin. in.

3. Cloth to be mounted squarely and to show no irregularities of spacing.

Method of Making the Fineness Test.—The method of using the sieves in the fineness test is to weigh out 50 g. of cement on a scale sensible at least to $\frac{1}{10}$ g. and to place it on the No. 200 sieve, on which it is shaken until not more than $\frac{1}{10}$ g. passes the sieve at the end of 1 min. of shaking. The arrival of this stage of completion can be watched either by using a pan under the sieve or by shaking over a piece of paper. The residue remaining on the sieve is weighed, placed on the No. 100 sieve and the operation repeated, again weighing the residue. The amount remaining on the No. 50 sieve is then determined similarly. The process of sifting can be accelerated by placing a small quantity of coarse shot or pebbles on the sieves with the cement during the shaking. These may be separated from the cement by passing the residue with the shot through a coarse sieve, such as the No. 20.

Results of Fineness Tests.—Portland cement should be ground to such a fineness that it will leave a residue of not more than 25%, by weight, on the No. 200 sieve, and not more than 8% on the No. 100 sieve. Of these two requirements, the first is the more important, because it is only that part of the cement passing the finest sieve that is active in the setting and hardening of the material. The amount remaining on the No. 100 sieve is also important, because this

part is most liable to cause unsoundness in the cement, and although specifications do not call for tests with the No. 50 sieve, it is usually employed for the same reason as the No. 100 sieve. Any appreciable residue on this sieve indicates that the material is much more liable to unsoundness. Any cement failing to pass the fineness test should be watched more carefully in the soundness and strength tests, but if these tests show good results up to 28 da., the cement can, as a rule, be used safely. It must be remembered, however, that only that part passing the No. 200 sieve is really cement, so that a coarsely ground shipment is practically equivalent to one adulterated with weak and unsound material.

TESTS FOR SPECIFIC GRAVITY

The object of the *specific gravity test* is to furnish indications of the degree of burning, the presence or absence of adulteration, and the amount of seasoning that the cement has received. When Portland cement is burned, the separate ingredients are in close contact and gradually combine by a process of diffusion. The greater the amount of this burning the more thoroughly are the elements combined. Thus, by their contraction they give, in volume, a higher density or specific gravity. Since, to secure good cement the burning must have been made within definite limits, it follows that the specific gravity must also lie within fixed limits if the cement has been properly manufactured.

The common adulterants of Portland cement, namely, limestone, natural cement, sand, slag, cinder, etc., all have specific gravities ranging from 2.6 to 2.75, while the specific gravity of Portland cement averages about 3.15. An appreciable amount of adulteration, therefore, is at once indicated in the results of the test.

Seasoning is indicated because the cement on standing gradually absorbs water and carbonic acid from the air. These ultimately combine with it and thus lower the specific gravity.

Apparatus for Specific-Gravity Test.—Of the many forms of apparatus employed for the specific-gravity test, the *Le Chatelier flask*, shown in Fig. 7, is the one most com-

monly used. It is also the one adopted by the technical societies as standard. It consists of a glass flask about 30 cm. high. The lower part up to mark *a* contains 120 cu. cm., and the bulb between the marks *a* and *b* contains exactly 20 cu. cm. The neck of the flask above the mark *b* is graduated into $\frac{1}{10}$ cu. cm. The funnel *c* inserted in the neck is to facilitate the introduction of the cement.

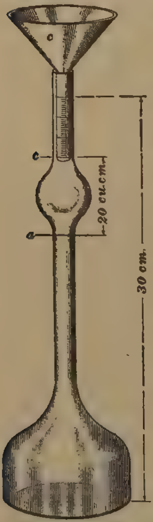


FIG. 7

Method of Making the Specific-Gravity Test.—The method of conducting the specific-gravity test is as follows: 64 g. of cement is carefully weighed on scales that should have a sensibility of at least .005 g. The flask, Fig. 7, is filled to the lower mark *a* with benzine or kerosene, which has no action on the cement, and carefully adjusted precisely to the mark by adding the liquid a drop at a time. The funnel is then placed in the neck of the flask and the weighed cement introduced slowly through it, the last traces of the cement being brushed through with a camel's-hair brush. The funnel is then removed and the height of the benzine read from the graduations, estimating to .01 cu. cm. The displaced volume is then 20 plus the reading in cubic centimeters, and the specific gravity of the cement is 64 divided by that quantity. For example, suppose that the reading on the flask is .54, then the displaced volume will be $20+.54=20.54$ and the specific gravity will be $64 \div 20.54=3.116$.

The apparatus must be protected from changes in temperature while in use; even touching the flask with the fingers will change the volume of the liquid noticeably. The flask is sometimes immersed in water during the tests to prevent these changes of temperature, but this precaution is unnecessary if proper care is exercised.

Results of Specific-Gravity Tests.—The specific gravity of well-burned Portland cement averages about 3.15 and should not fall below 3.1. If it falls below 3.1, tests should also be made on dried and on ignited samples to ascertain whether or not this condition has been produced by reason of excessive seasoning. As a rule, low specific gravity merely indicates well-seasoned cement, and if sound and sufficiently strong, such cement is the best sort of material for use, as its durability is scarcely open to question.

TESTS OF NATURAL AND SLAG CEMENTS

The methods of conducting tests of *natural* and *slag cements* are, in all important particulars, identical with those employed for Portland cement, although the results obtained and the interpretation to be put on them are often radically different. In the testing, the only essential difference is in the amount of water required by these cements to produce normal consistency, natural cement requiring from 23 to 35% and slag cement taking about 18%, or an average of 2 or 3% less than Portland. Tests of natural cement for tensile strength are also frequently made on 1–1 and 1–2 mortars, but recent practice is to test mortars of all kinds of cement in 1–3 mixtures. For these cements, moreover, the specific-gravity test has practically no significance, except in determining the uniformity with which the different brands are made.

CEMENT SPECIFICATIONS

The common requirements for high-grade Portland, natural, and slag cements are given in the table on page 228, and following the table is given a good example of a complete modern specification for Portland cement.

SPECIFICATIONS FOR PORTLAND CEMENT

Kind.—All cement shall be Portland of the best quality, dry and free from lumps. By Portland cement is meant the finely pulverized product resulting from the calcination to incipient fusion of an intimate mixture of properly pro-

portioned argillaceous and calcareous materials to which no addition greater than 3% has been made subsequent to calcination.

REQUIREMENTS FOR HIGH-GRADE CEMENTS

Requirements	Portland Cement	Natural Cement	Slag Cement
Specific gravity:			
Not less than........	3.1	2.8	2.7
Fineness:			
Residue on No. 100 sieve, not over.....	8%	10%	3%
Residue on No. 200 sieve, not over.....	25%	30%	10%
Time of setting:			
Initial, not less than .	20 min.	10 min.	20 min.
Hard, not less than ..	1 hr.	30 min.	1 hr.
Hard, not more than.	10 hr.	3 hr.	10 hr.
Tensile strength per sq. in.			
7 da., neat, not less than..............	500 lb.	125 lb.	350 lb.
28 da., neat, not less than..............	600 lb.	225 lb.	450 lb.
7 da., 1–3 quartz, not less than..........	170 lb.	50 lb.	125 lb.
28 da., 1–3 quartz, not less than	240 lb.	110 lb.	200 lb.
Soundness:			
Normal pats in air and water for 28 da. to be........	sound and hard	sound and hard	sound and hard
Boiling test to be ..	sound and hard		sound and hard
Analysis:			
Magnesia, MgO, not over..............	4%		4%
Anhydrous sulphuric acid, SO_3, not over	1.75%		
Sulphur, S, not over. .			1.3%

Packages.—Cement shall be packed in strong cloth or canvas bags, or in sound barrels lined with paper, which shall be plainly marked with the brand and the name of the manufacturer. Bags shall contain 94 lb. net and barrels shall contain 376 lb. net.

Inspection.—All cement must be inspected, and may be reinspected at any time. The contractor must submit the cement, and afford every facility for inspection and testing, at least 12 da. before desiring to use it. The chief engineer shall be notified at once on receipt of each shipment at the work. No cement will be inspected or allowed to be used unless delivered in suitable packages properly branded. Rejected cement must be immediately removed from the work.

Protection.—The cement must be protected in a suitable building having a wooden floor raised from the ground, or on a wooden platform properly protected with canvas. It shall be stored so that each shipment will be separate, and each lot shall be tagged with identifying number and date of receipt.

Quality.—The acceptance or rejection of a cement to be used will be based on the following requirements:

Specific gravity: Not less than 3.1.

Ultimate tensile strength per square inch:	Pounds
7 da. (1 da. in air, 6 da. in water)	500
28 da. (1 da. in air, 27 da. in water)	600
7 da. (1 da. in air, 6 da. in water), 1 part cement to 3 parts of standard quartz sand	170
28 da. (1 da. in air, 27 da. in water), 1 part of cement to 3 parts of standard quartz sand	240

Fineness: Residue on No. 100 sieve not over 8%, by weight; residue on No. 200 sieve not over 25%, by weight.

Set: It shall require at least 20 min. to develop initial set, and shall develop hard set in not less than 1 hr. nor more than 10 hr. These requirements may be modified where the conditions of use make it desirable.

Constancy of Volume: Pats of cement 3 in. in diameter, ½ in. thick at center, tapering to thin edge, immersed in water after 24 hr. in moist air, shall show no signs of cracking, distortion, or disintegration. Similar pats in air shall also remain sound and hard. The cement shall pass such accelerated tests as the chief engineer may determine.

Analysis: Sulphuric anhydride, SO_3, not more than 1.75%; magnesia, MgO, not more than 4%.

PLAIN CONCRETE

MATERIALS USED IN CONCRETE

DEFINITIONS AND TERMS

Concrete is usually made of cement, sand, and broken stone. The cement in a plastic state, either by itself or with the sand that is generally mixed with it, is called the *matrix*, and the broken stone, gravel, or other material used as a filler is called the *aggregate*. The sand is correctly classed as a part of the aggregate, although some engineers include it with the matrix. The aggregate is used to cheapen concrete. Pure, or neat, cement, when wet with water, would in a way fulfil all the physical requirements of concrete, but it would be too expensive.

In the concrete of today, hydraulic cement is used almost exclusively. For this reason, the term concrete, as commonly used, refers only to that variety. In specifying any other kind of concrete, the usual custom is to mention it by its full name, as *bituminous concrete*, *lime concrete*, etc. Such varieties, however, are of comparatively little importance.

The term concrete, besides being restricted to hydraulic-cement concrete, has another restriction: the aggregate must not be sand alone, although it may be partly sand. A mixture of hydraulic cement, sand, and water is called by the special name of *mortar*.

Concrete is usually named from the kind of aggregate used. For example, *stone concrete* embodies the use of broken stone or coarse pebbles, while in *cinder concrete*, the aggregate consists of cinders or broken slag.

The proportion of cement and sand to the broken stone depends on the spaces between the stones, which are known as *voids*. In all instances, there must be sufficient mortar to fill the voids entirely and to cover all surfaces of the separate stones.

AGGREGATES OTHER THAN SAND

The aggregates or broken stone used in concrete work should possess three qualities: (1) They should be hard and strong, so as to resist crushing and shearing or transverse stresses; (2) they should have surface texture that will permit the cement mortar to adhere to their surfaces; and (3) where the concrete is to be used for building construction, such as in reinforced-concrete work, and for fireproofing, they should possess refractory, or fire-resisting, qualities. Usually, aggregates that break in such a way as to allow the smallest spaces, or interstices, between the particles, will make the strongest concrete for construction purposes because the voids can be most economically filled with cement mortar.

Size of Aggregates.—In measuring broken stone, the size of the stone is determined by the size of the ring through which it will pass. For instance, a 2-in. stone is one that will pass through a ring, or hole, that is 2 in. in diameter.

The broken stone used in concrete work varies in size with the nature of the work. For foundation and mass construction, it is the custom to use broken stone of a size that will pass through the 2- or 2½-in. ring. For filling the spandrels of bridges or the spaces between walls, where mere mass is desired, broken stone of a much larger size is used.

In reinforced-concrete work, the broken stone must be small, owing to the narrow spaces in the forms. For columns and wall work, stone that will pass through a 1- or ¾-in. ring is suitable, while for filling the beam and girder forms, where numerous reinforcing rods occur, the broken stone is sometimes so small as to pass through a ½-in. ring.

The latest practice in making concrete is to use stone as it comes from the crusher, without screening it. While such stone, termed the *run of crusher*, contains broken stone of a size specified, it also has smaller particles of stone and such stone dust as is carried along with the broken stone from the crusher. Where the run of crusher is used, the proportion of the cement and sand must be changed, because the stone dust takes the place of some of the sand. In

using run of crusher the very finest dust should be washed or screened out as it tends to coat the large pieces and to prevent the cement from adhering to them.

Selection of Aggregates.—Usually the character of the aggregates used in mixing concrete depends on the availability of the supply. Where there is much choice in the selection of the aggregates those which are hardest and which break with a cubical fracture will make the best concrete although rounded pebbles are considered by some engineers to possess great advantages.

The size of the aggregates has much to do with the quality and strength of the concrete. It can, however, be stated as a general proposition that the larger the stones the stronger will be the concrete. This fact is shown by the accompanying table, which gives the results of tests made at the Watertown Arsenal in 1898. It is interesting to note that the concrete becomes heavier per cubic foot, or, in other words, more dense, the larger the stone used. All these tests were made with concrete manufactured in the proportion of 1 part of cement, 1 part of sand, and 3 parts of broken stones, or a 1–1–3 (1 to 1 to 3) mixture, as it is usually expressed. The figures on cinder concrete in the table are added simply to give a comparison of weights, for it will be noted that the cinder concrete is older than the other concretes and therefore stronger in proportion.

Aggregates that consist of stone of varying sizes are best for making concrete, owing to the fact that they pack closer. It is well however, to screen all the fine particles, such as ¼-in. sizes, and use them with the sand, as otherwise they will not mix properly with the cement.

Broken trap rock is the best aggregate for concrete work, the next in value is broken granite, while the third in order of merit is good clean gravel. In fact, these three aggregates can be classified together, and the item of cost only should influence the selection of any one of them. Marble, limestone, and slag make good aggregates, in the order named, but marble and limestone are objectionable if the concrete is to be used as a fireproofing. The poorest aggregates are sandstone, slate, and shale. The sandstone is inefficient

COMPRESSIVE STRENGTH OF CONCRETE MADE OF DIFFERENT-SIZED STONES

Material	First Group			Second Group			Third Group			Fourth Group		
	Age. Days	Weight per Cubic Foot Pounds	Compressive Strength. Pounds per Square Inch	Age. Days	Weight per Cubic Foot Pounds	Compressive Strength. Pounds per Square Inch	Age. Days	Weight per Cubic Foot Pounds	Compressive Strength. Pounds per Square Inch	Age. Days	Weight per Cubic Foot Pounds	Compressive Strength. Pounds per Square Inch
Trap, ½ in.	7	145.56	1,391	19	149.00	2,220	32	146.44	2,800	76	153.34	5,021
Trap, ¾ in.	8	147.01	1,900	20	150.12	2,769	32	148.27	3,200			
Trap, 1 in.	7	159.26	3,390	20	160.65	4,254	34	160.88	4,917	73	158.54	5,272
Trap, 1½ in. . . .	11	157.80	3,189	26	160.56	4,006	41	161.14	4,562			
							48	157.39	2,583			
Trap, 2½ in. . . .	7	158.37	2,400	22	159.27	4,143	32	161.44	4,140	65	161.76	4,523
Pebbles, ⅜ in. .	7	146.76	1,298	21	150.51	1,298	34	147.02	2,992	70	148.76	3,870
Pebbles, 1½ in.	7	149.63	2,276	22	151.75	2,276	29	151.51	3,817	61	150.89	4,018
Pebbles, 3 in. .	11	151.36	2,800	26	150.63	2,800	41	153.60	4,200			
							46	153.21	3,400			
Cinders.	31	115.90	2,329	90	114.12	2,834						

on account of its lack of hardness and its liability to crumble, and from the fact that its surface is likely to be unstable. Slate and shale are hard, but they are of a laminated structure and break in such flaky shapes that they will not pack closely. Cinders are frequently used as the aggregate for concrete. Cinder concrete, however, does not possess sufficient strength for structural purposes, and is generally used for filling or for fireproofing.

Based on percentages of efficiency, with trap rock taken at 100, the following table gives a fair representation of the comparative values of the different aggregates:

COMPARATIVE VALUE OF DIFFERENT AGGREGATES USED IN CONCRETE

Material	Value
Trap rock	100
Granite	90
Gravel (quartz)	90
Limestone (hard, like marble)	80
Limestone (soft)	75
Slag	75
Slate	60
Shale	55
Cinders	50

PROPORTIONING OF INGREDIENTS

Effect on Strength and Imperviousness.—The strength of concrete depends on the strength of the cement and the thoroughness with which the cement binds together the various pieces of aggregate. The more completely the voids are filled, the more completely will the aggregate be held together. Therefore, the more solid and condensed the concrete is, the less voids it will have, and the stronger it will be. The same is true with regard to making concrete waterproof: the more dense the concrete is, the more nearly waterproof it is.

A mixture of 1 part of cement, 1½ parts of sand, and 3 parts of stone, which would be considered extravagantly rich for

a dry place, is probably as dense a concrete, and as good for waterproofing qualities, as can be made.

When a concrete is made of cement, sand, and stone, and the stone is of such a size that it will pass through a 3-in. ring, but will not pass through a 2½-in. ring, the concrete is weaker and requires more cement than one made with graded stone from 3-in. down. When the stone is graded in size, the stones of smaller size fill the voids between the larger stones and thus reduce the quantity of cement and sand required.

Proportioning by Weight.—A method of proportioning the materials, that is simple and fairly accurate, is as follows: A batch of concrete is mixed in known proportions. The same quantity of water is used that it is proposed to use on the work, and the mixture is rammed and tamped in the receptacle in a uniform manner. The receptacle should preferably be of metal; a tin washtub, or a short section of 12-in. pipe, capped at one end, will answer. When the receptacle is full, it is weighed, and if the weight of the receptacle itself has previously been found, the weight of the concrete may be obtained. Various other mixtures of concrete are tried in the same manner, and since the denser the mixture the stronger it will be, the heaviest concrete is the strongest for the particular work. Each batch of concrete must be weighed and taken out of the receptacle before it has time to set; otherwise, some difficulty might be experienced in getting it loose.

Usual Proportions of Materials.—The strongest concrete does not always have to be used, as it may be required to withstand only slight stresses and be simply used for its weight. The strongest concrete would then be unnecessarily expensive. Therefore, the foregoing method for proportioning concrete is seldom employed The engineer usually specifies a mixture from his own experience without testing the aggregates in any way, except to see that the stone is under the specified maximum size and that the sand is in large grains and free from dirt and loam. A common proportion for unimportant work is 1–3–6. This proportion may be used for foundations below ground, in engine bases,

in the foundations for asphalt pavements, and for similar purposes. A richer mixture, 1–2–4 is used in piers, in dams, in important reinforced-concrete work, and in other places where great strength is desired.

Water for Concrete.—The wetter the concrete is, the easier it will be put in place, but mixtures that are too wet are not so strong as medium mixtures. The amount of water that will make the best mixture is such that after the concrete has been put in place and rammed it will quake like jelly when struck with a spade, and water will come to the surface. If the concrete is wetter than this, the water will have a slight chemical effect on the cement, and, moreover, the sand and cement will tend to separate from the broken stone.

In cinder concrete, owing to the porosity of the cinders, it is necessary to use a little more water, so that the cement will be liquid enough to fill the little cavities in each cinder. This precaution is indispensable when the concrete is to be used with steel, as otherwise the steel will be rapidly corroded by the action of air reaching it through the pores in the cinder.

Dry Concrete.—With the advent of the concrete block, a great deal is heard about *dry concrete*. This name is given to concrete in which as little water as possible is mixed. In the concrete-block manufacturing business, the mold in which each block is made is required as soon as possible, so that it can be used over again and thus increase the capacity of the machine to which it belongs. For this reason, the concrete-block manufacturers use, often, dry concrete, and attempt to supply the remainder of the water required for the complete crystallization, or setting, of the cement by *curing* the blocks; that is, by sprinkling them with water for a week or so. The results of recent tests seem to indicate that dry concrete will show higher compression values for a limited time after it is made, but that the rate of increase of strength is not so great as with wet concrete. After 1 yr. or 6 mo., the strength of the wet concrete will be found to have attained, and perhaps surpassed, that of the dry mixture.

A serious objection to dry concrete is that it cannot be rammed to so dense a mass as wet concrete. Therefore, if a waterproof concrete is desired, it should be mixed wet.

PROPERTIES OF CONCRETE

GENERAL CHARACTERISTICS

Corrosion of Steel in Concrete.—It has been conclusively proved by experience and test that steel or iron completely embedded in the usual mixture of concrete will not corrode seriously. Portland cement contains free alkali, and steel or iron will not rust in the presence of an alkali. Corrosion will occur only where the concrete has been carelessly placed and where voids in the concrete have exposed the metal.

Wet concrete offers more protection to iron or steel than a dry mixture, because the metal is better coated with the cement mixture. Cinder concrete, when of a rich mixture—at least one sufficiently rich in cement to coat every particle of the cinder—will protect ironwork or steelwork as well as stone concrete; but, if it is not properly mixed and particles of the uncoated coal or cinder come in contact with the steel, rapid corrosion is likely to take place.

Effect of Fire on Concrete.—Concrete is essentially a fireproof material. All the ingredients of which it is composed are of a highly refractory nature, the aggregates being the elements of the mixture that are most quickly affected by intense heat. This is especially true of granite and limestone aggregates, the former being likely to crack or burst when heated, and the latter liable to calcine. After cement has set, the chemical union of its particles is liable to destruction by fire, because intense heat robs the cement of the water of crystallization, or *dehydrates* the cement, thus softening the material and making it crumbly. If concrete in a mass is subjected to intense heat, this action of dehydration extends into the concrete for a depth of only $\frac{1}{8}$ to $\frac{1}{2}$ inch, and is not likely to penetrate farther.

Considère, a French concrete expert, has found by experiment that a 1–3 mortar will shrink from about .05 to .15%

in setting in air, and that the shrinkage will be two to three times as great with neat cement. The shrinkage in concrete will be much less than with neat cement or cement mortar.

The shrinkage of concrete is lessened by embedding in it steel rods or bars, as these, by their tensile resistance, prevent the shrinkage of the material in setting. By the experiments of Considère, it is found that with 1–3 mortar reinforced with steel the shrinkage in setting is about one-fifth that of the same mortar without the steel reinforcement.

Effect of Thermal Changes in Concrete.—Nearly all materials expand slightly as they become heated. Concrete and steel also follow this law. The contraction or the expansion of concrete due to changes in temperature is about the same as that of steel. The average coefficient of expansion of a 1–2–4 concrete for each Fahrenheit degree in change of temperature is .0000055. Experiments made on 1–3–6 concrete give a coefficient of expansion of .0000065, which is practically the same as the coefficient of steel.

Effect of Vibration on Concrete.—The effect of constant vibration on concrete structures has not been definitely determined. Many buildings and bridges constructed of concrete reinforced with steel rods and bars have withstood heavy and constant vibration, either continuous or intermittent, for an extended period of years with no apparent deterioration in strength. Fresh concrete is always, however, subject to deterioration by vibration, and the strength of concrete subjected to jar or shock when setting is materially reduced, because the process of crystallization between the particles, and the consequent cohesion of the mass, seems to be partly destroyed.

WORKING STRESSES AND STRENGTH VALUES OF CONCRETE

The ultimate strength of concrete varies so with the proportion of the mixture, manner of working, character of ingredients, and age of material, that it is necessary to assume low unit working stresses for it.

The usual working stress for plain concrete under compression is from 250 to 300 lb. per sq. in., although, in masses,

as in footings, a 1–2½–5 concrete would safely sustain as much as 500 lb. per sq. in. When reinforced concrete is subjected to compression from loads causing bending, it is customary to figure the safe allowable unit compressive stress in the compression portion of a reinforced-concrete beam at from 500 to 600 or even 750 lb. per sq. in.

In tension, concrete has little value; in fact, it cannot be relied on to resist this stress. Generally, the tensile strength of plain concrete is about one-tenth of its compressive strength.

The modulus of rupture, or the unit value for figuring the transverse strength of plain concrete, is much lower than the modulus of rupture of any of the good building stones. The safe unit bending stress for plain concrete, based on a factor of safety of 4, from values of the modulus of rupture obtained from recent tests made on concrete 30 da. old, is about 110 lb. This value is for concrete composed of 1 part of cement, 2 parts of sand, and 4 parts of broken stone. With a poorer mixture, as a 1–2–5 concrete, a safe bending stress of about 95 lb. should be used, while with a 1–3–5 mixture, the safe bending stress is barely 70 lb., and this value shows a corresponding decrease as the mixture becomes leaner.

The safe unit shearing stress of plain concrete is, in practice, taken at a very low figure when compared with recent tests giving the ultimate shearing resistance of this material. This low figure is probably due to the fact that few tests have been made to determine the value of plain concrete in shear; or perhaps it is due to the unreliability of concrete, as found in practice, to resist this stress. The conservative safe unit shearing stress of plain concrete is taken at 50 lb., although the value may be increased for rich mixtures and careful workmanship to 75 lb.

The safe *grip*, or *bond*, as it is called, of concrete on steel rods or bars with plain surfaces embedded in it, is taken, for purposes of calculation, at 50 lb. per sq. in. of the surface of the metal in contact with the concrete.

All the values just mentioned are based on concrete at least 1 mo. old. There is great diversity of opinion regard-

ing the safe unit values of plain concrete, and there is no uniformity in the building laws of the several cities regarding the strength of this material. This is shown by the following table, which gives the working values of concrete allowed by the building laws of several cities:

UNIT WORKING VALUES OF CONCRETE ALLOWED BY VARIOUS CITIES

Name of City	Direct Compression Pounds per Square Inch	Shear Pounds per Square Inch	Unit Compressive Stress Under Bending Loads Pounds per Square Inch
New York........	350	50	500
Philadelphia......	250	50	600
Cleveland........	400	50	500
San Francisco	450	75	500
Buffalo..........	350	50	500
Toronto..........	340	50	500

An average ultimate unit compressive strength of 2,000 lb. would be conservative for a 1–2–4 concrete, from 1 to 3 mo. old, and 1,600 lb. for a 1–3–6 concrete of the same age. Mixtures varying in richness between these limits would have proportional values.

The tensile strength of concrete is more affected by the quality of the mixture than is its compressive stress. Therefore, a conservative ultimate-tensile-strength value of a 1–2–4 mixture would be about 200 lb., while for a 1–3–6 mixture, it would be about 125 lb.

The shearing strength of concrete is usually much less than the compressive strength. For a 1–2–4 mixture an average ultimate shearing strength of 1,480 lb. per sq. in. has been determined by tests, while a 1–3–5 mixture has given 1,180 lb., and a 1–3–6 mixture has given a value in shear of 1,150 lb. per sq. in.

The average modulus of rupture for plain concrete that was from 33 to 35 da. old has been found to be 439 lb. per sq. in

AVERAGE ULTIMATE STRENGTH OF CONCRETE MADE FROM PORTLAND CEMENT, SAND, AND CRUSHED STONE

Proportion of Ingredients			Tension Pounds per Square Inch				Compression Pounds per Square Inch				Shear Pounds per Square Inch			
Cement	Sand	Stone	7 da.	1 mo.	3 mo.	6 mo.	7 da.	1 mo.	3 mo.	6 mo.	7 da.	1 mo.	3 mo.	6 mo.
1	2.0	4	160	210	240	250	1,600	2,150	2,400	2,500	200	269	300	313
1	2.5	5	143	195	225	235	1,430	1,950	2,250	2,350	179	244	281	294
1	3.0	6	125	180	210	220	1,250	1,800	2,100	2,200	156	225	263	275
1	3.5	7	110	166	196	208	1,100	1,660	1,960	2,080	138	208	245	260
1	4.0	8	98	152	182	195	980	1,520	1,820	1,950	123	190	228	244
1	4.5	9	85	140	169	184	850	1,400	1,690	1,840	106	175	211	230
1	5.0	10	75	126	155	172	750	1,260	1,550	1,720	94	158	194	215
1	5.5	11	65	112	142	160	650	1,120	1,420	1,600	81	140	178	200
1	6.0	12	60	100	130	150	600	1,000	1,300	1,500	75	125	163	188

for a 1–2–4 mixture, 380 lb. per sq. in. for a 1–2–5 mixture, and 285 lb. per sq. in. for a 1–3–5 mixture, while a 1–3–6 mixture gave a result of 226 lb. per sq. in.

The values given in the accompanying table are recommended as ultimate values by W. Purves Taylor, engineer in charge of the municipal testing laboratory of Philadelphia. The figures represent values obtained from six hundred experiments made on concrete properly mixed with good Portland cement. It will be noticed that the values given for shear are considerably lower than those just given. The results obtained depend to a large extent on the method of testing. Some engineers prefer the lower values.

MIXING AND WORKING OF CONCRETE

CONCRETE MIXTURES

Methods of Measuring Ingredients.—After deciding what proportions of ingredients will be used for the concrete, the engineer must be able to calculate the exact quantity of each material that he must order. An ordinary box car holds from 400 to 600 bags of cement. The purchaser is charged for the bags by the manufacturer, unless they are of paper, but he gets a rebate for those which are returned.

Cement is usually measured by the barrel just as it comes from the manufacturer, or as 4 bags to the barrel, while broken stone and sand are measured loose in a barrel. Portland cement, after it is taken out of its original package and stirred up, fills a larger volume than when packed. It is therefore necessary to state just how the cement is to be measured; and, as said before, the custom is to measure it by the barrel, compact. A cement barrel contains about 3.8 cu. ft.

Fuller's Rule for Quantities.—A practical rule has been devised by W. B. Fuller whereby, after the proportions of ingredients have been fixed, the quantity of material for a certain work may be obtained. It is called *Fuller's rule for quantities*, and may be expressed in mathematical symbols as follows:

Let c be the number of parts of cement; s, the number of parts of sand; g, the number of parts of gravel or broken stone; C, the number of barrels of Portland cement required for 1 cu. yd. of concrete; S, the number of cubic yards of sand required for 1 cu. yd. of concrete; and G, the number of cubic yards of stone or gravel required for 1 cu. yd. of concrete. Then

$$C=\frac{11}{c+s+g}, S=\frac{3.8}{27}C\,s, \text{ and } G=\frac{3.8}{27}C\,g$$

If the broken stone is of uniformly large size, with no smaller stone in it, the voids will be greater than if the stone were graded. Therefore, 5% must be added to each value found by the preceding formulas.

EXAMPLE.—If a 1–2–4 mixture is considered, what will be: (*a*) the number of barrels of cement, (*b*) the number of cubic yards of sand, and (*c*) the number of cubic yards of stone required for 1 cu. yd. of concrete.

SOLUTION.—(*a*) Here $c=1$, $s=2$, and $g=4$. Substituting these values in the first formula,

$$C=\frac{11}{1+2+4}=1.57$$

(*b*) Substituting the values of C and s in the second formula,

$$S=\frac{3.8}{27}\times 1.57\times 2=.44$$

(*c*) Substituting the values of C and g in the third formula,

$$G=\frac{3.8}{27}\times 1.57\times 4=.88$$

Table of Quantities.—The table on pages 244 and 245, giving the quantities of ingredients for concrete of various proportions, was prepared by Edwin Thacher. It will be noted in this table that the difference in the character and size of the stone or gravel used has been taken into account. These values will be found to agree fairly well with values found by Fuller's rule.

QUANTITIES OF INGREDIENTS FOR CONCRETE OF VARIOUS PROPORTIONS

Proportion of Ingredients			Ingredients Required for 1 Cubic Yard of Rammed Concrete											
			Stone, 1 Inch and Under—Dust Screened Out			Stone, 2½ Inches and Under—Dust Screened Out			Stone, 2½ Inches—With Most Small Stone Screened Out			Gravel, ¾ Inch and Under		
Cement	Sand	Stone	Cement bbl.	Sand cu. yd.	Stone cu. yd.	Cement bbl.	Sand cu. yd.	Stone cu. yd.	Cement bbl.	Sand cu. yd.	Stone cu. yd.	Cement bbl.	Sand cu. yd.	Gravel cu. yd.
1	1.0	2.0	2.57	.39	.78	2.63	.40	.80	2.72	.41	.83	2.30	.35	.74
1	1.0	2.5	2.29	.35	.70	2.34	.36	.89	2.41	.37	.92	2.10	.32	.80
1	1.0	3.0	2.06	.31	.94	2.10	.32	.96	2.16	.33	.98	1.89	.29	.86
1	1.0	3.5	1.84	.28	.98	1.88	.29	1.00	1.88	.29	1.05	1.71	.26	.91
1	1.5	2.5	2.05	.47	.78	2.09	.48	.80	2.16	.49	.82	1.83	.42	.73
1	1.5	3.0	1.85	.42	.84	1.90	.43	.87	1.96	.45	.89	1.71	.39	.78
1	1.5	3.5	1.72	.39	.91	1.74	.40	.93	1.79	.41	.96	1.57	.36	.83
1	1.5	4.0	1.57	.36	.96	1.61	.37	.98	1.64	.38	1.00	1.46	.33	.88
1	1.5	4.5	1.43	.33	.98	1.46	.33	1.00	1.51	.35	1.06	1.34	.31	.91
1	2.0	3.0	1.70	.52	.77	1.73	.53	.79	1.78	.54	.81	1.54	.47	.73
1	2.0	3.5	1.57	.48	.83	1.61	.49	.85	1.66	.50	.88	1.44	.44	.77
1	2.0	4.0	1.46	.44	.89	1.48	.45	.90	1.53	.47	.93	1.34	.41	.81
1	2.0	4.5	1.36	.42	.93	1.38	.42	.95	1.43	.43	.98	1.26	.38	.86
1	2.0	5.0	1.27	.39	.97	1.29	.39	.98	1.33	.39	1.03	1.17	.36	.89
1	2.5	3.5	1.45	.55	.77	1.48	.56	.79	1.51	.58	.81	1.32	.50	.70

1	2.5	4.0	1.35	.52	.82	1.38	.53	.84	1.42	.54	.87	1.24	.47	.75
1	2.5	4.5	1.27	.48	.87	1.29	.49	.88	1.33	.51	.91	1.16	.44	.80
1	2.5	5.0	1.19	.46	.91	1.21	.46	.92	1.26	.48	.96	1.10	.42	.83
1	2.5	5.5	1.13	.43	.94	1.15	.44	.96	1.18	.44	.99	1.03	.39	.86
1	2.5	6.0	1.07	.41	.97	1.07	.41	.98	1.10	.41	1.03	.98	.37	.89
1	3.0	4.0	1.26	.58	.77	1.28	.58	.78	1.32	.60	.80	1.15	.52	.72
1	3.0	4.5	1.18	.54	.81	1.20	.55	.82	1.24	.57	.85	1.09	.50	.75
1	3.0	5.0	1.11	.51	.85	1.14	.52	.87	1.17	.54	.89	1.03	.47	.78
1	3.0	5.5	1.06	.48	.89	1.07	.49	.90	1.11	.51	.93	.97	.44	.81
1	3.0	6.0	1.01	.46	.92	1.02	.47	.93	1.06	.48	.97	.92	.42	.84
1	3.0	6.5	.96	.44	.95	.98	.44	.96	1.00	.45	1.01	.88	.40	.87
1	3.0	7.0	.91	.42	.97	.92	.42	.98	.94	.42	1.05	.84	.38	.89
1	3.5	5.0	1.05	.56	.80	1.07	.57	.82	1.11	.59	.85	.96	.50	.76
1	3.5	5.5	1.00	.53	.84	1.02	.54	.85	1.06	.56	.89	.92	.48	.78
1	3.5	6.0	.95	.50	.87	.97	.51	.89	1.00	.53	.92	.88	.46	.80
1	3.5	6.5	.92	.49	.91	.93	.49	.92	.96	.51	.95	.83	.44	.82
1	3.5	7.0	.87	.47	.93	.89	.47	.95	.91	.49	.98	.80	.43	.85
1	3.5	7.5	.84	.45	.96	.86	.45	.98	.86	.47	1.01	.76	.41	.87
1	3.5	8.0	.80	.42	.97	.82	.43	1.01	.81	.45	1.04	.73	.39	.89
1	4.0	6.0	.90	.55	.82	.92	.56	.84	.95	.58	.87	.83	.51	.77
1	4.0	6.5	.87	.53	.85	.88	.53	.87	.91	.55	.90	.80	.49	.79
1	4.0	7.0	.83	.51	.89	.84	.51	.90	.87	.53	.93	.77	.47	.81
1	4.0	7.5	.80	.49	.91	.81	.50	.93	.84	.51	.96	.73	.44	.83
1	4.0	8.0	.77	.47	.93	.78	.48	.95	.81	.49	.98	.71	.43	.86
1	4.0	8.5	.74	.45	.95	.76	.46	.98	.78	.47	1.01	.68	.42	.88
1	4.0	9.0	.71	.43	.97	.73	.44	1.01	.75	.45	1.04	.65	.40	.89
1	5.0	9.0	.66	.50	.90	.67	.52	.93	.70	.53	.96	.61	.46	.83
1	5.0	10.0	.62	.47	.95	.63	.48	.96	.65	.50	1.00	.57	.43	.87

WORKING OF CONCRETE

Mixing of Concrete.—Concrete may be mixed either by hand or by machine. For small work, the concrete is mixed by hand in small batches, such as would be made up from 1 or 2 bags of cement. In mixing, hand work should be performed on a flat, water-tight platform. The sand, after it has been measured, is spread over the platform in an even layer. Upon the sand is placed the cement, and these two materials are turned over with shovels at least three times, or until the uniform color of the mixture indicates that they are thoroughly incorporated. The stones, or aggregates, having previously been well wetted, are then placed on the top of the mixture of sand and cement and these materials are also turned at least three times, water being added after the first turning. The water should always be added in small quantities. If a hose is used for this purpose, it should be fitted with a sprinkling nozzle, as otherwise much of the cement is liable to be washed out of the mixture. The concrete, when ready for placing, should be of uniform consistency, either mealy for a dry mix or mushy for a wet mix.

In large work, the mixing should be done by machine.

Retempering of Concrete.—If the cement of the concrete has attained its initial set before being placed—that is, if the concrete has commenced to harden—remixing with water, or *retempering of concrete*, as it is called, should not be allowed; and if concrete treated in this manner has been deposited in the forms, it should be taken out and removed from the site of the operation, because concrete cannot be retempered properly, except in small quantities for laboratory tests.

Concreting at High Temperatures.—If the weather is extremely warm, the stone and sand are liable to become heated to a high temperature. Then, in mixing the materials, the water necessary for the crystallization of the cement is rapidly absorbed by the stone and the sand, or else rapidly evaporated by contact with them. Again, the extreme heat will hasten the setting of the cement, and this tends to cause the concrete to cake in the mixing machine, producing lumpy and inferior concrete. In order to overcome

such difficulties, the stone should be thoroughly wetted with a hose, and the sand and stone should be kept under cover, away from the direct rays of the sun. Likewise, the mixing platform or machine should be roofed over. It is well, also, to wet down the finished concrete work with a hose several times a day in extremely hot weather, and less frequently in moderate temperatures.

Concreting in Freezing Weather.—Although it is practicable to mix and place concrete at a temperature as low as 27° F., it is not advisable to lay concrete work when the temperature is below 32°; neither should it be mixed and placed even at this temperature, if there is a possibility that the temperature will fall. If concrete is frozen, its setting is retarded and it is liable to become worthless, never properly setting and obtaining the requisite hardness and strength. There is, however, no certainty of the action of frost on concrete, as frozen concrete will frequently thaw out and set, with apparently little loss of strength.

To prevent the freezing of concrete when the temperature has fallen below 32° F., salt is sometimes used in the mixture. The addition of $1\frac{1}{4}$ lb. of salt to the water used with 1 bag of cement will not decrease the strength of the concrete; or, a 10% solution of salt can be used in the water employed in mixing the concrete. The addition of salt, however, is never advisable if a surface finish is required, as it is liable to cause efflorescence, or a white deposit, on the surface causing the work to become very unsightly.

Aggregates that are coated with ice or that have been exposed to severe weather for a long time should be heated or thawed out before being used. Concrete that is exposed to freezing after it was set should always be protected by placing over it a layer of boards and straw, or salt hay, or cement bags; or, where the work is in the nature of a reinforced-concrete floor system, by heating the interior of the structure by means of salamanders or fires.

Joining of Old Concrete With New.—New and old concrete can be joined only with difficulty, and the strength of such a connection is always uncertain. The joining of old and new concrete work is best done by thoroughly chipping, or

cutting away, the old surface, saturating it with water, and working into it thin coats of a 1–1 Portland-cement mortar, and, then, while the coating is still fresh, placing against it the new concrete.

There are some high-grade, imported cements that, in the form of cement mortar, more readily adhere to old concrete work than the usual Portland cements. These cements are frequently used for patching and piecing out work already in place.

ELEMENTS OF STEEL REINFORCEMENT

THEORY OF STEEL REINFORCEMENT

Principles of Construction.—When a beam is subjected to transverse stress, the portion of the beam section above the neutral axis is in compression, while in that portion below the neutral axis, tensile stresses are created. Ordinarily, concrete is about ten times as strong in compression as it is in tension. Thus, it can readily be seen that a beam of plain concrete without steel reinforcement would fail primarily from lack of tensile resistance, without realizing its full compressive strength. In order, therefore, to make concrete an economical material to use in construction, its deficiency in tensile resistance must be made up by embedding steel rods, bars, or some other form of metallic reinforcement in that portion of the beam section subjected to tensile stress.

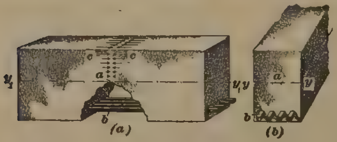


FIG. 1

In order to explain more fully this primary principle of reinforced concrete, reference is made to the reinforced, rectangular concrete beam shown in Fig. 1. The neutral line of the section is shown at y_1 y_1 in the side view (a), while the neutral axis is represented by y y in the end view (b). When the concrete beam is under transverse stress, there is neither

tensile nor compressive stress at the neutral axis. Therefore, the point in the beam marked *a*, which is on the neutral axis, is subjected to zero stress.

Imagine that the concrete is cut away below the neutral plane, leaving the steel reinforcing rods, or bars, exposed as at *b*. It is evident that the strength of the beam is not much affected by the cutting away of the concrete in this manner, as the necessary tension below the neutral axis is supplied by the reinforcing rods of steel, while the necessary compression

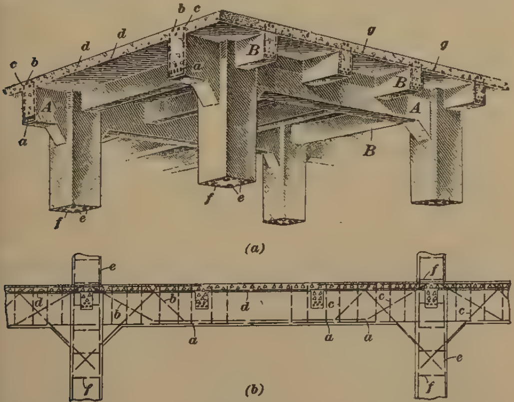


FIG. 2

above it is furnished by the concrete, as at *c*, *c*. The amount of compression in each square inch of concrete above the neutral axis varies from zero at the axis to maximum at the extreme upper surface of the beam. The concrete below the neutral axis $y\,y$ is usually so filled with very fine cracks that all the tension must be carried by the steel alone.

In ordinary reinforced-concrete column construction, merely vertical rods are employed. They are tied, however, at intervals with wire or other ties.

The principle of *hooped columns* is best explained as follows: It is well known that a column of sand will not resist compression, because it will spread, and it is likewise certain that a cylinder of, say, very thin metal will sustain only a small load. However, if the cylinder is filled with sand, the combination, in which the tensile strength of the cylinder is realized with the compressive resistance of the sand, will result in a strong column, or post, capable of resisting considerable compression. This principle is applied to the reinforcement of concrete columns by binding, or tying, together the concrete with cylindrical hoops, or helical, or spiral, windings of steel.

Parts of Steel Reinforcement Defined.—In Fig. 2 (*a*) is shown a perspective view of a complete bay of a reinforced-concrete floor system, and in (*b*), a diagrammatic representation of a typical system of reinforcement for a concrete girder and column. In (*a*), the heavy members *A* running between columns are commonly known as *girders*, and the lighter members *B* running between girders, as *beams*. In both (*a*) and (*b*), the rods, or bars, *a* are the *main reinforcing bars*, or *rods*, of the girders. The beams, of course, have similar main reinforcing bars. Of these main reinforcing bars, several are bent up, as at *b*, to form *trussed bars*. The web reinforcement of the girders is shown at *c*, and consists of **U**-shaped pieces of iron or steel, called *stirrups*. The rods that reinforce the slab of the reinforced-concrete floor system, called *slab rods*, are shown at *d*. This slab reinforcement may consist of straight rods, expanded metal, woven-wire lath, or any other metallic reinforcement. The rods of the columns, shown at *e*, are called *longitudinal column rods*, and the hooped separators, or ties, shown at *f*, *column ties*.

Any rod, or bar, used to resist shearing stresses is designated as a *shear bar*. A rod, or bar, used to resist the shrinkage of the concrete in setting, or to provide against cracks due to thermal changes, is called a *shrinkage rod*. Shrinkage rods are shown at *g* in view (*a*). A rod used to connect abutting beams or girders is called a *tension bar* or a *tie-bar*. The short rods used at the splice when longitudinal column rods are butted are called *splice rods*, or *bars*.

Members to Resist Lines of Failure.—In Fig. 3 are illustrated a typical beam having the usual type of steel reinforcement and the several methods of failure that might occur. At *a* are shown cracks, or lines of failure, that would be caused by lack of tensile resistance in the main reinforcing rods *b*. These cracks, although usually invisible, generally extend from the bottom surface to the neutral axis. They are nearly always present in concrete, but, of course, so long as the steel holds, the beam will not fail.

If the main reinforcing rods do not extend to the bearings, failure by vertical shear may occur near the abutments, along the line *x x*. Ordinarily, however, failures of this kind seldom happen, because the main rods usually extend across all

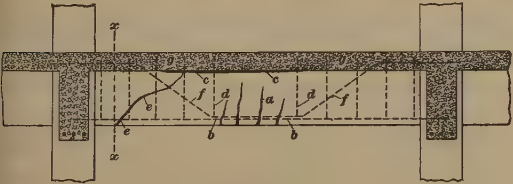


Fig. 3

such lines of vertical shear, and add greatly to the shearing resistance of the beam.

If the slab concrete is not placed at the same time that the concrete of the beam section is poured, failure by shearing usually occurs at the junction of the beam with the slab, as shown at *c c*. The shearing resistance at this junction may be increased, however, by extending stirrups *d* into the slab. If the crack *c c* opens, it usually joins with a crack like *e e* at each end of the beam, as suggested in the preceding paragraph.

The lines of failure indicated at *e e* are those which usually occur from diagonal tension stresses that cross these lines of failure at right angles. A beam is held against failure in this manner by placing stirrups in the concrete either vertically or obliquely. The bending up of the main reinforcing rods

to form the trussed bar, as shown at *f f*, will also assist in resisting such stresses, and, besides, will provide against negative bending moment where tension instead of compression is created at *g g*. The line of fracture shown at *e e* is typical of nearly all reinforced-concrete failures.

METALLIC REINFORCEMENT

CHARACTERISTICS OF METALS USED FOR REINFORCEMENT

Mild, or Soft, Steel.—The commercial *mild*, or *soft*, *steel*, which is an excellent material for reinforcing rods, especially in concrete structures subjected to sudden strain or shock, should have a unit ultimate tensile strength of from 52,000 to 62,000 lb. and an elastic limit of not less than one-half this amount. It should have an elongation of at least 25% in 8 in., and should be capable of being bent cold through 180° and hammered flat on itself without evidence of fracture on the outside circumference of the bend.

Medium Steel.—The grade of steel known as *medium* more fully meets all the requirements of reinforced-concrete construction than any other grade. Commercial medium steel should have an ultimate unit tensile stress of 60,000 to 70,000 lb., and an elastic limit of not less than one-half this amount. Upon testing, the elongation should be found to be at least 22% in a length of 8 in., and it should withstand bending through 180° around a diameter equal to the diameter or the thickness of the pieces tested. Such a test piece should not show fracture on the outside circumference of the bent portion.

High-Carbon Steel.—The name *high-carbon steel* is applied to steels rolled particularly for the reinforcement of concrete. Such steels contain a higher percentage of carbon than either medium or soft steel. The product as a rule is brittle, and possesses a unit ultimate tensile strength of from 80,000 to 100,000 lb., with an elastic limit of about one-half of its ultimate strength. The amount of elongation in 8 in. is sometimes as low as 5%, and seldom exceeds 15 to 18%. Ordinarily, it will not bend much beyond a right angle with-

out showing fracture. On account of the brittleness and general unreliability of high-carbon steel, the material should be carefully tested and inspected before being used in reinforced-concrete construction.

AREAS AND WEIGHTS OF SQUARE AND ROUND BARS

Size Inches	Square ■		Round ●	
	Area Inches	Weight per Foot Pounds	Area Inches	Weight per Foot Pounds
$\frac{1}{16}$	.0039	.013	.0031	.010
$\frac{1}{8}$	.0156	.053	.0123	.042
$\frac{3}{16}$	.0352	.120	.0276	.094
$\frac{1}{4}$	.0625	.213	.0491	.167
$\frac{5}{16}$	.0977	.332	.0767	.261
$\frac{3}{8}$	.1406	.478	.1104	.376
$\frac{7}{16}$	.1914	.651	.1503	.511
$\frac{1}{2}$	.2500	.850	.1963	.668
$\frac{9}{16}$	.3164	1.076	.2485	.845
$\frac{5}{8}$	.3906	1.328	.3068	1.043
$\frac{11}{16}$	.4727	1.607	.3712	1.262
$\frac{3}{4}$	.5625	1.913	.4418	1.502
$\frac{13}{16}$	.6602	2.245	.5185	1.763
$\frac{7}{8}$	.7656	2.603	.6013	2.044
$\frac{15}{16}$	.8789	2.989	.6903	2.347
1	1.0000	3.400	.7854	2.670
$1\frac{1}{16}$	1.1289	3.838	.8866	3.014
$1\frac{1}{8}$	1.2656	4.303	.9940	3.379
$1\frac{3}{16}$	1.4102	4.795	1.1075	3.766
$1\frac{1}{4}$	1.5625	5.312	1.2272	4.173
$1\frac{5}{16}$	1.7227	5.857	1.3530	4.600
$1\frac{3}{8}$	1.8906	6.428	1.4849	5.049
$1\frac{7}{16}$	2.0664	7.026	1.6230	5.518
$1\frac{1}{2}$	2.2500	7.650	1.7671	6.008
$1\frac{9}{16}$	2.4414	8.301	1.9175	6.520
$1\frac{5}{8}$	2.6406	8.978	2.0739	7.051
$1\frac{11}{16}$	2.8477	9.682	2.2365	7.604
$1\frac{3}{4}$	3.0625	10.413	2.4053	8.178
$1\frac{13}{16}$	3.2852	11.170	2.5802	8.773
$1\frac{7}{8}$	3.5156	11.953	2.7612	9.388
$1\frac{15}{16}$	3.7539	12.763	2.9483	10.024
2	4.0000	13.600	3.1416	10.681

There are to be had on the market twisted bars of square and hexagonal section. Such bars are from 8 to 25% stronger than the bars in their original form. The percentage of increase of strength is greatest with the bars of small section and least with those of large section.

Rerolled Bars.—Bars made from old steel rails can be obtained for reinforcing concrete. In making such bars, the flange, web, and bulb of the rails are cut apart, and rerolled into square and round sections. The square sections may be had twisted, the twisting being performed while the material is hot.

Much of this material has a high elastic limit and tensile strength, but it is inclined to be brittle and variable in strength and ductility. Besides, any lapping or folding of the original material when passed through the rolls is liable to develop laminations throughout the bar, tending to diminish its strength.

TYPES OF STEEL REINFORCEMENT

Plain Bar Iron.—The cheapest form of metallic reinforcement for concrete is the *plain, round, rolled bar*. Bars of this kind can be obtained in any part of the United States. The pound price of round, rolled bars is lower than that of any other form of rolled steel, so they offer the cheapest and most available material. For slabs, $\frac{3}{8}$- to $\frac{1}{2}$-in. round bars are used, while for beam, girder, and column reinforcement, from $\frac{5}{8}$- to $1\frac{1}{8}$-in. bars are ordinarily employed.

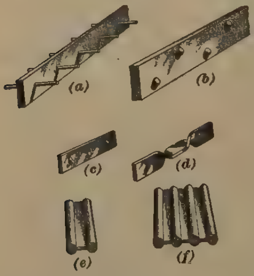


FIG. 1

The principal objection to the use of plain, round bars in reinforced-concrete work is that they are not gripped, or held, well by the concrete.

Plain square and flat bars are sometimes used for the reinforcement of concrete, though, generally, both of these sections, when so used, are deformed by twisting.

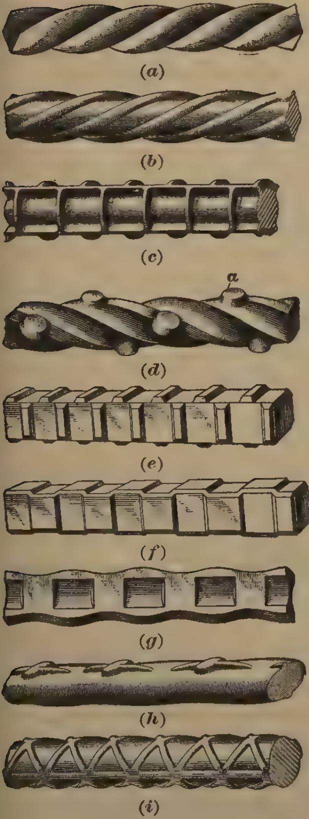


Fig. 2

In the nomenclature of reinforced concrete, round, rolled sections are designated as *rods;* square sections as *bars;* and rectangular sections, as *flats*, or *flat bars*.

In the table on page 253 are given the areas and weights of square and round bars from $\frac{1}{16}$- to 2-in. sizes.

Bars of Special Construction.—Some early forms of bars used in reinforcement of concrete are shown in Fig. 1. That shown in (*a*) is known as the *Hyatt bar*. An early form of the *Thacher bar* is shown in (*b*). In (*c*) is shown the *Staff bar*. It consists of a flat bar, through which a countersunk punch has been partly driven, thus forcing the metal out on the opposite side so as to form projections. The *De Mann bar* is shown in (*d*). Two forms of the *unit bar* are shown in (*e*) and (*f*); that in (*e*) is known as the *Siamese bar* and that at (*f*) as the *quad bar*.

Square-Twisted Bars.—The *square-twisted bar* consists of a square bar that is twisted by giving it a certain number of turns around its axis, either while it is hot or while it is cold. This bar is often known as the *Ransome bar*.

By twisting the bar to the screw shape, as shown in Fig. 2 (*a*), a form is obtained that has great resistance to pulling from a mass of concrete.

If the square bars are twisted cold, their elastic limit and ultimate strength are increased from 8 to 25%.

PHYSICAL PROPERTIES OF THE RANSOME BAR

Size of Bar Inches	Number of Twists per Foot of Length	Area of Bar Square Inches	Weight per Foot Pounds	Elastic Limit Pounds per Square Inch	Ultimate Tensile Strength Pounds per Square Inch	Elastic Limit of One Bar Pounds	Ultimate Tensile Strength of One Bar Pounds
$\frac{1}{4}$	5	.0625	.213	62,350	86,700	3,897	5,419
$\frac{3}{8}$	3	.1406	.478	61,800	86,600	8,689	12,176
$\frac{1}{2}$	2	.2500	.850	60,120	86,850	15,030	21,713
$\frac{5}{8}$	2	.3906	1.328	57,890	85,820	22,612	33,520
$\frac{3}{4}$	1	.5625	1.913	56,720	85,240	31,905	47,948
$\frac{7}{8}$	1	.7656	2.603	56,150	84,730	42,988	64,869
1	$\frac{3}{4}$	1.0000	3.400	55,760	84,275	55,760	84,275
$1\frac{1}{4}$	$\frac{1}{2}$	1.5625	5.312	55,450	83,150	86,641	129,921

The physical properties of the Ransome bar are given in the accompanying table.

Spiral Bar.—In Fig. 2 (*b*) is shown a type of twisted bar called the *spiral bar*. It is made by rolling steel and then twisting it. The twisted bars have elastic limits of from 65,000 to 80,000 lb. The section of the bar is practically round, with four attached half-round beads. These beads assume the spiral form on twisting the bar, and the same advantage with regard to the bond is secured as with square-

twisted bars. An added advantage is assumed to exist in the elimination of all the sharp corners that occur in the square bar.

Kahn Cup Bar.—One of the latest types of reinforcing bars to be commercially used is the *Kahn cup bar* illustrated in Fig. 2 (*c*). The ribs are connected by cross-ribs, forming cups, or depressions, of such a shape as to allow the concrete to flow into them readily. In this way, a positive mechanical bond in the concrete is provided. The properties of the Kahn cup bar are given in the accompanying table.

PROPERTIES OF THE KAHN CUP BAR

Nominal Size of Bar Inches	Sectional Area Square Inches	Weight per Foot Pounds	Elastic Limit of Each Bar Pounds	Ultimate Tensile Strength of Each Bar Pounds
$\frac{3}{8}$	.1406	.502	5,000	9,800
$\frac{1}{2}$	.2500	.893	9,000	17,500
$\frac{5}{8}$	.3906	1.394	14,000	27,300
$\frac{3}{4}$	.5625	2.008	21,000	39,400
$\frac{7}{8}$	.7656	2.733	28,000	53,600
1	1.0000	3.570	37,000	70,000
$1\frac{1}{8}$	1.2656	4.518	47,000	88,600
$1\frac{1}{4}$	1.5625	5.578	58,000	109,400

Square-Twisted Lug Bars.—The *twisted lug bar* shown in Fig. 2 (*d*) is a development of the square-twisted bar. The square bar in this case is rolled with rounded corners, and with projections, or lugs, *a* at intervals, thus providing additional mechanical bond in the concrete. The rounded corners of this bar eliminate the sharp angles of the ordinary square-twisted bar. These bars have a high elastic limit. Their safe working strength is based on a safe unit stress of 20,000 lb. The properties of twisted lug bars are given in the accompanying table.

Corrugated Bar.—In Fig. 2 (*e*) and (*f*) are shown two styles of deformed bar of the corrugated type known as the *Johnson*

PROPERTIES OF TWISTED LUG BARS

Size of Bar Inches	Net Sectional Area Square Inches	Weight per Foot Pounds	Safe Working Stress for Each Bar Pounds
$\frac{1}{4}$	.0625	.222	1,250
$\frac{3}{8}$	.1406	.492	2,810
$\frac{1}{2}$	.2500	.870	5,000
$\frac{5}{8}$	.3906	1.350	7,810
$\frac{3}{4}$	.5625	1.940	11,250
$\frac{7}{8}$	.7656	2.640	15,310
1	1.0000	3.450	20,000
$1\frac{1}{8}$	1.2656	4.350	25,310
$1\frac{1}{4}$	1.5625	5.370	31,250

bar, named after its inventor, A. L. Johnson. The old style of bar is shown in (*e*), and the new style in (*f*). High-carbon steel having an elastic limit of from 65,000 to 70,000 lb. per sq. in. is used in making the Johnson bar. This type of bar was one of the first deformed bars to be manufactured. The following table gives the size, net section, and weight of Johnson bars :

SIZE, NET SECTION, AND WEIGHT OF CORRUGATED, OR JOHNSON, BARS

Size of Bars Inches	Weight per Foot Pounds	Net Section Square Inches
$\frac{1}{4}$	.78	.19
$\frac{3}{4}$	1.56	.38
$\frac{7}{8}$	2.25	.55
1	2.90	.70
$1\frac{1}{4}$	4.56	1.10

Universal Bar.—In Fig. 2 (*g*) is shown a type of flat deformed bar, called the *Universal bar*, that has been used to some extent for reinforcement. The net section and weight of the various sizes of the Universal bar are given in the following table.

NET SECTIONS AND WEIGHTS OF UNIVERSAL BARS

No.	Size of Bar Inches	Weight per Foot Pounds	Net Section Square Inch
1	$\frac{1}{4}\times 1$	.73	.19
2	$\frac{3}{8}\times 1\frac{3}{8}$	1.35	.41
3	$\frac{3}{8}\times 1\frac{3}{4}$	1.97	.54
4	$\frac{3}{8}\times 2$	2.27	.65
5	$\frac{3}{8}\times 2\frac{1}{2}$	2.85	.80

Thacher Bar.—The deformed bar illustrated in Fig. 2 (*h*), called the *Thacher bar*, was devised in order to obtain a bar so deformed that the net section throughout the bar would be uniform. By having the section uniform, the bar is of the same strength at every point. In forming this bar, an

SIZE, WEIGHT, AND ULTIMATE TENSILE STRENGTH OF THACHER BARS

(*Medium Steel*)

Diameter of Bar Inches	Weight per Foot Pounds	Area of Net Section Inches	Average Ultimate Tensile Strength for Each Bar Pounds
$\frac{1}{4}$	.16	.047	3,000
$\frac{3}{8}$	.34	.10	6,400
$\frac{1}{2}$	.61	.18	11,500
$\frac{5}{8}$	.95	.28	17,900
$\frac{3}{4}$	1.39	.41	26,200
$\frac{7}{8}$	1.87	.55	35,200
1	2.41	.71	45,400
$1\frac{1}{8}$	3.06	.90	57,600
$1\frac{1}{4}$	3.74	1.10	70,400
$1\frac{3}{8}$	4.49	1.32	84,500
$1\frac{1}{2}$	5.30	1.56	99,800
$1\frac{5}{8}$	6.15	1.81	115,800
$1\frac{3}{4}$	7.07	2.08	133,100
$1\frac{7}{8}$	7.99	2.35	150,400
2	9.01	2.65	169,600

effort is made to eliminate all sharp corners. The Thacher bar is rolled from medium steel and has an elastic limit of about 35,000 lb. The accompanying table gives the size, weight, and ultimate tensile strength of the Thacher bar.

SIZE, AREA, AND WEIGHT OF DIAMOND BARS

Nominal Size of Bar Inches	Area of Section Square Inches	Weight per Foot Pounds
$\frac{1}{4}$	.062	.22
$\frac{3}{8}$	.14	.48
$\frac{7}{16}$	.19	.65
$\frac{1}{2}$	.25	.85
$\frac{5}{8}$	.39	1.33
$\frac{3}{4}$	.56	1.91
$\frac{7}{8}$	.76	2.60
1	1.00	3.40
$1\frac{1}{4}$	1.56	5.31

Diamond Bar.—One of the most recent forms of deformed bars is the *Diamond bar*, which is shown in Fig. 2 (*i*). This bar is rolled with a cross-section of constant area. The nominal size, area, and weight of Diamond bars are given in the accompanying table.

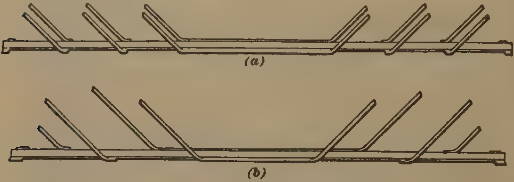


FIG. 3

Kahn Trussed Bar.—Fig. 3 shows two styles of a deformed bar known as the *Kahn trussed bar*. In the old style of bar, shown in (*a*), the prongs are opposite each other; in the new style, shown in (*b*), they are staggered. The shapes of the bar

in section are shown in Fig. 4. The fins are partly sheared across and also in a direction parallel with the axis of the bar, and are bent up, as shown in Fig. 3 so as to form a grip with the concrete and to provide the stirrups, or web members, necessary to resist diagonal stresses.

The Kahn bars are made in the several sizes indicated in Fig. 4. The respective sectional areas of the bars are also

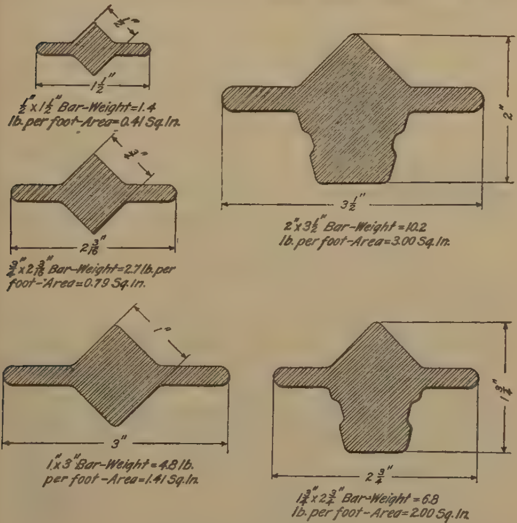


FIG. 4

given in this figure. These bars are rolled of mild steel, as this metal will best stand the shearing stresses to which the bar is subjected in manufacture. The elastic limit of the metal in the bars ranges from 33,000 to 35,000 lb. per sq. in.

Trus-Con Bar.—Another deformed bar, known as the *trus-con bar*, is illustrated in Fig. 5. This bar is round in

section and has notched shoulders for holding washers.

FIG. 5

The washers are punched so that they will slip over the projections, and, then by turning, they are locked with the bar. The trus-con bar is made in $\frac{5}{8}$-, $\frac{3}{4}$-, 1, $1\frac{1}{4}$-, and $1\frac{1}{2}$-in. sizes.

FIG. 6

Monolith Steel Bars.—In Fig. 6 is shown the *monolith steel bar*. This type of bar has no sharp corners, and is not reduced in area or strength by deformations. It also has large surface area. The bar is grooved on the sides so that round iron stirrups, as shown at a, may be inserted. These stirrups are held in place in the bar by swedging the flanges of the bar together. The monolith steel bars are made in sections equivalent to $\frac{1}{2}$-, $\frac{5}{8}$-, 1-, and $1\frac{1}{2}$-in. square bars, and are rolled for stirrups of $\frac{3}{16}$-, $\frac{5}{16}$-, $\frac{3}{8}$-, and $\frac{1}{2}$-in. diameter.

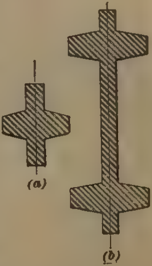


FIG. 7

Columbian Bar.—A rolled shape known as the *Columbian bar* is extensively used in the construction of the Columbian fireproof floor systems and reinforced-concrete structures. The typical forms of this type of bar are shown in Fig. 7. Mild steel is used for rolling the Columbian bar. This kind of steel has an ultimate strength of from 60,000 to 70,000 lb. per sq. in. and an elastic limit of one-half the ultimate strength. The smallest bar, shown in (a), is known as the 1-in. bar, while the type of bar shown in (b), is made in 2-, $2\frac{1}{2}$-, $3\frac{1}{2}$-, and $4\frac{1}{4}$-in. sizes. The 5-in. bar carries a double rib at the bottom.

U Bars.—In Fig. 8 is shown a section of **U** *bar* that can be used to advantage in reinforced-concrete construction either as a tension or as a compression member, being particularly efficient for the latter purpose. These bars are rolled from high elastic-limit steel, and are in some instances made from rerolled steel rails. They are $\frac{1}{4}$, $\frac{5}{16}$, $\frac{3}{8}$, and $\frac{1}{2}$ in. in thickness, weigh from 3 to 9 lb. per ft., and can be obtained in lengths up to 60 ft.

FIG. 8

Structural Shapes Used as Steel Reinforcement.—The usual rolled-steel structural shapes have been used extensively for reinforced-concrete construction. They have an advantage in that they can be readily obtained. Structural shapes as metallic reinforcement for concrete work are being superseded by either plain or deformed rolled bars. One advantage of the use of structural shapes is that a rigid framework can be built up, after the manner of skeleton construction, though of much lighter section, and the concrete then filled in around it.

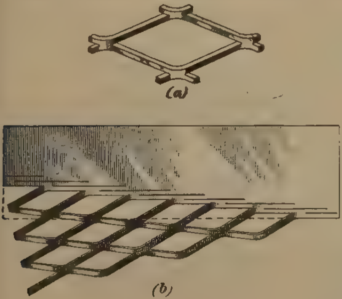


FIG. 9

Expanded Metal. Among the earlier forms of metallic reinforcement for concrete is the distorted steel plate known as *expanded metal*, a familiar illustration of which is shown in Fig. 9 (*a*). This form of reinforcement is manufactured by partly shearing a sheet of steel in parallel rows, as shown in Fig. 9 (*b*), and then pulling the material sidewise, thus forming a diamond mesh. In this way, the area of a sheet is increased about eight times, with a corresponding decrease in weight per unit area and without any waste of material. The steel from which expanded metal is

SIZES OF EXPANDED METAL

U. S. Standard Gauge of Sheet Steel	Sectional Area per Foot of Width Square Inch	Weight Pounds per Square Foot	Elastic Limit per Foot of Width Pounds	Length of Sheets Feet	Thickness Metal Inch	Length of Mesh Inches	Width of Mesh Inches	Width of Separate Strands Inch
18	.203	.69	12,200	8	$\frac{1}{20}$	$1\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{64}$
12	.253	.86	15,200	8	$\frac{7}{64}$	2	$\frac{3}{4}$	$\frac{5}{64}$
12	.188	.64	11,300	12	$\frac{7}{64}$	3	$1\frac{1}{2}$	$\frac{7}{64}$
16	.087	.29	5,220	8	$\frac{1}{16}$	6	$2\frac{1}{4}$	$\frac{1}{8}$
16	.130	.44	7,800	8	$\frac{1}{16}$	6	$2\frac{1}{4}$	$\frac{3}{16}$
16	.059	.20	3,540	12	$\frac{1}{16}$	8	3	$\frac{7}{64}$
12	.109	.37	6,550	12	$\frac{7}{64}$	8	3	$\frac{1}{8}$
10	.162	.55	9,700	12	$\frac{9}{64}$	8	3	$\frac{9}{64}$
10	.243	.81	14,600	12	$\frac{9}{64}$	8	3	$\frac{7}{32}$
10	.324	1.07	19,400	12	$\frac{9}{64}$	8	3	$\frac{9}{32}$
7	.400	1.36	24,000	12	$\frac{3}{16}$	8	3	$\frac{13}{64}$
4	.245	.84	14,700	12	$\frac{15}{64}$	12	6	$\frac{1}{4}$
4	.368	1.26	22,100	12	$\frac{15}{64}$	12	6	$\frac{3}{8}$

manufactured has a unit tensile strength of about 65,000 lb. and an average elastic limit of about 30,000 lb. It is

FIG. 10

principally used in reinforced-concrete construction as slab reinforcement. The sizes of expanded metal are given in the accompanying table.

Kahn Expanded Metal.—Another concrete-slab reinforcement, called *Kahn expanded metal*, is shown in Fig. 10. This

PROPERTIES OF KAHN EXPANDED METAL

Number	Distance Between Ribs Inches	Sectional Area of Metal per Foot in Width. Square Inches	Safe Tensile Stress per Foot in Width Pounds	Ultimate Stress per Foot in Width Pounds	Width of Sheet Inches	Weight per Square Foot Pounds
2	2	.48	8,640	33,600	17	2.13
3	3	.32	5,760	22,400	25	1.43
4	4	.24	4,320	16,800	33	1.08
5	5	.19	3,420	13,300	41	.87
6	6	.16	2,880	11,200	49	.72
7	7	.14	2,520	9,800	57	.62
8	8	.12	2,160	8,400	65	.55

form of reinforcement is manufactured by shearing a rolled section along lines parallel with its length, and then pulling

the metal so sheared in a direction at right angles to the lines of shear. In this type of expanded metal, the main bars

FIG. 11

of the section represent the material available for the reinforcement of the slab, while the light cross-bars act as spacing bars and also as shrinkage rods when embedded in concrete. The advantage claimed for the Kahn expanded metal is that it transmits the load directly to the supports without any tendency to elongate or distort. The properties of this reinforcing metal are given in the accompanying table.

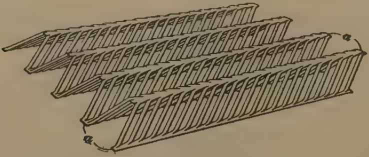


FIG. 12

Herring-Bone Metal Lath.—In Fig. 11 is shown a type of expanded metal known as *herring-bone metal lath*. This

metal is particularly interesting, because, when bent into the form shown in Fig. 12, it provides a self-centering material that is useful for light roof construction. When bent in this manner, the herring-bone expanded metal is known to the manufacturers as *trussit*. The longitudinal ribs, as at a, give it at least sufficient transverse resistance to allow the placing of a thin roof slab of concrete on it without other centering or support.

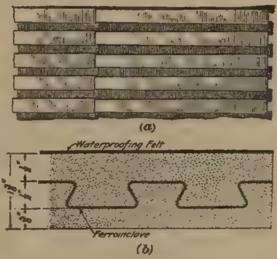


FIG. 13

Sheet-Metal Reinforcement.—The type of metal reinforcement known as *Ferroinclave* is used for making floor slabs and for stair and roof construction. This reinforcement consists of sheet metal that is bent into grooves, as indicated in Fig. 13 (a), annealed sheet steel generally of No. 24 U. S. gauge, being used in its manufacture. The corrugations are made dovetailed in section so that the end of one sheet can slip into another, as shown. The sheets are 20⅝ in. in width and 10 ft. in length. Fig. 13 (b) shows how this type of metallic reinforcement is used in constructing a slab for a light roof.

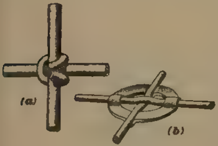


FIG. 14

Lock-Woven and Tie-Locked Wire Fabric.—In Fig. 14 (a) is illustrated the form of wire netting known as *lock-woven wire fabric*. The cross-wires of this fabric are joined by means of a staple of light wire, which is bent so as to embrace the crossing wires at their intersection.

Another type of junction for cross-wires is shown in Fig. 14 (b), which illustrates the principal features of the *tie-locked fabric*. Here, the cross-wires are secured, or

locked, in position by means of a small disk, or washer, and by kinking the wires.

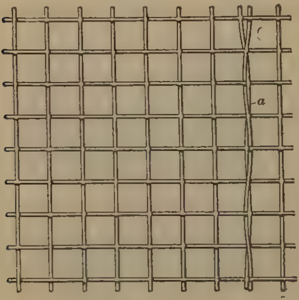
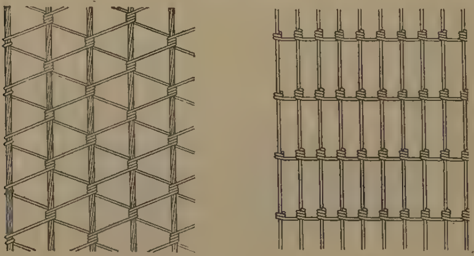


FIG. 15

Electrically Welded Fabric.—The *Clinton wire cloth* is a fabric that is secured at the intersections by a perfect electric weld, and it has at intervals a double wire that twines in and out, as shown at *a*, Fig. 15.

Triangular- and Square-Mesh Wire Reinforcement.—The two types of wire reinforcement shown in Fig. 16 are extensively manufactured for reinforced-concrete construction. The reinforcement shown in (*a*) is known as the *triangular-mesh reinforcement*, and that shown in (*b*), as the *square-mesh reinforcement*. The latter consists of heavy longitudinal wires and cross-wires, or spacing wires. The cross-wires are carried through and



(*a*) (*b*)

FIG. 16

twisted around the longitudinal wires so as to form the rectangular spaces.

SYSTEMS OF STEEL REINFORCEMENT

LOOSE-ROD SYSTEMS

A complete floor system constructed of *loose rods* is shown in Fig. 1. The beam reinforcement consists of three reinforcing rods. Two of these rods run straight through the entire length of the beam, as at *a*, while the third one is bent upwards at the ends, as at *b*. This bent member provides tensile resistance at the top of the beams and thus takes care of the negative bending moment, which occurs in all beams fixed at the end. The bend in such rods is usually made at an angle of about 30° with the horizontal. The rods should be straight at the center of the span for at least one-third the distance between the supports.

A tie-rod *c* that is 4 or 5 ft. in length, and sometimes bent down at the ends, should be placed over the top of the beam juncture.

The girder reinforcement consists of five rods, two of them being bent up, as shown at *e*, to provide against negative bending moment.

In the best work, two short rods *f* are located transversely through the column. These rods tie the adjoining girders together and provide additional rigidity at the junction of the girders with the column.

The slab rods, shown at *h*, are generally spaced at about 6 inches from center to center. They should bond with the stirrups, or web reinforcement, of the beams, and may be threaded through, interlocked, or wired to them. It is customary to provide shrinkage rods that extend at right angles to the regular slab reinforcement, in order to prevent shrinkage cracks in the concrete. For this purpose, $\frac{1}{4}$-in. round or square rods *j* are generally used, and these are spaced about 2 ft. from center to center. In order to bond the concrete over the main girders securely, it is also good practice to provide over these important members rods *d* of about the same size as the slab rods. These rods should run through holes punched in the top of the stirrup, as

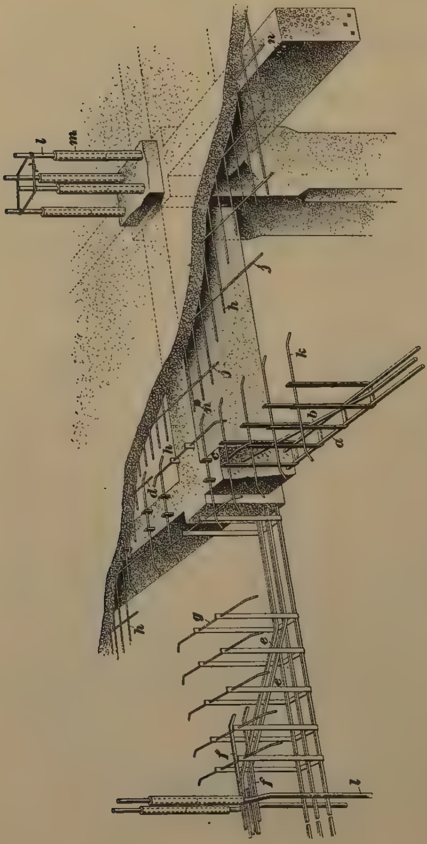


FIG. 1

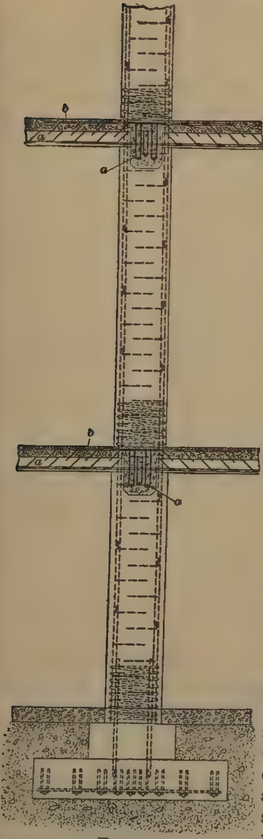


FIG. 2

illustrated at *g*, and should extend at right angles to the axis of the girder. Sometimes, similar rods are used in the slab over beams, as shown at *k*.

The longitudinal reinforcement of the concrete columns consists of four round rods *l*. It is customary to project them above the concrete of each story about a foot and to splice them by lapping and wiring or by using pipe sockets *m*, as illustrated. Frequently, it is not possible to lay out beforehand the electric-light or power wiring, but if this installation is to be adopted 1¼-in. pipes to serve as a passageway should be embedded near the center of the span of all beams and girders, close to the under side of the slab construction, as at *n*.

Kahn System.—The *Kahn system* of reinforced-concrete construction is based on the use of the Kahn trussed bar. A typical monolithic construction based on the use of the Kahn system is shown in Fig. 2. The main reinforcing members of the beams and girders are shown at *a*, and they consist of

FIG. 3

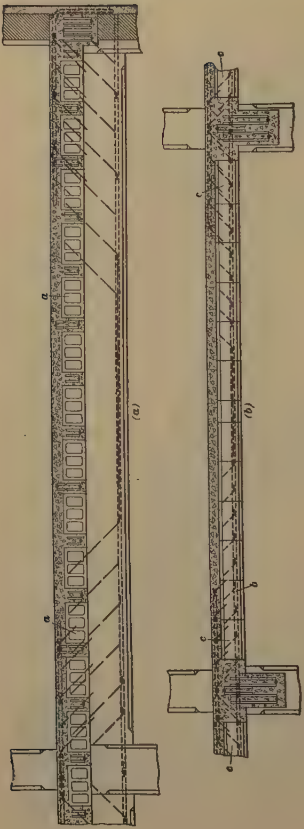


FIG. 4

Kahn bars with the prongs bent upwards so as to form stirrups. The bars are usually placed in the forms on a 2-in. bed of concrete, and after being centered, or registered in their proper positions, they are secured by wiring or by blocking. In order that the girders and beam connections over columns may have continuity, an inverted Kahn bar *b* is used over such junctions.

Kahn bars are spaced in each corner of the column, as shown in Fig. 3, with the prongs bent at right angles. The bars are usually tied together with heavy wire or light rods, wound around them as shown part way up the columns in Fig. 2.

The Kahn bar is frequently used with concrete and hollow terra-cotta tile to form floor systems designed for light loads. This type of construction is illustrated in Fig. 4. In view (*a*) is shown a

cross-section of the construction through the beams or secondary members, of the floor system. Usually, the tiles are from 12 to 16 in. in width and from 6 to 12 in. in depth, the hollow part having the cross-section indicated at *a*. The tiles are spaced about $5\frac{3}{4}$ in. apart, and in the space between them is laid concrete that is reinforced with Kahn bars. The concrete is carried over the top of the tile to a depth of about 2 in. The Kahn bar shown at *b* in (*b*) is provided with the usual prongs, or stirrups, which are bent upwards toward the abutments. The tiles *c* are serrated, or corrugated, along the sides, so as to provide some shearing strength in addition to the adhesion of the concrete. During the construction of this floor system, the

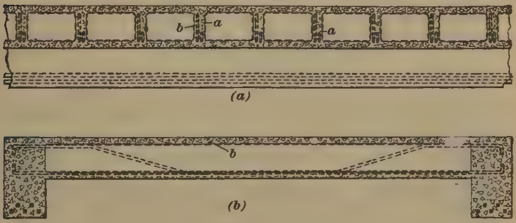


FIG. 5

tiles should be held temporarily in position either by placing blocking between them or by nailing them to the forms. It is not customary in this type of construction to use shrinkage rods in the slab that is laid over the top of the tile. Since the tiles are quickly laid on the centering and, consequently, minimize the quantity of concrete required for the construction of the floor, and, also, since the concrete placed in conjunction with the tiles sets more rapidly than when placed alone, this particular type of construction can be carried on with considerable rapidity. Such a floor construction as this, embodying tile having a hollow air space, acts as a good deadener of sound and also tends to preven heat from passing from one floor to another.

Merrick System.—In Fig. 5 is shown the *Merrick floor system* of reinforced-concrete construction. The space between girders is occupied by narrow concrete beams *a*, as shown in the transverse section (*a*). Between these beams are placed boxes made of one of the many styles of metal fabric on the market. These boxes run the entire length of the clear span of the floorbeams, as shown in the longitudinal section (*b*), and serve to make the floor lighter. They are indicated in both views by the heavy dotted lines *b*. Above and below the metal-fabric boxes is a layer of concrete. This layer is usually made about 2 in. thick, so as to give a flat ceiling and a flat floor surface.

Gabriel System.—The *Gabriel system* of reinforced-concrete construction consists of steel reinforcing bars to which are attached round-iron stirrups. These stirrups are formed by wrapping a wire *a* several times around the reinforcing bar

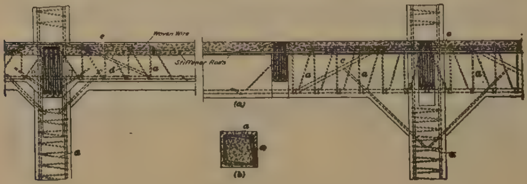


FIG. 6

and extending it up into the slab, as illustrated in Fig. 6, which shows this particular type of construction complete and in its several details. The wiring of the columns is continuous, and extends from the bottom to the top, as indicated in the figure, particularly in the sectional view (*b*). Several of the reinforcing rods of both the girders and the beams bend upwards, as shown at *c* and *d*, respectively. They also lap over the center of the column to form an additional bond in the concrete, as shown at *e*. In this system of construction a lap bar is sometimes provided at the top of the junction, as shown. The slab is shown reinforced with woven wire and shrinkage rods of round iron.

With the Gabriel system of floor construction, hollow tile is also used, as illustrated in Fig. 7. The system shown in view (*a*) is suitable only for very light construction. The hollow tiles *a* are sandwiched between reinforced-concrete joists, as at *b*, and the reinforcement of the floor consists of a main reinforcing bar *c*, around which a round-iron stirrup is wrapped so as to extend continuously. A similar construction is shown in view (*b*). Here, however, a slab of

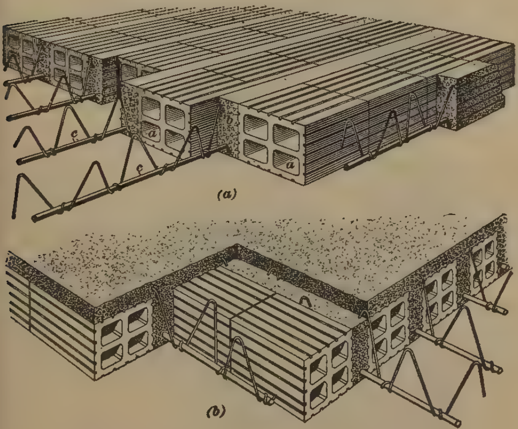


FIG. 7

concrete is extended over the top of the tile, thus giving additional compressive strength at the top of the concrete beams and permitting this construction to be used for heavier loads.

Mushroom System of Reinforcement.—A system of reinforced-concrete construction that differs from all others in that no beams nor girders are used throughout is illustrated in Fig. 8. This system of construction, termed the *mushroom*

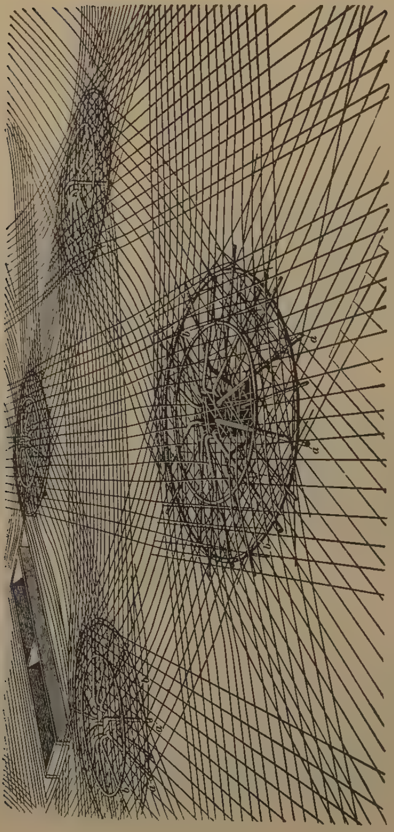


Fig. 8

system, consists of reinforced columns that have a spread cap of heavy reinforcing bars radiating from the column, as illustrated at *a*, Fig. 8. These rods are held radially by circular reinforcing bars *b*, to which they are securely wired. By means of this heavy reinforcement, the column is greatly strengthened at the top, and by spreading in this way, it supports a large portion of the floor slab, after the manner of radiating cantilevers. The flat slab that covers the column supports is reinforced in several directions with light reinforcing rods.

FABRICATED SYSTEMS

Unit System.—The girder frame in the *unit system* is now made of plain bars, and the stirrups in the latest development are in the form of an inverted **U**, the construction of the frame being as shown in Fig. 9.

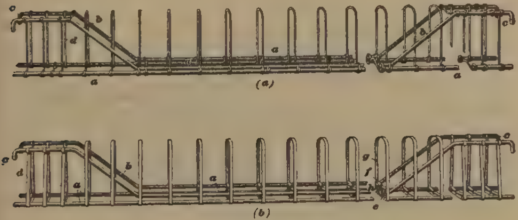


FIG. 9

The frame shown in view (*a*) is fabricated, or fastened together, by a shrinking process; that is, each stirrup and tie-rod is hot-shrunk to the main members. This girder frame is designed to withstand the hard usage of shipment and unloading. The type of frame shown in view (*b*) is designed for use in districts located so far away from the shops of the manufacturer as to make the freight rates for shipping complete frames prohibitory.

In the frames shown in Fig. 9 there are four main reinforcing rods *a*, although the frame may be made up with any

number of rods necessary to secure the desired sectional area. The two rods at the top are bent upwards, as shown at *b*, and the ends *c* are turned over so as to add to their grip in the concrete. The stirrups in both frames are shown at *d*, $\frac{3}{8}$-in. round bar iron being used for the stirrups of the frame shown in view (*a*), and light strap, about $\frac{1}{8}$ in. $\times$ 1 in., for the stirrups of the construction shown in view (*b*). In the latter frame, the section at the right of the figure shows the reinforcing bars held in the clamping device, which consists of the casting *e*, the clip *f*, and the stirrup *g*. These parts are secured by means of a bolt *h*.

Pin-Connected Girder Frame.—Another type of rod reinforcement built up in the form of a girder frame is illustrated

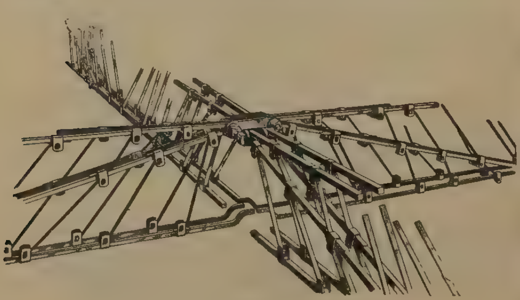


FIG. 10

in Fig. 10. This reinforcement is known as the *pin-connected girder frame*, and it is arranged so that the frames forming the reinforcement of the beams and girders may be connected by pins and links at intersections and over column supports.

Cummings System of Reinforced Concrete.—The system of steel reinforcement shown in Fig. 11, known as the *Cummings loop-truss girder*, consists of a series of main reinforcing bars *a*, to which are attached a set of smaller bars *b* that turn up at the ends and form a loop welded at the ends. These

loops are secured to the main reinforcing rods by means of metal clips *c*.

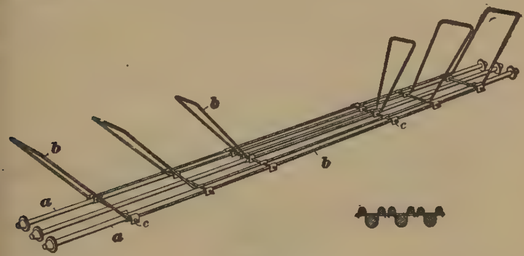


FIG. 11

Shear-Frame System of Steel Reinforcement.—The *shear-frame system* is employed to resist the negative, or reversed, bending moments that occur at the points of support of monolithic beams and girders. This type of reinforcement consists of a built-up frame constructed as shown in Fig. 12. The shear frame consists of straight top rods *a* and bent bottom rods *b*. These two sets of rods are securely clamped together by means of flat bar-iron plates, or clips, *c*, which also act as stirrups. Additional stirrups are provided at *g*,

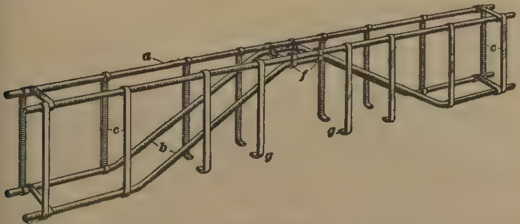


FIG. 12

and where the top and bottom rods join they are firmly fastened together by means of special clamps *f*.

This type of reinforcement provides a positive tie, or junction, between beams and at the intersection of beams, girders, and columns, insuring against failure at these points by providing resistance to shear and to failure by horizontal

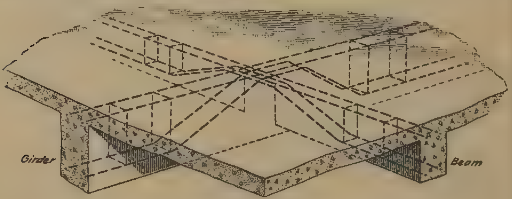


FIG. 13

or oblique tension cracks. The frame may be used with any type of reinforcement, and is placed at the junction of beams with girders, as shown in Fig. 13, and at the intersection of the girder and column supports.

MISCELLANEOUS SYSTEMS

Brayton System.—The system of reinforced concrete known as the *Brayton* is illustrated in Fig. 14. The main reinforcing member is a standard, rolled-steel, I-beam section, and the entire reinforcement, including the steelwork, is put together in a manner similar to the usual-steel-frame construction. The structural-steel beams that form the reinforcement carry the dead load of the centering and the weight of the concrete. Thus, shores need not be used in the erection.

The I-beam *a* forming the main reinforcing member of the concrete beam is shown in view (*a*). As shown at *b*, the stirrups extend beyond the top flange of the I beam in the form of loops, and are either riveted through the web or fastened by clipping them to the lower flange, as indicated in (*b*). The I beam is supported on a column built up of angles and provided with brackets *d*. The slab rods are

fastened to the stirrups by means of clips and wiring, as shown in detail in view (*c*).

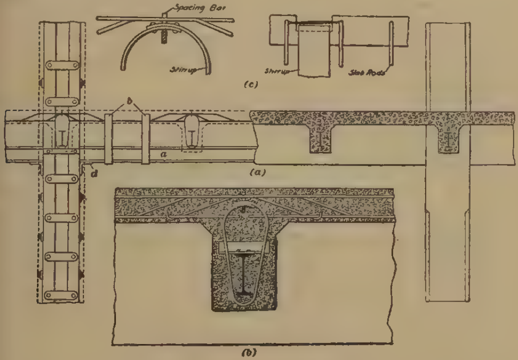


FIG. 14

Visintini System.—Another system of concrete construction is the *Visintini system*. A general view of a girder constructed according to this system is shown in Fig. 15. The girders can be made of any width and breadth required to withstand the stresses. In France, the columns of buildings are also built by this system, but in the United States solid columns have been preferred. The beams are made along

FIG. 15

the lines of latticed steel girders. In light work, the vertical webs *a*, which are in compression, are not always reinforced,

but the slanting webs b, which are in tension, always have steel in them. The upper and lower chords contain each a set of three reinforcing rods, as shown at d and e, respectively. The beams may be made in any convenient location and put in place on the job afterwards. When solid reinforced-concrete columns are used with this system, these columns are cast in place and are provided with brackets to hold the principal girders. These girders are then hoisted on to these brackets. Part of the end of the girder is made solid, as shown at c, in order to resist the stresses in shear and direct compression due to the reaction of the support.

DESIGN OF CONCRETE STRUCTURAL MEMBERS

PLAIN CONCRETE

CONCRETE BEAMS

Method of Design.—Stone and plain-concrete beams are designed by exactly the same method as any other kind of beam, except that the weight of the beam itself can hardly ever be neglected. The formula employed is as follows:

$$M = \frac{s\,I}{c} = S\,s,$$

in which M is the bending moment; s, the unit stress produced; I, the moment of inertia; c, the distance from the neutral axis to the most remote fiber; and S, the section modulus.

In a beam of rectangular section, $S = \frac{b\,d^2}{6}$, in which b is the breadth of the beam and d is its depth of the beam.

The modulus of rupture for various kinds of stones and other materials is given in the accompanying table.

With the values in this table, a factor of safety of from 10 to 20 is usually employed when problems dealing with the

MODULI OF RUPTURE OF VARIOUS MINERALS

Material	Modulus of Rupture
Slate	5,000
Glass	3,000
Bluestone flagging	2,250
Marble, white, Italian	2,090
Marble, white, Vermont	1,850
Marble, gray, Vermont	1,260
Granite, Quincy, Massachusetts	1,800
Granite, New York	1,800
Granite, Connecticut	1,500
Sandstone, Massachusetts	1,080
Sandstone, Middletown, Connecticut	1,000
Sandstone, Ohio	500
Freestone, Little Falls, New York	1,730
Freestone, Belleville, New Jersey	1,440
Freestone, Dorchester, Massachusetts	800
Freestone, Hubeginy	650
Freestone, Caen, Normandy	450
Limestone, average value	1,500
Brick, common, or Philadelphia pressed	600
Brick, best hard	800
Rubble masonry in cement	200

safe load instead of the ultimate breaking load are being solved. The weight of the stone itself must almost always be taken into account.

Example.—Design a Quincy granite lintel 6 in. wide on a 5-ft. span to carry a uniform load, which includes its own weight, of 300 lb. per ft., with a factor of safety of 10.

Solution.—In this example, $W = 300 \times 5 = 1{,}500$ lb. and $l = 5 \times 12 = 60$ in. Therefore,

$$M = \frac{W\,l}{8} = \frac{1{,}500 \times 60}{8} = 11{,}250 \text{ in.-lb.}$$

According to the table, the modulus of rupture for Quincy granite is 1,800, and if a factor of safety of 10 is used, the value of s is $1{,}800 \div 10 = 180$ lb. per sq. in. Therefore,

$$S = \frac{b\,d^2}{6} = \frac{6\,d^2}{6} = d^2$$

MODULI OF RUPTURE OF CONCRETE

Material	Mixture	Consistency	Modulus of Rupture at Various Ages					
			1 da.	1 wk.	4 wk.	13 wk.	26 wk.	1 yr.
Neat cement		Water, 21% weight of dry materials	761	1,512	1,674	1,953	2,023	
Mortar	1–3	Water, 11.5% weight of dry materials		588	888	1,026	1,008	1,080
Mortar	1–4	Water, 11% weight of dry materials		390	636	852	864	870
Cinder concrete	1–2–5	Wet			175	240	246	
Cinder concrete	1–2–5	Medium			198	231	277	
Cinder concrete	1–2–5	Damp			198	225	250	
Granite concrete	1–2–4	Wet			375	501	539	
Granite concrete	1–2–4	Medium			475	536	566	
Granite concrete	1–2–4	Damp			499	591	618	
Gravel concrete	1–2–4	Wet			391	380	435	
Gravel concrete	1–2–4	Medium			451	477	520	
Gravel concrete	1–2–4	Damp			426	495	496	
Limestone concrete	1–2–4	Wet			422	487	507	
Limestone concrete	1–2–4	Medium			458	541	566	
Limestone concrete	1–2–4	Damp			537	521	589	

Substituting these quantities in the formula $M = S\,s$, then $11{,}250 = d^2 \times 180$. Thus, $d^2 = 62.5$ and $d = 7.906$, say 8, in.

The modulus of rupture of concrete is usually less than that of stone.

The table on page 284 gives values found by the United States Geological Survey. Three degrees of wetness are recognized in mixing the concrete, namely, wet, medium, and damp. *Wet concrete* is such that sufficient water is added to make it semiliquid; *damp concrete* is decidedly granular, with little tendency to lump; while *medium concrete* is of a consistency between the other two mixtures. The values given in the table are mostly for 1–2–4 mixtures. A 1–3–6 mixture gives values at least 15% lower than the 1–2–4 mixture.

The factor of safety employed is sometimes 4, but usually 6 or higher, as the strength of concrete is uncertain.

Example.—Design a concrete beam 12 in. wide on a 12-ft. span to carry 800 lb. at its center. The safe working stress is to be 100 lb. per sq. in.

Solution.—The moment is equal to $\frac{Wl}{4} = \frac{800 \times 12 \times 12}{4}$ $= 28{,}800$ in.-lb. Assume the beam itself weighs 260 lb. per ft. Then the moment due to the dead load is $\frac{260 \times 12 \times 12 \times 12}{8} = 56{,}160$ in.-lb. The total moment is $56{,}160 + 28{,}800 = 84{,}960$ in.-lb. Substituting the correct values in the formula $M = Ss$, then $84{,}960 = \frac{12 \times d^2}{6} \times 100$. Therefore, $d^2 = 424.8$ and $d = 20.6$, say, 21 in.

CONCRETE COLUMNS

Plain-concrete and stone columns may be divided into two classes, namely, those which are centrally loaded, and those which are eccentrically loaded. The height of the column should never be more than twelve times the least dimension of the cross-section and even less for stone and brick.

Column Centrally Loaded.—For a *centrally loaded column* the allowable compressive stress per square inch is multiplied by the area of the cross-section of the column to find the

allowable load. Thus, if it is decided to allow an intensity of stress of 300 lb. per sq. in., and the column is of square section 10 in. on a side, the allowable load will be $10 \times 10 \times 300 = 30{,}000$ lb. The breaking load on columns between two and twelve times as high as the least dimension of their cross-section seems to be independent of their height. A column between these two limits, however, cannot withstand as high an intensity of stress as a cube, for it is more

ULTIMATE UNIT CRUSHING STRENGTH OF STONE CONCRETE WITH PORTLAND-CEMENT MORTAR

Proportion of Ingredients			Compression Pounds per Square Inch			
Cement	Sand	Stone	7 da.	1 mo.	3 mo.	6 mo.
1	2.0	4	1,600	2,150	2,400	2,500
1	2.5	5	1,430	1,950	2,250	2,350
1	3.0	6	1,250	1,800	2,100	2,200
1	3.5	7	1,100	1,660	1,960	2,080
1	4.0	8	980	1,520	1,820	1,950
1	4.5	9	850	1,400	1,690	1,840
1	5.0	10	750	1,260	1,550	1,720
1	5.5	11	650	1,120	1,420	1,600
1	6.0	12	600	1,000	1,300	1,500

NOTE.—For gravel concrete, use 75% of the figures given in the table.

likely to break by shearing. For this reason, when employing values taken from the tables on pages 286 and 287, for column calculations, a larger factor of safety should be used than with other work. This factor is usually taken as at least 6, and 10 or higher for masonry.

EXAMPLE.—What is the allowable working load on a concrete column that is 10 ft. high and 12 in. in diameter and made of 1–2–4 stone concrete 6 mo. old with a factor of safety of 6.

SOLUTION.—The cross-sectional area of the column is $.7854 \times 12^2 = 113.1$ sq. in. From the table the ultimate

ULTIMATE UNIT CRUSHING STRENGTH OF VARIOUS STONES AND STONE MASONRY PIERS

Material	Compressive Strength, Pounds per Square Inch	Material	Compressive Strength, Pounds per Square Inch
Granite, Colo.	15,000	Limestone, Marquette, Mich.	8,000
Granite, Conn.	14,000	Limestone, Conshohocken, Pa.	15,000
Granite, Mass.	16,000	Marble, Montgomery Co., Pa.	11,000
Granite, Me.	15,000	Marble, Lee (dolomite), Mass.	22,800
Granite, Minn.	25,000	Marble, Pleasantville (dolomite), N. Y.	22,000
Granite, N. Y.	16,000	Marble, Italian	12,000
Granite, N. H.	12,000	Marble, Vt.	10,000
Bluestone	15,000	Slate	10,000
Sandstone, Middletown, Conn.	7,000	Piers, ashlar, bluestone	2,100
Sandstone, Longmeadow, Mass.	10,000	Granite, ashlar piers	2,100
Sandstone, Hudson River, N. Y.	12,000	Piers, ashlar, limestone	1,500
Sandstone, Little Falls (brown), N. Y.	10,000	Piers, ashlar, common sandstone	1,050
Sandstone, Ohio	8,000	Piers rubble, cement mortar	900
Sandstone, Hummelstown (brown), Pa.	12,000	Piers, rubble, lime mortar	480
Limestone, Kingston, N. Y.	12,000		
Limestone, Garrison Station, N. Y.	18,000		
Limestone, Bedford (oolitic), Ind.	8,000		

crushing strength of 1–2–4 concrete 6 mo. old is 2,500 lb. per sq. in. Using a factor of safety of 6, the safe intensity of stress is $\frac{2,500}{6}$. Then the safe total load the column can carry is $113.1 \times \frac{2,500}{6} = 47,125$ lb

ULTIMATE CRUSHING STRENGTH OF BRICK MASONRY PIERS

(*Average Age of Brickwork, 6 Months*)

Material	Composition of Mortar	Compressive Strength, Pounds per Square Inch
Wire-cut brick..........	1 cement, 5 sand	3,000
Dry-pressed brick.......	1 cement, 5 sand	3,400
Dry-pressed brick.......	1 cement, 1 lime, 3 sand	2,300
Repressed brick........	1 cement, 5 sand	1,700
Light-hard, sand-struck brick................	1 cement, 5 sand	1,900
Light-hard, sand-struck brick................	1 cement, 7 sand	853
Hard, sand-struck brick.	1 cement, 1 sand	2,100
Hard, sand-struck brick.	1 cement, 1 lime, 3 sand	1,500
Hard, sand-struck brick.	1 cement, 5 sand	1,200
Sand-lime brick.........	1 cement, 3 sand	1,100
Sand-lime brick.........	1 lime, 3 sand	450
Sand-lime brick.........	Neat cement	1,400
Terra-cotta work.......	1 cement, 3 sand	2,000

Eccentrically Loaded Column.—The stress on an *eccentrically loaded column* is computed by the following formulas:

For circular columns,

$$s=\frac{P}{A}+\frac{8eP_e}{Ad}$$

For rectangular columns,

$$s=\frac{P}{A}+\frac{6eP_e}{Ad}$$

In these formulas, s is the stress, in pounds per square inch, developed in the column; P, the total load on the column, in pounds; A, the area of column section, in square inches; e, the eccentricity of eccentric part of load, in inches; P_e, the eccentric part of load in pounds; d, the diameter of column, or dimensions measured in the plane of the eccentricity, in inches.

If the total load is eccentric, the formulas just given reduce to the following:

For circular columns

$$s=\frac{P_e}{A}\left(1+\frac{8e}{d}\right)$$

For rectangular columns,

$$s=\frac{P_e}{A}\left(1+\frac{6e}{d}\right)$$

According to these formulas, it is necessary first, in designing a column that will stand a given load, to select by inspection the section of column that seems to be about correct, and then to solve the equation for s. This value of s must be less than the allowable working stress it is proposed to use. If it is larger, a larger area of column must be selected, and the problem worked out again; if it is very much smaller, possibly too large a section for economy has been selected, and in this case a smaller section should be assumed and the equation again solved for s. It should be borne in mind that the height of the column must not be greater than twelve times the least dimension of the cross-section. The eccentricity should not be so great as to cause tension in the column.

Example.—Design a cylindrical column of 1–2–4 stone concrete, 18 ft. high, to carry with a factor of 6, a central load of 100,000 lb. and an eccentric load of 100,000 lb., the eccentricity being 4 in.

Solution.—Since the column is 18 ft. high, it should be at least 18 in. in diameter. The ultimate crushing strength may be taken as 2,500 lb. per sq. in. in 6 mo. The safe working stress would therefore be $2{,}500 \div 6 = 417$ lb. per sq. in.

A column 28 in. in diameter will be tried first. The area of the cross-section is $.7854 \times 28^2$, or 615.75 sq. in. To apply the first formula, $A=615.75$, $P=200{,}000$, $P_e=100\,000$, $e=4$, and $d=28$. Substituting in the formula,

$$s=\frac{200{,}000}{615.75}+\frac{8\times 4\times 100{,}000}{615.75\times 28}=325+186=511 \text{ lb. per sq. in.}$$

This stress is larger than the allowable stress, which shows that the column section selected is too small. If a section 31 in. in diameter is assumed, then,

$$A=.7854\times 31^2=754.77 \text{ sq. in.}$$

Substituting in the formula,

$$s=\frac{200,000}{754.77}+\frac{8\times 4\times 100,000}{754.77\times 31}=402 \text{ lb. per sq. in.}$$

Since this is less than 417 lb., a column of this diameter is safe.

In a fire, a column is apt to be injured by the heat to a distance below the surface of $1\frac{1}{2}$ in. Therefore, in designing columns by the preceding methods, $1\frac{1}{2}$ in. should be added all around the column proper.

REINFORCED CONCRETE

BEAMS

The design of reinforced concrete is not an exact science. The majority of the recommendations and formulas herein given are taken from the excellent report of the "Joint Committee." The *Joint Committee* is a committee of members of the American Society of Civil Engineers, the American Society for Testing Materials, the American Engineering and Maintenance of Way Association, and the Association of American Portland Cement Manufacturers formed "for the purpose of investigating current practice and providing definite information concerning the properties of concrete and reinforced concrete."

Only Portland cement is suitable for reinforced concrete.

The aggregate is divided into two classes, namely, fine and coarse. *Fine aggregate* consists of sand, crushed stone, or gravel screenings, and which, when dry, passes through a screen having holes $\frac{1}{4}$ in. in diameter. *Coarse aggregate* consists of crushed stone or gravel which is retained on a screen having holes $\frac{1}{4}$ in. in diameter, but which passes through a screen having holes 1 in. in diameter or smaller. In both fine and coarse aggregates, a gradation of size of the particles is generally desirable.

Cinder concrete is not suitable for reinforced-concrete structures.

For reinforced-concrete work, a mixture based on the proportion of 1–6 is generally used; that is, 1 part of cement to a total of 6 parts of fine and coarse aggregates, measured separately. The fine and coarse aggregates are often in the proportion of 1–2, which makes a concrete that is commonly called a *1–2–4 mixture*. For columns, richer mixtures are often required, and leaner mixtures can often be used in mass work.

Moving live loads and suddenly applied loads require special consideration. These loads can often be taken care of by increasing the amount of live load used in the calculation. The weight of the beam or the floor slab itself should always be considered when estimating the dead load.

The span length of beams should be taken as the length from center to center of supports aad not of the clear span, but it need not be considered longer than the clear span plus the depth of the beam. Brackets are not considered as reducing the clear span.

In the formulas about to be given, rather than employ ultimate stresses and divide the result by the factor of safety, working, or safe, stresses should be used.

Rectangular Beams.—The following notation will be used in the design of beams and columns:

F_s = tensile stress in steel, in pounds per square inch;

F_c = compressive stress in concrete, in pounds per square inch;

E_s = modulus of elasticity of steel;

E_c = modulus of elasticity of concrete;

$n = E_s \div E_c$;

M = moment of resistance of beam, in inch-pounds;

A = area of steel, in square inches;

b = breadth of beam, in inches;

d = depth of beam, in inches, from top to center of steel reinforcement;

k = coefficient;

j = coefficient;

p = ratio of area of steel to $bd = \frac{A}{bd}$.

First assume values for F_s, F_c, and n. These values are usually controlled by building ordinances. The following values are recommended as safe working values by the Joint Committee and are used throughout the text to serve as examples in working out problems used as illustrations: $F_s=16{,}000$; $F_c=650$; $n=15$ for concrete capable of developing an average compressive stress of 2,000 pounds in 28 days when tested in cylinders of specified shape.

After values of F_s, F_c, and n are decided on, substitute them in the following formula and solve for p. This formula gives the value of p that makes F_s and F_c reach their full values under the same load.

$$p=\tfrac{1}{2}\times\frac{1}{\dfrac{F_s}{F_c}\left(\dfrac{F_s}{nF_c}+1\right)}$$

Substituting the values mentioned above

$$p=\tfrac{1}{2}\times\frac{1}{\dfrac{16{,}000}{650}\left(\dfrac{16{,}000}{15\times 650}+1\right)}=.00769$$

The value of k is now found by the formula

$$k=\sqrt{2pn+(pn)^2}-pn$$

Substituting the values for p and n gives

$$k=\sqrt{2\times .00769\times 15+(.00769\times 15)^2}-.00769\times 15=.379$$

From this value, j is found by the following formula:

$$j=1-\tfrac{1}{3}k$$

Substituting the value of k just found,

$$j=1-\tfrac{1}{3}\times .379=.874,\text{ or approximately }\tfrac{7}{8}.$$

For any value of F_s, F_c, and n employed, the preceding formulas must be solved to obtain p, k, and j before the problem proper can be attacked. For example, the values of k and j may be taken as $\frac{3}{8}$ and $\frac{7}{8}$, respectively, because these values are close to the values found from the values of F_s, F_c, and n, assumed.

The resisting moment M may then be found by transposing in either of the following formulas:

$$F_s=\frac{M}{Ajd}=\frac{M}{pjbd^2},\text{ or }F_c=\frac{2M}{jkbd^2}$$

The first of these will be found to be the more convenient to use in most cases, and it may be transposed to the following:

$$M = F_s A j d.$$

As an example, design a girder on a 20-ft. span to carry a load of 700 lb. per ft. This load includes the weight of the girder. The total load is $20 \times 700 = 14{,}000$ lb. The bending moment is $\frac{Wl}{8} = \frac{14{,}000 \times 20}{8} = 35{,}000$ ft.-lb., or $35{,}000 \times 12 = 420{,}000$ in.-lb. Assume the effective depth of the girder—that is, the distance from the top to the center of the steel reinforcement, or d—to be 18 in. Substituting the correct values in the equation $M = F_s A j d$, $420{,}000 = 16{,}000 \times A \times \frac{7}{8} \times 18$, and $A = 1.667$ sq. in.

Since $A = pbd$, and since p is limited to .00769, $1.667 = .00769 \times b \times 18$ and $b = 12.04$, say $12\frac{1}{2}$, in.

All that now remains is to decide how much concrete to put below the steel. This does not affect directly the strength of the beam. Its principal uses are to hold the steel in place and protect it from fire and rust. The Joint Committee recommends a thickness of 2 in. for girders, $1\frac{1}{2}$ in. for beams, and 1 in. for slabs. Therefore, the girder just designed would be $12\frac{1}{2}$ in. broad and 20 in. in total depth, and it would have 1.667 sq. in. of steel 18 in. from the top.

EXAMPLE.—Design a floor slab on a 10-ft. span to carry a load of 250 lb. per sq. ft. This load includes its own weight.

SOLUTION.—Consider a section of the slab 12 in. wide. The total load is $250 \times 10 = 2{,}500$ lb. The maximum moment is $\frac{Wl}{8} = \frac{2{,}500 \times 10}{8} = 3{,}125$ ft.-lb., or 37,500 in.-lb. $A = .00769\ bd$, but $b = 12$ in., as a strip that wide is considered. Therefore, $A = .00769 \times 12\ d = .09\ d$. Substituting the values for M, A, F_s and j in the equation for the moment, $37{,}500 = 16{,}000 \times .09\ d \times \frac{7}{8} \times d$; $d^2 = 29.76$, and $d = 5.5$. Therefore, $A = .09 \times 5.5 = .495$, say $\frac{1}{2}$, sq. in. The slab must therefore be $6\frac{1}{2}$ in. thick and have $\frac{1}{2}$ sq. in. of steel every foot, which must be placed $5\frac{1}{2}$ in. from the top. Of course, all the steel must not be put in one rod, but in several rods spaced at equal distances so that it will average $\frac{1}{2}$ sq. in. per ft.

These solutions give the most economical design; that is, when the allowable unit stress in the steel and the allowable unit stress in the concrete are realized under the proposed load. This condition is determined by the value used for p. However, other values of p are sometimes used, and in such cases the procedure is as follows:

Assume a value of n. Find the value of k by the formula $k=\sqrt{2\,pn+(pn)^2}-pn$. Then find the value of j by the formula $j=1-\frac{1}{3}k$. Next assume values for F_s and F_c. Find M from the conditions of the problem. Assume values for either b or d, and find the other by means of the formula $F_s=M\div pjbd^2$. Also find the same dimension by the formula $F_c=2\,M\div jkbd^2$, and use the largest value found by either of these equations.

Sometimes the problem is thus: b, d, and M are given. Assume F_s, then find A by the formula $M=F_sAjd$, using $\frac{7}{8}$ for j. Solve $p=A\div bd$ for p, which should, by changing b, if required, be kept less than the value of p that gives the most economical design mentioned above. Then to check, find k and j as on page 292 from the value of p obtained and find F_s and F_c accurately by the formulas at the foot of page 292. These values must not be excessive.

To investigate a beam already built, proceed as follows. Measure the value of b, d, and A. Calculate the value of p as well as the moment of the loads on the beam, or that are to be put on the beam, including the weight of the beam itself. Assume a value for n, and find the value of k by the formula $k=\sqrt{2\,pn+(pn)^2}-pn$. Find the value of j by the formula $j=1-\frac{1}{3}k$. Find the stress in the steel by the formula $F_s=M\div Ajd$, and then the stress in the concrete by the formula $F_c=2\,M\div jkbd^2$. Neither of these values should exceed the safe allowable limit.

Continuous Beams.—If W is the total uniform load on a beam and l is its length, then, for a simple beam, the moment is $Wl\div 8$. In building construction, many beams and floor slabs are continuous, and in this case the external moment at the center of the span is decreased and there is produced a negative moment over each support. Sufficient steel

should be placed over each support at the top of the beam to withstand this moment. This steel should extend far enough on each side of the support to reach the point where the bending moment changes sign. In many cases one-quarter of the span each way will be sufficient. This steel is often made up partly of rods bent up from the bottom of the beam and partly from extra rods inserted at the top.

For both beams and slabs for interior spans of continuous beams the Joint Committee recommends that the bending moment be taken as $\frac{Wl}{12}$, both at the center of the span and over the support. In beams for end spans and the adjoining support, the bending moment should be taken at $\frac{Wl}{10}$. After the bending moment is found, the beam is designed in the manner already stated.

Some engineers claim this reduction in bending moment is unwarranted and use $\frac{Wl}{8}$ for all cases. No matter which method is followed, the beam should be reinforced over the support.

Caution must be exercised in designing continuous **T** beams. At the support, the compression in the concrete is apt to be excessive, as it comes on the stem of the **T**. The Joint Committee allows a higher stress in the concrete here and recommends a value approaching 750 lb. It is often necessary to leave some steel at the bottom of **T** beams near the supports to assist in withstanding this compression.

Beams Reinforced at Top and Bottom.—For beams reinforced at the top and bottom, the preceding notation, with the following additional characters, is employed:

A' = area of compressive steel at top of beam, in square inches;

p' = ratio of area of compressive steel to $bd = \frac{A'}{bd}$;

F_s' = compressive stress in steel, in pounds per square inch;

d' = distance from top of beam to top steel, in inches.

To design such a beam, assume values for p, p', d', d, and n. Then find k by the formula

$$k=\sqrt{2\,n\left(p+p'\frac{d'}{d}\right)+n^2(p+p')^2}-n(p+p')$$

After k is found, assume a value for b, and find F_c by the formula

$$F_c=\frac{6\,M}{bd^2\left[3\,k-k^2+\frac{6\,p'n}{k}\left(k-\frac{d'}{d}\right)\left(1-\frac{d'}{d}\right)\right]}$$

Also, find F_s by the formula

$$F_s=nF_c\frac{1-k}{k}$$

and F_s' by the formula

$$F_s'=nF_c\frac{k-\frac{d'}{d}}{k}$$

The values of F_c, F_s, and F_s' should all be within safe limits. F_s' will usually be low, which shows that the steel is not used economically. On account of this defect, double reinforced beams are seldom used except in places where

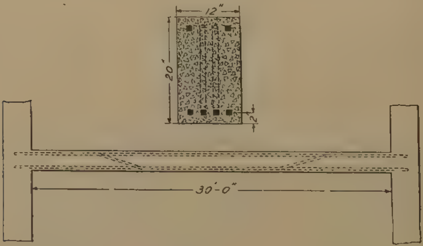


FIG. 1

the size of the beam is limited by the conditions of the problem.

As an example, design a girder, as shown in Fig. 1, to carry besides its own weight 450 lb. per ft. The section of

the girder is limited to that shown. The steel in the bottom should be kept up 2 in. to protect it from fire, so that the effective depth, or d, is only 18 in. In the first place, the beam itself weighs $150 \times \frac{12}{12} \times \frac{20}{12} \times 1 = 250$ lb. per ft. The total load to be carried is therefore $450 + 250 = 700$ lb. per ft., or $700 \times 30 = 21{,}000$ lb. in all. The span is here taken, for convenience, as the clear distance between supports and not the distance from center to center of support, which is more correct. The maximum bending moment is $\frac{Wl}{8} = \frac{21{,}000 \times 30}{8} = 78{,}750$ ft.-lb., or $78{,}750 \times 12 = 945{,}000$ in.-lb.

First find A by the formula given for rectangular beams, $F_s = \frac{M}{Ajd}$. Assume that $j = \frac{7}{8}$ and that $F_s = 16{,}000$. Then, $16{,}000 = \frac{945{,}000}{A \times \frac{7}{8} + 18}$, or $A = 3.75$ sq. in.

If the beam were reinforced at the bottom only, the allowable steel would be about $.0075 \times bd = .0075 \times 12 \times 18 = 1.62$ sq. in. The excess of steel is then $3.75 - 1.62 = 2.13$ sq. in. It is for this reason that reinforcement must be put at the top of the beam; if it were not put there, the stress in the concrete at that place would be too great. The amount of steel required at the top is usually about two and one-quarter times the *excess* at the bottom, or, in the problem at hand, $2\frac{1}{4} \times 2.13 = 4.79$, say $4\frac{3}{4}$, sq. in. Assume that this steel is placed 2 in. from the top.

This completes the approximate design of the beam; that is, its size and the amount and location of the steel at the top and bottom have been assumed. It now remains to see whether or not the values assumed will be safe. First k must be found by the formula given, which is

$$k = \sqrt{2n\left(p + p'\frac{d'}{d}\right) + n^2(p + p')^2} - n(p + p')$$

The values of p, p', d, and d' are: $p = \frac{3.75}{12 \times 18} = .01736$,

$p'=\frac{4.75}{12\times 18}=.02199$, $d=18$, and $d'=2$. Assuming that $n=15$ and substituting these values in the formula,

$$k=\sqrt{2\times 15\left(.01736+.02199\times\frac{2}{18}\right)+15^2(.01736+.02199)^2}-15(.01736+.02199)=.380.$$

Now,
$$F_c=\frac{6\,M}{bd^2\left[3\,k-k^2+\frac{6\,p'n}{k}\left(k-\frac{d'}{d}\right)\left(1-\frac{d'}{d}\right)\right]}$$

Substituting the values in this equation,

$$F_c=\frac{6\times 945{,}000}{12\times 18^2\left[3\times .380-.380^2+\frac{6\times .02199\times 15}{.380}(.380-\frac{2}{18})(1-\frac{2}{18})\right]}$$
$$=651 \text{ lb. per sq. in.},$$

which, according to the stresses used in these examples, is about safe.

The stress in the steel in tension is found by the formula,

$$F_s=nF_c\frac{1-k}{k}$$

Substituting the correct values,

$$F_s=15\times 651\times\frac{1-.380}{.380}=15{,}930 \text{ lb. per sq. in.},$$

which is also safe.

The stress in the steel in compression is found by the formula

$$F_s'=nF_c\frac{k-\frac{d'}{d}}{k}$$

Substituting the correct values,

$$F_s'=15\times 651\times\frac{.380-\frac{2}{18}}{.380}=6{,}913 \text{ lb. per sq. in.}$$

This value is low, but it cannot be helped. It is this fact that makes the double reinforced beam uneconomical.

T Beams.—The Joint Committee makes the following suggestions in regard to **T** beams:

"In beam and slab construction, an effective bond should be provided at the junction of the beam and slab. When

the principal slab reinforcement is parallel to the beam, transverse reinforcement should be used extending over the beam and well into the slab.

"Where adequate bond between slab and web of beam is provided, the slab may be considered as an integral part of the beam, but its effective width shall be determined by the following rules:

"1. It shall not exceed one-fourth of the span length of the beam.

"2. Its overhang width on either side of the web shall not exceed four times the thickness of the slab.

"3. In the design of T beams acting as continuous beams, due consideration should be given to the compressive stresses at the support."

The notation used for T-beam formulas is the same as before, with the following exceptions:

b = width of flange, in inches;

b' = width of stem or beam proper, in inches;

t = thickness of slab, in inches.

First assume values for F_s, F_c and n. Also, assume values for d, b', and t. The value to be taken for b is determined partly by the rules just given. Assume an approximate value of A from the formula $M = A(d - \frac{1}{2}t)F_s$. Solve the following formula for kd:

$$kd = \frac{2\,ndA + bt^2}{2\,nA + 2\,bt}$$

If kd is less than t, design the same as a rectangular beam; that is, again find k by the formula

$$k = \sqrt{2\,pn + (pn)^2} - pn \text{ where } p = \frac{A}{bd} \text{ not } \frac{A}{b'd}$$

Then find j by the formula

$$j = 1 - \tfrac{1}{3}\,k$$

and M by the formula

$$M = F_s A j d,$$

which must not exceed $\dfrac{F_c jkbd^2}{2}$.

If kd is greater than t, proceed as follows. Find jd by the formula

$$jd = d - \frac{3\,kd - 2\,t}{2\,kd - t} \times \frac{t}{3}$$

Find F_s by the formula
$$F_s=\frac{M}{Ajd}$$
and find F_c by the formula
$$F_c=\frac{Mkd}{bt(kd-\frac{1}{2}t)jd}$$

These values of F_s and F_c must be less than the allowable values.

These formulas neglect the compression in the stem. For approximate results, the formulas for rectangular beams may be used.

As an example, design a beam to carry 5,184 lb. per ft. on a span of 10 ft. The load given includes the weight of the beam, and the floor slab is 5 in. thick. The bending moment is $\frac{5{,}184\times 10\times 10}{8}=64{,}800$ ft.-lb., or $64{,}800\times 12=777{,}600$ in.-lb. Assume that $F_s=16{,}000$ lb., $F_c=650$ lb., and $n=15$, and that $d=16$ in. and $b^1=16$ in; b is governed by the preceding rules. It must not exceed one-fourth the span, or 30 in., and its overhang must be less than four times the thickness of the slab, which would limit its width to $16+2\times 4\times 5=56$ in. Therefore b is taken at 30 in. Solving for A in the formula, $M=A\ (d-\frac{1}{2}t)F_s$, $777{,}600=A\ (16-2\frac{1}{2})$ 16,000.

Therefore, $A=3.6$ sq. in.; $kd=\frac{2\times 15\times 16\times 3.6+30\times 25}{2\times 15\times 3.6+2\times 30\times 5}$ $=6.073$, and $jd=16-\frac{3\times 6.073-2\times 5}{2\times 6.073-5}\times\frac{5}{3}=14.1$. Then,
$$F_s=\frac{777{,}600}{3.6\times 14.1}=15{,}319 \text{ lb. per sq. in.,}$$
which is safe, and
$$F_c=\frac{777{,}600\times 6.073}{30\times 5(6.073-2\frac{1}{2})14.1}=625 \text{ lb. per sq. in.,}$$
which is also safe.

Shear and Bond.—One method of failure of beams that is common is shown in Fig. 2. Such cracks are caused by shear or diagonal tension. They are prevented by bending up some of the main reinforcing rods to make truss rods and by the use of **U**-shaped stirrups. The size and the

method of placing these stirrups are still matters of discussion that depend much on the experience of the designer. One important thing to be remembered is that they are comparatively short and that there is danger of their pulling out of the concrete. For these reasons they are frequently securely fastened to the slab reinforcement or bent over at the ends. Sometimes deformed bars are used, as they give a more secure grip on the concrete.

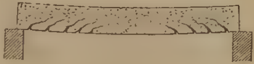


FIG. 2

When inclined stirrups are used, they should be fastened to the main reinforcing bars in such a way that they will not slip.

An approximate formula for shear in a rectangular beam reinforced at the bottom is $v=\frac{V}{bjd}$, where v = unit shearing stress in pounds per square inch at any point, V = total vertical shear at that point in pounds, and the other terms have the same meaning as before. The Joint Committee recommends for a grade of concrete mentioned above a unit safe shear of 40 pounds. If V is greater than this, stirrups must be used. The same formula may be used for **T** beams with b' inserted instead of b and jd referring to **T** beams. Even with web reinforcement, v should never exceed 120 pounds.

No absolute rules can be given for the size and placement of stirrups. In many instances $\frac{3}{4}$ in. square or $\frac{3}{16}'' \times 1''$ flats are used, but not in all cases. The spacing of stirrups is greater at the center of the span than at the supports. The Joint Committee recommends a maximum limit of spacing of three-fourths the depth of the beam. The spacing at the supports is often one-fifth or one-sixth of the depth of the beam. Great care is required in the design of stirrups of **T** beams to insure that the slab and the beam proper are securely tied together.

An approximate formula for ordinary rectangular beams with vertical stirrups is $P=\frac{Vs}{jd}$, where P = stress in one stirrup, s = spacing of stirrups, V = proportion of shear supposed to be carried by reinforcement, usually $\frac{2}{3}$, and j and d have the values already given.

The Joint Committee recommends that the bonding stress between plain reinforcing bars and concrete be assumed to be 80 lb. and in the case of drawn wire, to be 40 lb. for the same grade of concrete as specified in discussing beams. The difference in stress in the tensile steel at two sections must be taken up by the bond to the concrete between these two sections and should be investigated.

COLUMNS

Concentrically Loaded Columns.—There are two general methods of reinforcing concrete columns with steel. One

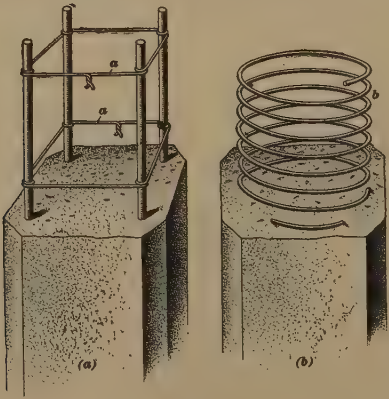


FIG. 3

method is known as *straight reinforcement* and the other as *hooped reinforcement.* These two styles of reinforcement are illustrated in Fig. 3. In (*a*) is shown straight reinforcement. This consists of steel rods that stand vertically in the concrete. Sometimes, the rods are placed directly in the middle of the column, but as a rule they are arranged around the outside of the column about 2 in. from the surface. These steel rods are tied together by wire ties, as shown at *a*. The

distance between two ties should not exceed the width, or diameter of the column. If the ties are spaced too far apart, the column is apt to fail by the reinforcement bulging.

In Fig. 3 (*b*) is shown a column reinforced with hooped reinforcement. This type of reinforcement consists of either a steel spiral or a separate steel hoop that is about 2 in. from the surface of the column, as shown at *b*.

Some columns have both styles of reinforcement just mentioned.

In the design, as given here, A_c is the *effective* area of the concrete. The effect of fire is to injure the concrete for a depth of about 1½ in. from the surface. Therefore, in investigating a column already built, first deduct 1½ in. all the way around the column from the total area of the concrete so as to get the effective area of the concrete. For the same reason, after a column is designed, add a coat 1½ in. on all sides for fire protection. The reinforcement to protect it from fire should be embedded at least 2 in. in the concrete.

In a hooped column, the effective area should not only be limited to 1½ in. from the surface, but should further be limited to the concrete within the hooping. Outside of the hooping, of course, at least 2 in. of concrete must be placed.

In height, columns should be less than fifteen times their least dimension.

Straight Reinforcement.—Let A_c represent the effective area of cross-section of concrete, in square inches; A_s, the area of cross-section of steel in square inches; E_c, the modulus of elasticity of concrete; E_s, the modulus of elasticity of steel; F_c, the safe compressive stress per unit area of concrete; F_s, the safe compressive stress per unit area of steel; n, the ratio $\frac{E_s}{E_c}$, and W, the total load on column.

The design formulas are as follows:

$$W = F_c(A_c + nA_s)$$

$$F_s = nF_c$$

Any values of F_c and n as may be required may be used. The Joint Committee uses 450 lb. for F_c and 15 for n for grade of concrete as specified in discussing beams. These values are used here for the sake of example.

If a column is 12 in. square and has 3 sq. in. of steel, determine the safe load that it will carry. Deducting $1\frac{1}{2}$ in. from each surface, the column will be 81 sq. in. But 3 sq. in. of this area is steel. Therefore, the effective area of the column is $81-3=78$ sq. in. By the formula, $W=450(78+15\times 3)=55{,}350$ lb. The stress in the steel is therefore $450\times 15=6{,}750$ lb. per sq. in., which is safe.

The use of steel in a column has two advantages. In the first place, the formulas do not take into account the length of the column; the longer a column is, the more it is apt to bend or bulge, and the steel helps materially to resist this tendency. Then again, the introduction of steel permits the column to be made much smaller in size, and this is often of great advantage.

Having shown how to investigate a column, the method of designing one to carry a certain load will now be considered. Thus, assume that it is desired to design a column to carry a load of 40 T. The percentage of reinforcement does not have to be a definite amount as is the case with beams, but may be any amount. Suppose that it is desired not to have the column too large; therefore, assume that the reinforcement will occupy 4% as much area as the concrete. Let A_c = area of concrete. Then, $A_s=4\%$ of $A_c=\frac{1}{25}A_c$. Substituting the correct values in the formula it will be found that $80{,}000=450(A_c+NA_s)=450(A_c+15\times\frac{1}{25}A_c)=720\ A_c$. $A_c=111$ sq. in. and $A_s=\frac{1}{25}A_c=4.4+$ sq. in. Therefore, the total area $=A_c+A_s=116$ sq. in.

If the column is square, it will be 10.77 in. on a side; say 11 in., to allow for chamfered corners. When $1\frac{1}{2}$ in. of fireproofing is put on, the column will be 14 in. square and will contain a little over 4.4 sq. in. of steel.

Empirical Rules for Straight Reinforcement.—One or two more or less empirical formulas are used in designing concrete columns with straight reinforcement. These formulas mostly originate from the building laws of various cities. The building laws of one large city, for instance, furnish good examples of such formulas. They stipulate that the safe allowable load shall be 500 lb. per sq. in. of column section, counting both concrete and steel the same. If

sufficient steel is inserted, this rule will of course be as safe as the formula already given.

In a large building, the loads on the columns in the lower floors become very great, and as it is usually not desirable to increase the outside size of the column, the percentage of reinforcement is increased. A favorite style of reinforcement employed under such conditions is shown in Fig. 4. It consists of four angles riveted back to back in the center of the column to form a steel core. The area of this steel core can be increased if desired by using packing plates between the angles. The laws of this city then assume that the steel core takes all the vertical load and that the concrete prevents it from bending sidewise. The rule for designing such a column is, therefore, to allow a safe stress of 16,000 lb. per sq. in. on the steel and nothing on the concrete. Thus, suppose 160 T. is to be safely carried. This equals $160 \times 2{,}000 = 320{,}000$ lb. The area required for the steel core, then, will be $320{,}000 \div 16{,}000 = 20$ sq. in. If four angles are to be used, each one will have to have an area of $20 \div 4 = 5$ sq. in. Angles 4 in. $\times$ 4 in. $\times \frac{11}{16}$ in. would be large enough.

FIG. 4

It is recommended that vertical rods be inserted in the concrete around the steel core to help stiffen it. The rule just given is of an empirical nature and is here presented to show the practice sometimes followed.

Hooped Reinforcement.—Hooping of concrete columns increases the ultimate strength of the concrete contained inside the hooping, but has little effect below the elastic limit. Therefore, with hooping, a greater unit stress may be used, although the steel in the hooping itself does not carry the load applied. The amount of hooping should not be less than 1% of the volume of the concrete inclosed. The clear spacing between the bands should not exceed one-fourth the diameter of the inclosed column. Adequate means must be provided to hold the hooping in place while the concrete is being placed. The Joint Committee allows a unit working stress of 540 lb. on hooped concrete columns,

and on hooped columns that contain in addition more than 1% and less than 4% of longitudinal reinforcement they allow a unit working stress of 650 lb. This is for the grade of concrete above specified. Some engineers consider these stresses rather high; nevertheless, in all cases the stress selected should be governed by the laws of the locality and the experience of the designer.

Eccentrically Loaded Columns.—It is difficult to find a column that is entirely concentrically loaded. If, in a building, the live load is transferred from one girder to another, the girders being carried by the same column, their deflection will put a twist in the column itself. Also, the outside columns of buildings are often eccentrically loaded. In the lower floors of buildings the eccentric load due to unequal distribution of load, in panels, may usually be neglected because of the large concentric load on the column, but in the top floors of a building the matter may at least deserve attention. If the load is eccentric, the following method will usually give safe results:

First, assume a section and find the stress in the concrete and steel due to bending as in a beam with double reinforcement. Next, find the stress in the concrete and steel due to direct compression and add the two algebraically. The sums obtained should not exceed the allowable stress for columns. Sometimes it is found advisable to put more steel in the tension side than is used in the other side. As this method is approximate, low unit stresses should be used.

ARCHES

In *reinforced-concrete arches*, the arch ring is nearly always thinner at the crown than near the haunches. The arch in reinforced-concrete work is often either true parabolic or the curve is made up of a series of circular arcs approaching the shape of a parabola. The reinforcement for arches is usually placed in two layers—one near the intrados and the other near the extrados—and these two layers of reinforcement are usually laced together with lighter rods

In constructing bridges of reinforced concrete, it is considered advisable to lay all the concrete at once, but if this is

impossible, the bridge may be constructed in parallel sections running lengthwise of the arch. By following this method of construction, the arch is not materially weakened, for each ring may be considered as a complete unit of the arch.

The manufacturers of Kahn bars have suggested an approximate method of reinforced-concrete arch design. This method is based on an article by F. F. Weld, C. E., published in the Engineering Record. It consists in using an empirical formula obtained from a study of many existing arches and original designs analyzed by more elaborate methods.

The first step in the design is to determine the rise of the arch. This should be at least one-tenth of the span.

The curve that the arch takes, especially where uniform loads are expected, is often a parabola. This parabolic curve is usually followed by the center line of the arch ring, and not by the curve of the intrados. This curve may be drawn by plotting a sufficient number of points determined by the following formula:

$$Y=H\left[1-\left(\frac{2X}{S}\right)^2\right],$$

in which Y is the rise of parabola, in feet, at any point under consideration; H, the rise, in feet, at the center of the arch; X, the horizontal distance, in feet, from vertical center line of arch to point under consideration; and S, the span of the arch, in feet.

As an example, lay out the parabolic curve for the center line of an arch where the span is 88 ft.

First, the rise must be determined; this is at least one-tenth of the span, which is 8.8, or practically 9, ft. Having the rise and the span, proceed to lay out the curve shown in Fig. 10 as follows: First lay off to a convenient scale the line $a\,b$ equal to the span. At the center of $a\,b$, or c, erect a perpendicular, and on it measure $c\,d$ equal to the rise H. Then, the points a, d, and b lie on the required curve.

To obtain other points, proceed as follows: Divide the span into any number of convenient parts, say in this case, eight, because eight is an even number, which makes one division point at c. Then, each division is 11 ft. long. Now, the first division immediately to the right of c is e.

From e erect a perpendicular. Then, returning to the formula, it is desired to find the vertical distance to the

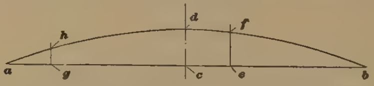


FIG. 10

parabolic curve. This distance is denoted by Y. The following values are known: $H=9$, $X=11$, and $S=88$. Substituting these values in the formula,

$$Y=9\times\left[1-\left(\frac{2\times 11}{88}\right)^2\right]=8.4375 \text{ ft.}$$

Lay off ef equal to 8.4375 ft. Then f is a point on the curve.

Suppose it is desired to locate the curve at the first division point from a, namely, g. Through g draw a perpendicular. Here, $H=9$, $X=33$, and $S=88$. Therefore,

$$Y=9\times\left[1-\left(\frac{2\times 33}{88}\right)^2\right]=3.9375 \text{ ft.}$$

From g, there is plotted upwards to scale 3.9375 ft. to h, which is a point on the curve. Other points may be obtained in the same manner and the parabolic curve drawn through them.

Instead of using the formula to determine the location of points along the parabola, as just shown, it is customary

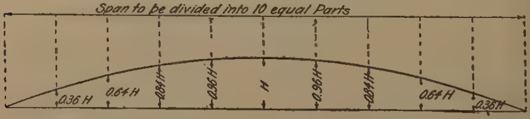


FIG. 11

to divide an arch into ten equal horizontal parts and then use the values given in Fig. 11. These values are derived from the formula on page 307, and enable a parabola to be plotted with ease when the span is divided into ten equal parts. The rise at each one of these parts is given in terms

of the total rise. As will be observed, these values are independent of the span.

As an example, consider an arch having a span of 70 ft. and a rise of 8 ft. In this case, $H=8$ and the height of the curve at $70 \div 10 = 7$ ft. from each end is $.36 \times 8 = 2.88$ ft. At 14 ft. from each end, the height of the curve above the spring line is $.64 \times 8 = 5.12$ ft., and so on, until the necessary number of points on the curve are located. The curve may then be drawn, passing through these points.

Having determined the rise to be given to the arch, the next step is to find the thickness of the arch ring at the crown. This may be found by the formula proposed by F. F. Weld, C. E., which is as follows:

$$D = \sqrt{S} + \frac{S}{10} + \frac{L}{200} + \frac{F}{400},$$

in which D is the crown thickness, in inches; S, the span of the arch, in feet; L, the live load per square foot; and F, the dead load at crown per square foot, exclusive of the weight of the arch itself.

Thus, for example, take an arch whose span is 64 ft., whose live load is 100 lb. per sq. ft., and whose dead load is 200 lb. per sq. ft., and determine its thickness at the crown.

In this case, $S=64$, $L=100$, and $F=200$. Then,

$$D = \sqrt{64} + \frac{64}{10} + \frac{100}{200} + \frac{200}{400} = 15.4 \text{ in.}$$

If there is no live load, the formula may still be used. Thus, find the thickness of an arch at the crown when the span is 40 ft. and the dead load at the crown is 150 lb. per sq. ft. Here, $S=40$, $L=0$, and $F=150$. Therefore,

$$D = \sqrt{40} + \frac{40}{10} + 0 + \frac{150}{400} = 10.6996, \text{ say } 11, \text{ in.}$$

The arch ring is made thinner at the crown than elsewhere. The custom is to increase gradually the thickness of the arch ring from the crown to the abutments. The thickness of the arch ring directly above points on the spring line, one-quarter of the span from the abutments, is made from one and one-fourth to one and one-half times the thickness at the crown. From these points to the abutments the arch-

ring thickness usually increases more rapidly, and while no definite proportions can be laid down, it is usually at least twice as thick at the skewbacks as at the crown.

One of the points yet to be considered is the amount of steel reinforcement to be used. As was said, this reinforcement usually is placed in two layers, one layer near the intrados and the other near the extrados. If the arch carried only a dead load and the conditions were absolutely uniform, only one layer of reinforcement would be necessary; but, in actual practice, as even the changes of temperature cause large variations in stress, two layers of reinforcement are used unless a very careful analysis of stresses has been made to prove that they are not needed. The amount of steel in each layer is kept uniform throughout its length. The cross-sectional area ot steel in each layer is equal to $\frac{4}{10}$ of 1% of the area of the arch-ring section at the crown.

Thus, suppose that the thickness of an arch at the crown is 24 in. For a width of 1 ft., the area of the arch-ring section is $24 \times 12 = 288$ sq. in. The area of the steel in the top or bottom layer per foot of arch width is therefore $288 \times \frac{\frac{4}{10}}{100} = 1.152$ sq. in. The area of a $\frac{7}{8}$-in. round bar is .6013 sq. in. Therefore, one $\frac{7}{8}$-in. round bar placed near the extrados and one $\frac{7}{8}$-in. round bar placed near the intrados, every 6 in. in width along the bridge, will be found sufficient.

The distance from the center of the bars to the surface of the intrados and extrados, that is, the depth to which the bars are to be embedded in the concrete, is another question that must be decided. The nearer the reinforcement is to the surface, the more efficient it will be. On the other hand, the reinforcement must be embedded deep enough, so that it will not tear loose and so that it will be protected from fire and rust. As a general rule, the distance from the center of the steel reinforcement to the surface of the arch ring should be from 2 to 3 in.

The two layers of reinforcement are laced together by light steel rods that are run from one layer to the other. No uniform method is followed in designing these *shear members*, as they are called, and the amount of material used also varies greatly.

As an example of a complete problem, the following case is suggested: Design a reinforced-concrete arch for a 65-ft. span. This arch is to carry a live load of 200 lb. per sq. ft. and a dead load of 300 lb. per sq. ft. at the crown.

First, the rise of the arch must be determined. This may be taken as one-tenth the span, or 6.5 ft. The curve of the arch may now be laid out. If the curve to be followed is a parabola, the method of constructing it is given on page 307. This curve may be used either as the center line or as the intrados of the arch, preferably the former.

The thickness of the arch ring at the crown may now be determined by the formula on page 309. In this case, $S=65$, $L=200$, and $F=300$. Therefore,

$$D=\sqrt{65}+\frac{65}{10}+\frac{200}{200}+\frac{300}{400}=16.3123 \text{ in.}$$

Calling this thickness 18 in., the thickness of the arch ring at the quarter points may be taken as $1\frac{1}{2}\times 18=27$ in., and its thickness at the haunches may be taken as $2\times 18=36$ in.

The amount of steel reinforcement required for each layer must now be determined. For each foot of width of the arch, this will be $18\times 12\times\frac{1}{100}\times\frac{4}{10}=.864$ sq. in. A $\frac{3}{4}$-in. round rod has an area of .4418 sq. in.; therefore, one $\frac{3}{4}$-in. rod every 6 in. near the extrados, and one $1\frac{1}{8}$-in. rod every 6 in. near the intrados will be sufficient. These rods should be embedded 2 or 3 in. in the concrete. The two layers of reinforcement should be either securely tied together or bonded into the concrete with shear or diagonal members.

FOUNDATIONS

BEARING VALUE OF FOUNDATION SOILS

There is some difference of opinion regarding the safe bearing value of foundation soils, due probably to the difficulty of arriving at any experimental results that will have a general application. Conservative engineering practice, however, dictates that the greatest unit pressure on the

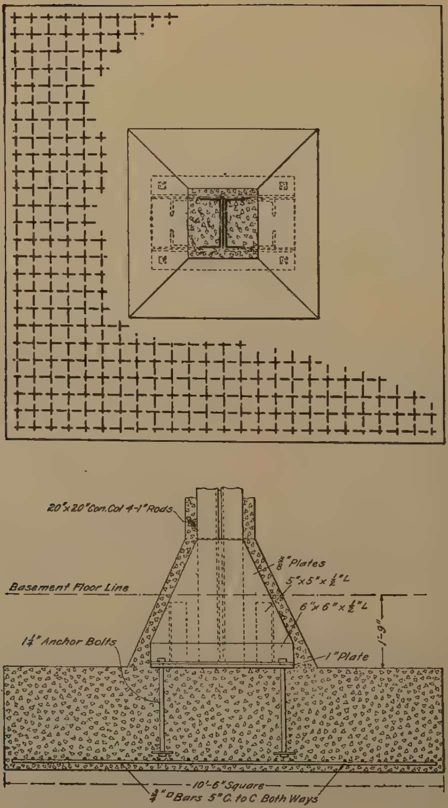

20"x20" Con. Col 4-1" Rods
3/8" Plates
5"x5"x1/2"L
Basement Floor Line
6"x6"x1/2"L
1'-9"
1 1/4" Anchor Bolts
1" Plate
10'-6" Square
3/4" □ Bars 5" C. to C Both Ways

FIG. 1

different foundation soils shall not exceed the values given in the accompanying table

The observance of the revised building laws of the several cities is considered good engineering practice, for they are usually the results of careful investigations and records of long experience. The following, taken from the New York Building Laws, is interesting and gives bearing values that are well within the safe limits:

SAFE BEARING VALUES OF DIFFERENT FOUNDATION SOILS

Material	Tons per Square Foot
Granite rock formation	30
Limestone, compact beds	25
Sandstone, compact beds	20
Shale formation, or soft friable rock	8 to 10
Gravel and sand, compact	6 to 10
Gravel, dry and coarse, packed and confined	6
Gravel and sand, mixed with dry clay	4 to 6
Clay, absolutely dry and in thick beds	4
Clay, moderately dry and in thick beds	3
Clay, soft (similar to Chicago clay)	1 to 1½
Sand, compact, well-cemented, and confined	4
Sand, clean and dry, in natural beds and confined	2
Earth, solid, dry, and in natural beds	4

Where no test of the sustaining power of the soil is made, different soils, excluding mud, at the bottom of the footings shall be deemed to sustain safely the following loads to the superficial foot:

Soft clay, 1 T. per sq. ft.

Ordinary clay and sand together, in layers, wet and springy, 2 T. per sq. ft.

Loam clay, or fine sand, firm and dry, 3 T. per sq. ft.

Very firm, coarse sand, stiff gravel, or hard clay, 4 T. per sq. ft., or as otherwise determined by the Commissioner of Buildings having jurisdiction.

Where a test is made of the sustaining power of the soil, the Commissioner of Buildings shall be notified so that he may be present in person or by representative. The record of the test shall be filed in the Department of Buildings.

SPREAD FOOTINGS

DESIGN AND CONSTRUCTION

The term *spread footings* is applied to either wall or column footings that have a considerable projection beyond the upper tier of the footing, wall, or column base, as the case may be.

The usual type of spread footing for the support of a column is illustrated in Fig. 1. This footing was designed to be used with a structural-steel column core, and is one that would ordinarily be used for a ten- or twelve-story building that is to be erected on unstable soil.

Placing Reinforcement in Column Footings.—The reinforcing rods, or bars, are placed from 2 to 4 in. from the bottom of the footing, and are arranged so as to cross each other at right angles. No attempt is made to interlace the bars or rods. It is good practice, however, to wire them together, for by so doing, any danger of misplacing the bars is avoided. Several tiers of bars, or rods, are used in heavy footings, although spread footings that support light loads are sometimes reinforced with expanded metal or woven-wire fabric.

In designing column footings in reinforced concrete, some steel reinforcement is often placed in the upper part of the footing directly under the base of the column. This reinforcement acts as a mattress to distribute the concentrated load from the column and also as a bond to tie the concrete together.

Two other methods of arranging the reinforcing rods in concrete column footings are shown in Fig. 2. In (*a*), the rods and bars are crossed at right angles; every other bar is made short so as to save metal.

In the method of placing the steel reinforcement shown in Figs. 1 and 2 (*a*), it will be observed that the corners of the footings are subjected to great moment. Therefore, in order to strengthen the corners of the footing, the reinforcing rods are frequently arranged as shown in Fig. 2 (*b*).

The steel reinforcement of column footings is never painted, even if the footings are to be placed in damp situations. Coating the bars will partly destroy the bond that it is necessary to maintain between the concrete and steel; besides, the concrete is sufficient protection for the steel against any serious corrosion. In all instances, however, the steel bars should be cut off enough to allow the ends to be entirely protected.

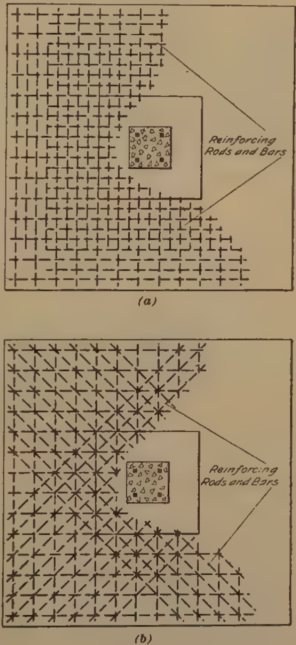


FIG. 2

Spread Footings for Outside Columns.—Instead of a continuous footing, *isolated spread footings* are frequently used in outside, or wall, columns. Such footings are illustrated in Fig. 3. The position that the footings occupy on the building plan is shown, the wall footing being at *a* and the corner footing at *b*.

FORMULAS FOR THE DESIGN OF FOOTINGS

The theoretical design of a spread reinforced-concrete footing consists first in determining the total load on the column to be supported by the footing, and then finding the required area of the bottom reinforced-concrete footing or layer of concrete by dividing the load by the assumed allowable bearing value of the soil. The next thing to find is the area of the base of the column where it bears on the upper tier, or layer, of concrete, because the size of this portion of the footing is fixed by the size of the column base. After the areas of the bottom and top tiers, or layers, of con-

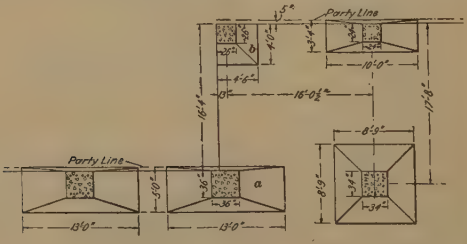


FIG. 3

crete have been ascertained, the projection of the bottom tier beyond the upper is known, and the bending-moment stress on the lower tier of concrete can be calculated. Sufficient steel rods may then be introduced, and the footing made of a depth that will resist this bending moment.

The usual formulas for bending moments and the resistance of reinforced-concrete rectangular sections may be applied in determining the strength of reinforced-concrete spread footings. However, in office practice, it is desirable to use direct formulas for finding the area of steel reinforcement required and for determining the unit compression created in the upper portion of the concrete footing.

The formula for finding the area of steel reinforcement required is expressed as follows:

$$A = \frac{Wx}{27{,}000t},$$

in which A is the sectional area of steel reinforcement, in square inches, required for each linear foot of spread footings along one side; W, the total load, in pounds, on a portion of projection of footing 1 ft. in width; x, the projection of footing, in inches; and t, the distance, in inches, from center of action of steel reinforcement to top surface of concrete footing.

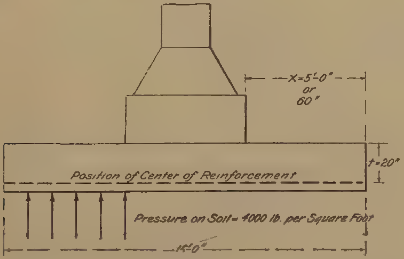


FIG. 4

The following formula is used to determine the compression created on the concrete due to bending stress:

$$C = \frac{Wx}{4.59t^2}$$

The value C is the compression, in pounds per square inch of section, and the other values are the same as in the first formula.

In order to explain the application of the preceding formulas, assume that the reinforced-concrete footing of a column is required to sustain 900,000 lb., and that, as the soil will safely support 4,000 lb. per sq. ft. of bearing area, the footing will be 900,000÷4,000=225 sq. ft. To obtain this area, the footing will have to be 15 ft. square, and the

design shown in Fig. 4 may be assumed in applying the formula.

In this figure, the projection of the footing, or the distance x, is 60 in., and the dimension t, or the distance from the center of action of the steel reinforcement to the top of the footing, is 20 in. The pressure on a projection of the footing 1 ft. in width is equal to the unit pressure on the soil, or, $4,000 \times 5 = 20,000$ lb., which is the value of W in the formula. These values may be substituted in the formula for determining the amount of the steel reinforcement, so that

$$A = \frac{20,000 \times 60}{27,000 \times 20} = 2.2 \text{ sq. in.}$$

This area of steel is to be included in each linear foot of the footing course; therefore, the reinforcement may consist of 1-in. square twisted bars spaced practically every 6 in. both ways, or other bars at a spacing to give this sectional area may be used.

The amount of the steel reinforcement having thus been determined, it remains to find out whether or not the concrete in the footing course is overstressed, and the second formula may be applied as follows:

$$C = \frac{20,000 \times 60}{4.59 \times 20 \times 20} = 653 \text{ lb.}$$

This result, which is the maximum compression, in pounds per square inch, on the concrete section, is somewhat high. If good concrete is used, the footing as reinforced and designed may be considered safe, though, in fact, the thickness of the concrete footing might be increased several inches.

CANTILEVER FOUNDATIONS

In reinforced-concrete construction, as well as in other types of construction, it is frequently necessary to place a new building close against the walls of an adjoining building. In many instances the wall of the adjoining property rests entirely on its own lot and is not a party wall built half on each side of the party line; also, the adjoining building may be of inferior construction or may be occupied by tenants

engaged in manufacture. Under such conditions it is undesirable to tear out the wall and build a party wall.

With buildings of ordinary height and load, provided the basement floor of the new building does not extend below that of the old, few difficulties are encountered in the design of the foundations for the new structure; but if the new building is to be many stories in height and requires extensive foundations along the wall lines, the problem of the design of the reinforced-concrete foundations becomes more complicated, because it is desirable to proportion the footings so that the center of action of the loads will coincide with the center of action from the pressure of the soil beneath. In order to accomplish this desired result in a steel structure, a cantilever-girder system of foundation construction would be employed, and a similar system can be constructed in reinforced concrete.

In Fig. 1 is shown the detail drawings of a reinforced-concrete cantilever-foundation construction for a six-story building, the floors of which are designed for light manufacturing purposes. In view (*a*) is shown a diagrammatic cross-section of the building, which illustrates the conditions of loading and the spans of the cantilever and other girders. The live load to be supported by the floor construction and the cantilever girder is 220 lb. per sq. ft., and the soil beneath the footings is capable of supporting safely 6,000 lb. per sq. ft.

The details of the construction and the reinforcement are shown in view (*b*). The footing is reinforced against failure from transverse stress by bars *a*, placed near the bottom of the footing, and further by a mattress of rods or bars placed directly beneath the bearing of the foundation wall column, as at *b*. The column, or wall pier, *c* is reinforced with vertical reinforcing bars, which are tied at close intervals with wire or loop ties *d*. Particular attention is called to the manner in which the outside rods of the foundation pier or column run straight through and up into the column above, being continued as the inside rods of the upper tiers. The outside rods of the first-floor wall column are bent so as to pass obliquely through the cantilever bracket of the foundation pier or column and the flare at the

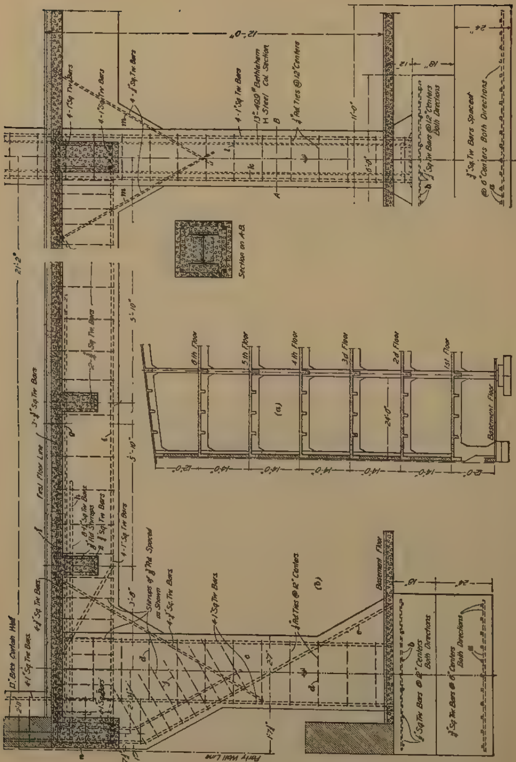

Section on A-B
6th Floor
5th Floor
4th Floor
3d Floor
2d Floor
1st Floor
Basement Floor
(a)
(b)
Party Wall Line
First Floor Line

FIG. 1

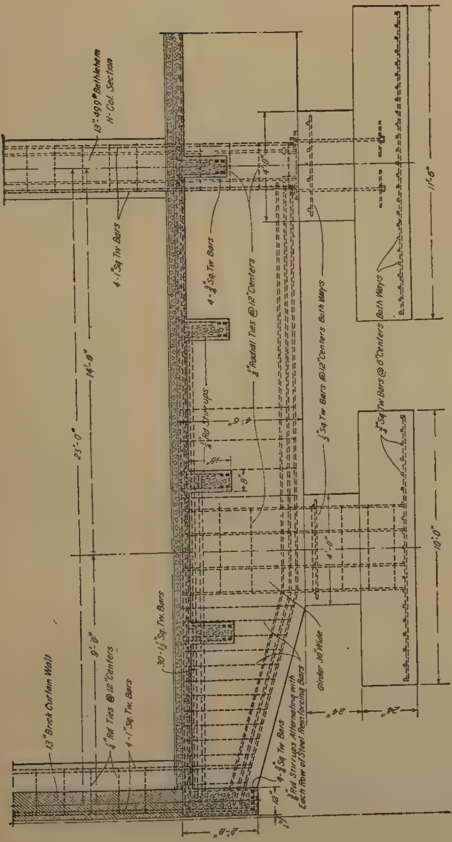

13" 499# Bethlehem H-Col. Section
4-1" Sq. Tw. Bars
4-3/4" Sq. Tw. Bars
1/4" Radial Ties @ 12" Centers
5/8" Sq. Tw. Bars @ 12" Centers Both Ways
3/4" Sq. Tw. Bars @ 6" Centers Both Ways
1/4" Rd Stirrups
13" Brick Curtain Wall
1/4" Rd Ties @ 12" Centers
4-1" Sq. Tw. Bars
30-1 1/2" Sq. Tw. Bars
4-3/4" Sq. Tw Bars
3/8" Rd Stirrups Alternating with Each Row of Steel Reinforcing Bars
Girder 30" Wide
23'-0"
14'-8"
9'-0"
11'-6"
10'-0"
4'-6"
4'-0"
2'-8"
24"

FIG. 2

bottom, as shown at *e*. The reinforcing rods of the cantilever girder are arranged as shown at *f*, *g*, *h*, and *i*. The rods in the upper part of the girder are arranged in three rows, or layers, and are stopped off in length in practically the same manner as the flange plates of a built-up girder, because the bending moment is reduced toward the interior column. Only one set of the reinforcing rods, as at *f*, extends entirely through into the interior column, because the bending moment is so reduced that this set is all that is required. The bottom rods, as at *i*, are inserted to improve the resistance to compression in the cantilever girder beam at the bottom.

Both the girder and the cantilever bracket are well supplied with stirrups, as shown at *j*. The vertical reinforcement of the interior column is shown at *k*, and the ties and bracket reinforcement, at *l* and *m*, respectively. The lintels, or connecting beams, at the end of the cantilever are shown at *n*.

In Fig. 2 is shown a reinforced-concrete cantilever-foundation construction designed to carry the same load on the end of the cantilever as was required of the construction illustrated in Fig. 1. As the cantilever girders have a considerable projection beyond the foundation footings, they must be very strongly reinforced in both the top and bottom, as shown. The several details of this construction are worked out in the illustration, and an analysis of these will show that the several principal and secondary stresses created in the structure are amply provided for

BUILDING DETAILS

LINTEL AND SPANDREL CONSTRUCTION

There are two ways of arranging the beams and girders in a building. These different ways bring different loads on the spandrels

In the construction shown in Fig. 1 (*a*), one-half of the floor load from the beams *a* and *b* is concentrated at two points upon the lintel, and the lintel has this load, as well as the weight of the spandrel wall, to sustain. Also, as the beams *a*

and b extend into the lintel, it must at least be equal in depth to the depth of these beams. Thus, the height of the window opening is materially reduced.

The method of framing shown in Fig. 2 (b) is frequently employed. The girders, instead of extending from column to column, as shown at c in view (a), extend from column to wall pier; consequently, the beams extend in a direction parallel with the lintel. The slab spans from the beam a to the lintel b. The lintel carries one-half of the slab load between these structural members and is required to sustain the weight of the spandrel as well. The advantage gained by this method of construction is that there are no beams abutting the lintel. Thus, this member may be reduced in depth to the very minimum for strength required to support the small floor load and the weight of the spandrel wall.

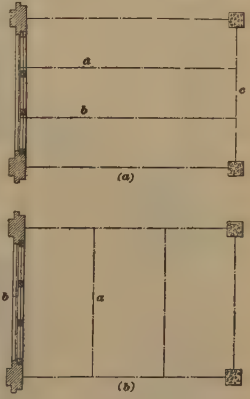


FIG. 1

The style of lintel shown in Fig. 2 (a) is the one in most common use. The girders are arranged, as shown in Fig. 1 (b), so that the lintel carries only part of the slab and the spandrel section. The lintel is made as shallow as possible, so as to admit maximum light by having the window head near the ceiling.

When no beams are used and the slab spans from girder to girder, the lintel carries no load except the weight of the spandrel. In this case, reinforced concrete is sometimes dispensed with and a brick arch employed, as shown in Fig. 2 (b). The arch, however, cuts off the top corners of the window, and thus somewhat reduces the amount of light that can be admitted.

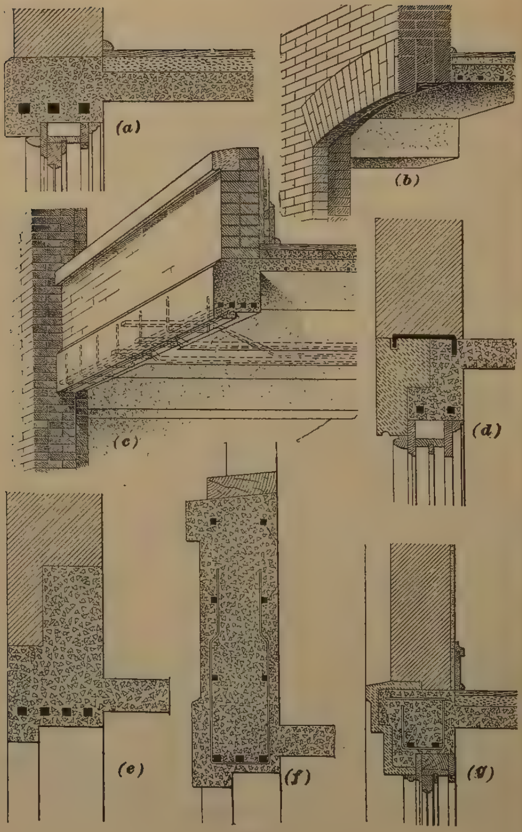


FIG. 2

When the lintel supports the ends of floor beams, as shown in Fig. 1 (*a*), the construction shown in Fig. 2 (*c*) is used.

Sometimes it is desired to cover the concrete lintel with a stone facing, as shown in (*d*). The stone is held to the concrete by means of an iron clamp at the top, as shown.

When the span of the lintel is very long or the load to be carried is heavy, it is necessary to make the lintel deep. So as not to lower the head of the window, the lintel is run up into the spandrel above, as shown at (*e*). In this case, the floor slab joins the lintel near its lower surface and therefore does not assist in resisting compression. If the lintel must be strengthened still further, sometimes the entire spandrel is made of reinforced concrete, as shown in (*f*).

In cases where a building is to have terra-cotta trimmings, the concrete lintels are often also covered with terra cotta. Such an arrangement is shown at (*g*). The brick spandrel is placed on top of the terra cotta and assists in holding it in place.

EAVES AND CORNICES

For ordinary factory construction, in suburban districts at least, where little artistic effect is desired, the simplest way to construct the eaves of the roof with a hanging gutter is shown in Fig. 3 (*a*). The gutter itself is often omitted and the rain allowed to fall to the ground, especially if there is no cellar and if the floor level is somewhat above grade. The slab reinforcing rods are shown running parallel to the wall. Where the overhang of the slab is considerable, the reinforcing rods must run at right angles to the wall and be bent up toward the top of the slab in the overhang so as to take up the reverse bending moment, as shown in (*b*).

Sometimes the gutter is constructed in the concrete by forming a gusset, as shown in (*c*). The roofing is extended directly over the gusset and secured by a wooden strip.

When a parapet wall is required, the construction shown in (*d*) may be used. The flashing is secured under the coping. Where the parapet wall is low enough, the best plan is to raise the gusset so that it will come to the under side of the coping. In this way the flashing may be omitted altogether and the roofing run under the coping. Such an arrangement

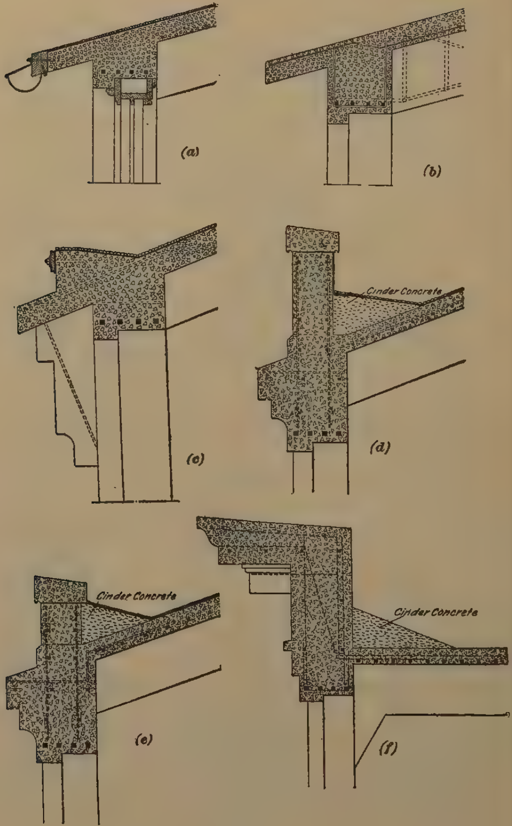


FIG. 3

is shown in (*c*). In both (*d*) and (*e*), the gusset is made of cinder concrete, as it is cheaper and lighter than stone concrete and is not required to carry a heavy load.

In (*f*) is shown the method of reinforcing a cornice that has a heavy overhang. This construction requires special care, as the cornice may be very top heavy.

BRICK FACING AND TERRA-COTTA STRING-COURSES

The method of tying a brick facing to a concrete pier by means of copper ties is illustrated in Fig. 4. The ties, which consist of strips of copper about $\frac{1}{16}$ in. thick, $\frac{3}{4}$ in. wide, and 7 in. long, are placed every seventh joint. These ties are spaced about 2 ft. apart horizontally, and are embedded in the concrete work to a depth of about 4 in., which gives them, when a $\frac{5}{8}$- or $\frac{3}{4}$-in. space is left between the facing and the concrete, a surface about $2\frac{1}{4}$ in. long in the bricks. It is customary in arranging the brick facing to allow about 5 in. for the width of the brick, and the space back of the brick is usually flushed in with mortar as the bricks are laid

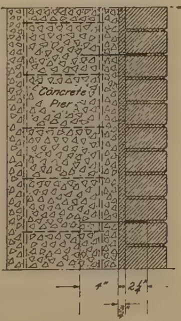


FIG. 4

FASTENINGS IN CONCRETE

There are numerous devices intended to be embedded in the concrete work for the purpose of holding **T**-headed or tap bolts in a secure manner. These necessarily must be arranged in the forms before the concrete is poured. Ordinarily, these sockets should be tapped out for $\frac{3}{4}$-in. bolts, but where heavy machinery or hoisting apparatus is to be employed, it will be better to use $\frac{7}{8}$-in. bolts. Sockets should be provided near the end bearings of each beam and girder, and intermediately, not farther apart than 4 to 5 ft.

A $\frac{3}{4}$-in. bolt will ordinarily test in a tensile pull to from 15,000 to 17,000 lb. Thus, it can be considered to sustain safely a load of about 4,000 lb.

Where it is necessary to bolt heavy machinery to the ceiling, the location should be ascertained, and heavy through bolts should then be embedded in the concrete work.

BEAM SOCKETS

Styles of Sockets.—The socket shown in Fig. 5 (*a*) is known as the *Unit socket.* The cast-iron part *a* rests on the bottom of the form load, and is bolted to it by the bolt *b*, which passes through a hole in the bottom of the forms. This socket, as shown, clamps to four reinforcing rods of the beam or girder in which it is placed. In (*b*) is shown a malleable-iron socket. It is tapped at *a* to receive a stud bolt. This socket is secured to the form boards by means of lugs *b*, through which nails are driven into the wood.

The socket shown in (*c*) is known as the *Jennings-Steinmetz socket.* It consists of a piece of pipe swaged out or broken at the upper end, as indicated at *a*, with a solid wrought-iron bar or block welded to the other end. The solid portion of the block is tapped out for the bolt *b*, which passes through the bottom board *c* of the form.

What is known as the *Hancock insert* is shown in (*d*). This device consists of a cast- or malleable-iron casing that is made in loose halves *a*, and is wired together through the lugs *b* cast on the side. Before the two halves of the insert are wired together, however, there is placed in the recess a nut *c*, into which a bolt can be screwed from beneath. The castings are arranged with lugs, or flanges, *d* on the face end and by means of these the insert may be screwed or nailed to the form board and thus secured in an upright and secure position when the concrete is placed. The flange *e* at the top of the insert furnishes additional bond, or key, with the concrete. The Hancock insert is made in 3-, 4-, and 6-in. lengths for $\frac{1}{2}$-, $\frac{5}{8}$-, $\frac{3}{4}$,- and 1-in. bolts.

In (*e*) is shown the *Bigelow socket.* This casting, which is made of malleable iron, is arranged with a slot *a* that is

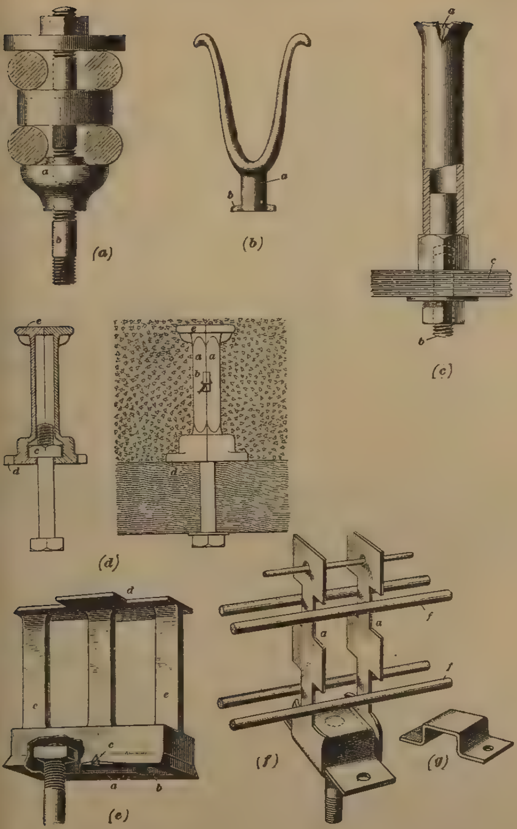


FIG. 5

enlarged at b so as to receive the head of the bolt. In order that the casting may be securely held in the concrete work, it is arranged with a plate d and connecting pieces e. On the sides of the socket at the face are cast notches c into which nails may be driven so that the device may be secured to the forms. The Bigelow socket is made for $\frac{1}{4}$-, $\frac{5}{16}$-, $\frac{3}{8}$-, $\frac{7}{16}$-, $\frac{1}{2}$-, and $\frac{3}{4}$-in. bolts. The smaller sizes are suitable for embedment in slab work, and the $\frac{7}{16}$-, $\frac{1}{2}$-, and $\frac{3}{4}$-in. sizes may be used in beams and girders.

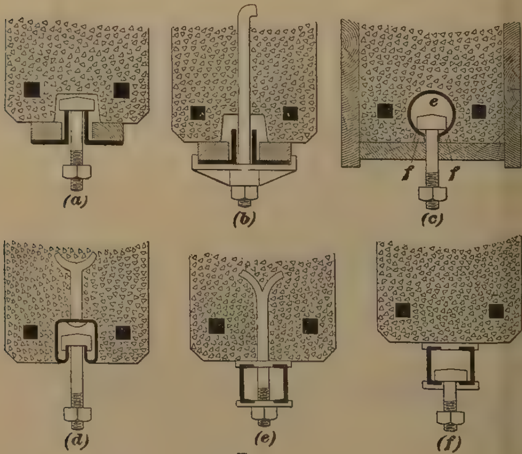


FIG. 6

In (f) is shown a *sheet-steel socket*. It is cut, as shown at a, so as to fit around the beam reinforcing bars f. In (g) is shown separately the cap that fits over the nut held by the socket into which the stud bolt fits.

Continuous Inserts for T-Headed Bolts.—In the construction of factory buildings it is sometimes desirable to arrange a slot in the bottom of the concrete beams and girders so

that a **T**-headed bolt can be introduced at any point along the entire length. To accomplish this, several methods are employed, the principal ones being illustrated in Fig. 6.

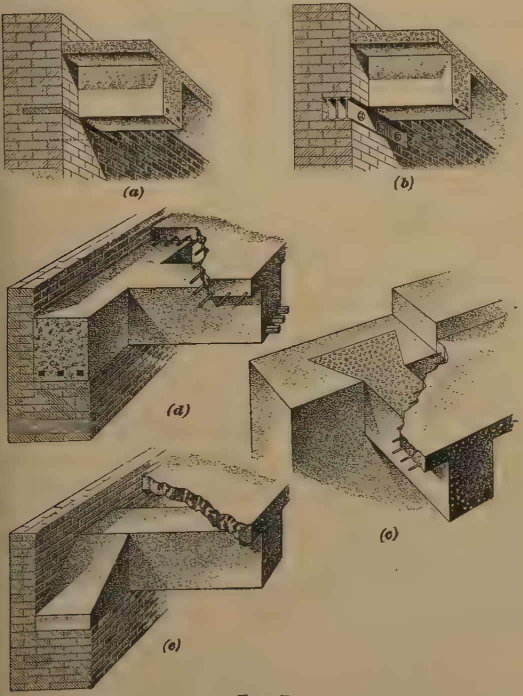


FIG. 7

Two views of one device are shown in (*a*) and (*b*). View (*a*) shows the **T**-headed bolt that carries the shafting

and view (*b*) shows the bolt that secures the continuous insert to the under side of the beam.

The device shown in (*c*) consists of a split pipe in which the **T**-headed bolts slide. The form boards are shown in place in this view so that it may be seen how the split pipe is held in place in the forms by the strips *f*.

In (*d*), the slide consists of a piece of bent steel embedded in the concrete. As shown, bolts are also used to hold the steel slide in place.

Two views of another device are shown in (*e*) and (*f*). View (*e*) shows how the slide for the **T**-headed bolts is held in place, and view (*f*) shows the bolt in position to support machinery.

BEARINGS FOR CONCRETE BEAMS AND GIRDERS

It is often necessary to increase the bearing area of concrete beams, especially when they rest on brick walls. The ways in which this can be done are shown in Fig. 7.

In view (*d*), the end of the beam is formed with projecting spurs at right angles to the axis of the beam, those spurs being of such length as to distribute the bearing over a considerable area. The reinforcing rods of the cross-beam are shown at *c*.

In order to save concrete, the bearing of the beams may be increased by the method shown in (*e*). This construction however, is not so practical as the one shown in (*d*), because the form is more troublesome to make and fill, and, besides the masonry does not build so readily with flat beds on the slope as it will with the construction shown in (*d*).

FORM WORK

CONSTRUCTION AND FINISH OF FORM WORK

In the erection of reinforced-concrete work, nothing requires more careful consideration than the construction of the *form work*, or molds, necessary to shape and support the concrete until it has thoroughly set and hardened. Throughout the practice of reinforced-concrete construction various methods of form constructions are in use.

The greatest economy is gained by constructing the forms so that they can be used over and over again in the structure. Economy in construction can also be gained by fastening the form work together with a minimum amount of nailing. Every nail that is driven gives trouble when the forms are taken down to be replaced for the upper floors. In many constructions, wedges and clamps are used instead of nails or screws if the forms are to be reused.

In some instances, both wooden and metal forms are coated on the side next to the concrete in order that the forms may be detached more readily. Coating the forms also serves to prevent the marking of the grain of the wooden forms on the finished concrete work.

Dead oil, or crude petroleum, has been used with success for this purpose. It is not unusual to soap wooden forms, and in some cases tallow and bacon fat have been employed. The latter is especially recommended for coating metal forms, as it seems to give the best results with forms of this material. In some instances, wooden forms have been covered on the inside with paper, and even canvas has been used, although it is usually found that the paper adheres to the concrete work and is detached only with difficulty. It is not customary, however, to oil or coat the forms unless they are to be used for fine exterior work.

FORMS FOR FLOOR SYSTEMS

Common Types of Form Work.—In Fig. 1 is shown a type of form work extensively used for the construction of rein-

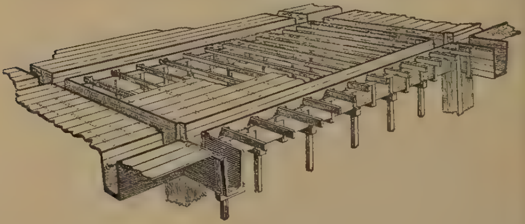


FIG. 1

forced-concrete floor systems, and in Fig. 2 is shown a perspective of the forms at the intersection of a beam and girder. This form work is designed so that light $\frac{7}{8}$-in. dressed tongued-and-grooved material may be used extensively in its construction. It is arranged so that the sides of the beams and girders, together with the slab form boards, may be removed without the necessity of removing the supports directly underneath the beams and girders.

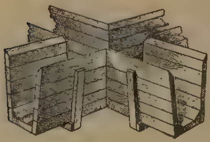


FIG. 2

The column form for this type of construction is shown in Fig. 3.

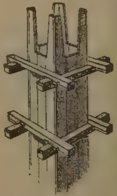


FIG. 3

Forms Constructed of Plank. — A superior type of form for a reinforced-concrete floor system is shown in Fig. 4. The wooden forms are supported by $3'' \times 4''$ studs. As it is important to bring the forms to a true level, a double adjustment wedge is provided at the bottom of the studs. The forms for the columns are made of $1\frac{1}{2}$- or 2-in.

material. In the construction of the beam and girder forms 2-in. planks are generally used for the sides. In order to form

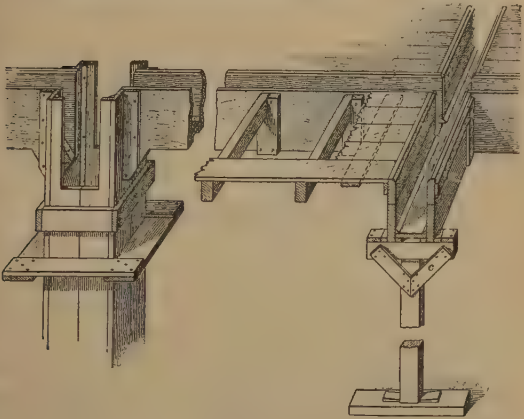


FIG. 4

a chamfer on the lower edges of the beams and girders, triangular fillet pieces are nailed in the forms. It is customary

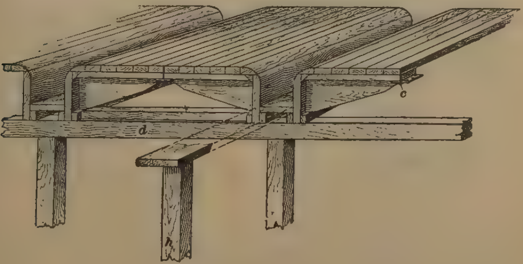


FIG. 5

in this type of construction to make the forms for the slabs of $\frac{7}{8}$-in. plain boards, frequently using tongued-and-grooved material.

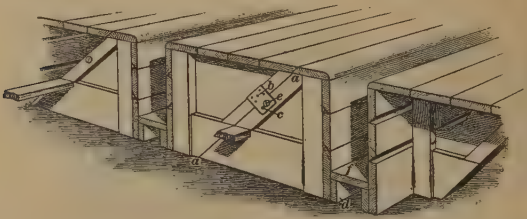


FIG. 6

Collapsible Forms.—Several attempts have been made to arrange the forms for a reinforced-concrete floor system so that they will partly collapse and thus be easy to take down or away from the concrete as soon as it has properly set.

One type of collapsible form is shown in Fig. 5. In this construction, the side forms for the beams are embodied with the slab centering by the construction of a collapsible box, which furnishes a form that is the exact shape of the space between two adjacent beams. These boxes are arranged with a hinge *c*, and when in place they rest upon cap pieces *d*.

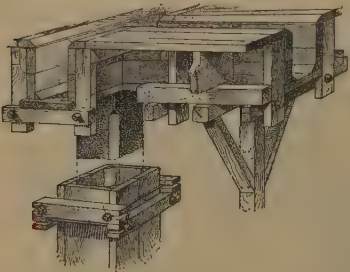


FIG. 7

In order to take down the forms, the struts are removed, and the form is allowed to double up on the hinges *c* and thus release itself from the concrete. After the collapsible centering has been removed,

the planks beneath the soffit of the beams and girders may be retained in position by means of struts *h*.

Another type of collapsible form is shown in Fig. 6. In this form, the box that supplies the form work for the sides of the beams and the centering of the slabs, instead of being hinged, is cut through along the line *a a* and is held in position by the slotted iron plate *b* and screw *e*. By releasing and loosening the screw, it will slip in the slot *c*. Then one-half of the form can be drawn downwards, so that it will recede from the face of the beam and thus allow the form to free itself from the concrete and be taken down. It will be observed that the sides of the collapsible forms make the sides for the beam forms, the battens *d* furnishing the support for the bottom form board *f* of the beams.

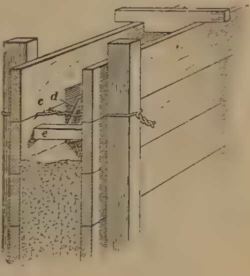


FIG. 8

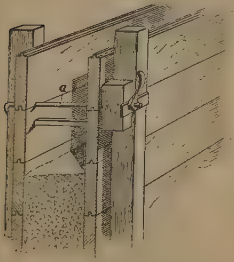


FIG. 9

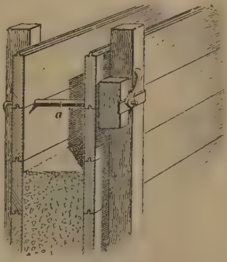


FIG. 10

Heavy Construction—Partly Collapsible Forms.—In Fig. 7 are shown the details of a form construction for a reinforced-

concrete floor system that is particularly well constructed. It may be readily set up and taken apart. The sheathing of the beam and girder forms and the plank used for the slab centering are 2 in. in thickness. The forms are supported by 4″×4″ studs.

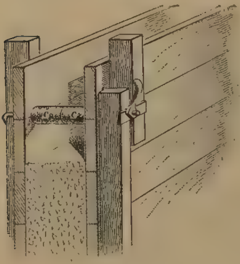


FIG. 11

Wall Forms With Wire Ties. A type of wall-form construction that is frequently used is illustrated in Fig. 8. In order to prevent the sides of the forms from spreading when the concrete is tamped in place, a wire tie *c* is used. This tie is made taut by twisting with a bar, or stick, *d*. To keep the form boards the proper distance apart for the thickness of the wall, a block or stick *e* of wood is sometimes inserted.

Wall-Form Construction With Clamp Bolts.—A wall-form construction similar to that shown in Fig. 8 is illustrated in Fig. 9. In this form, however, a clamp bolt *a*, instead of a wire tie, is used to prevent spreading. If a bolt of this character is used, it must be knocked out before the concrete has finally set and when the form boards are to be raised to form the next course of concrete. The bolt is preferable to the wire tie, because it is removed from the concrete. Wire ties are usually cut off close to the concrete work after the form boards have been removed, and as the ends frequently project, they rust and thus stain the wall.

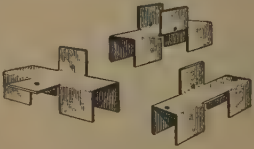


FIG. 12

In Fig. 10 is shown the construction of a wall form in which a pipe separator *a* is used with the clamp bolt. The pipes may be driven out of the concrete after it has obtained its initial set, or they may be left in place.

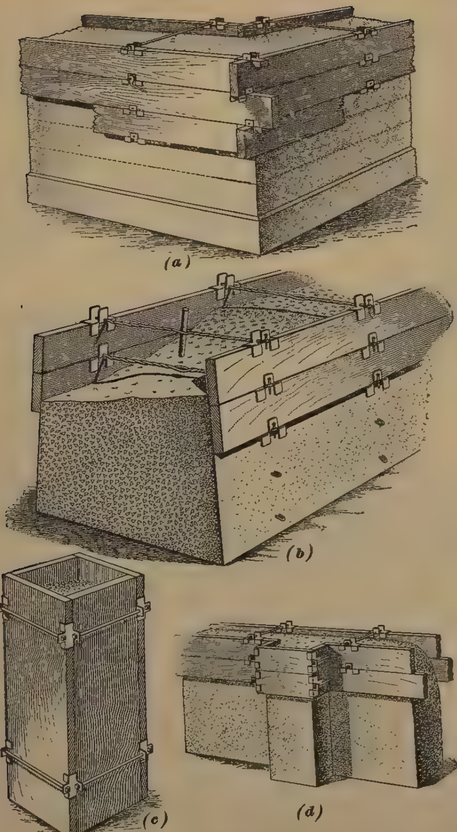


FIG. 13

One of the best methods of constructing wall forms is to use concrete separators. The separator known as the

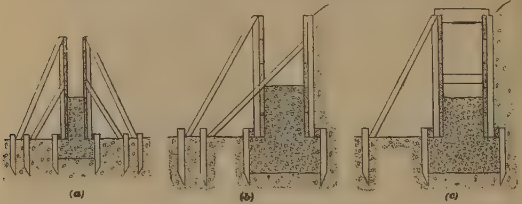


FIG. 14

McCarty separator, used in constructing the form work, is illustrated in Fig. 11. The separator *c* is cast previous to the construction of the forms.

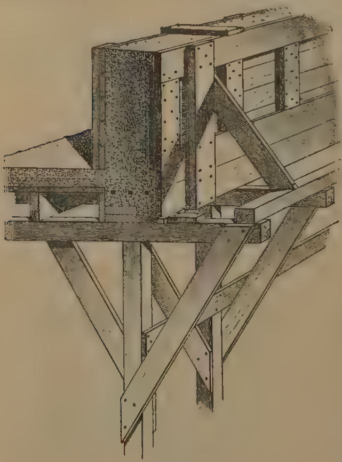


FIG. 15

Clamping Devices and Plank Holders for Wall Forms.—Many devices that aid in the construction of concrete walls have been invented. One of the most useful of these devices is the *Sullivan pressed-steel plank holder*, various forms of which are shown in Fig. 12. These holders are formed from an iron plate by shearing and bending it so as to form clips. The application of this type of plank holder is illustrated in Fig. 13.

Braces for Wall Forms.—If a wall is to be constructed in a place where there is no embankment, a double set of forms braced as shown in Fig. 14 (a) must be used. If the soil of an embankment against which a concrete wall is to be built is unstable, it is necessary to sheath and brace it. This may

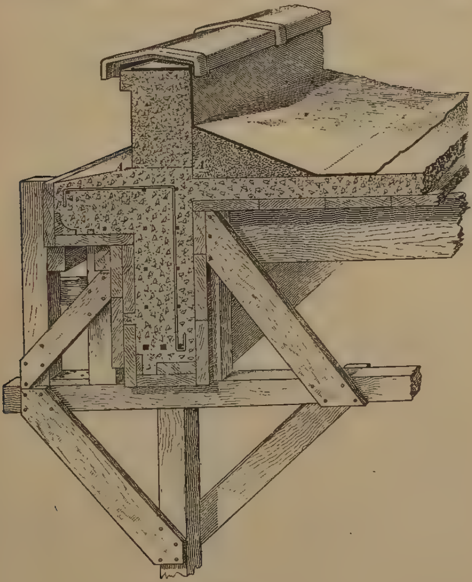


FIG. 16

be accomplished in either the manner shown in Fig. 14 (b) or that shown in (c).

Spandrel-Wall Forms.—In the molding of spandrel walls of buildings a special type of form construction must be used. A typical spandrel-wall form is shown in Fig. 15. Attention

is called to the fact that the spandrel wall cannot be carried up at the time that the lintel and slab forms are filled, especially if wet concrete is used. Therefore, the concrete must first be deposited to the level of the top of the slab, and when this has its initial set, the balance of the wall may be filled. If this precaution is not taken, the hydrostatic pressure of the wet concrete in the spandrel wall will force the concrete from underneath the form at the slab and prevent the filling of the wall form.

Cornice Construction.—In Fig. 16 is shown a typical form construction for a reinforced-concrete cornice. The cornice work is carefully constructed of 1⅝-in. dressed material. In filling the form, the concrete is carried to the level *A A*, and the parapet wall is afterwards constructed above this point.

WATERPROOFING OF CONCRETE

CLASSIFICATION OF SYSTEMS

There are three principal methods, or systems, employed in the waterproofing of concrete. They may be termed the *integral*, the *superficial*, and the *membrane method.*

The integral method consists in adding something to the concrete when it is placed, or in mixing the concrete in certain proportions, so as to make it waterproof throughout. The superficial method consists in coating the concrete with paint or some other material. This material adheres to the concrete and hardens, or dries, on it. The membrane method consists in putting on the concrete a coating that is distinct from it. While the coating may adhere to the concrete, it does not crack when the concrete cracks, but is in a distinct membrane usually strengthened by felt or some other fiber cloth.

INTEGRAL METHOD

Mixing of Concrete.—According to some authorities, if concrete is properly mixed, it will be impervious to water. The exact mixture to use will depend on the quality of sand

and broken stone. This most waterproof mixture is also the densest mixture. Therefore, in searching for the most waterproof mixture of concrete, the engineer really finds the densest mixture. For many grades of sand and broken stone, a 1–1½–3 mixture is used. To be impervious to water, concrete must be placed in a wet condition and be well rammed into place

Concrete, particularly that which is very dense, becomes more impervious to water as it grows older. The first water that penetrates it carries particles of clay and other material that stops up its pores and gradually makes it more waterproof.

Adding of Lime or Clay.—By *hydrated lime* is meant lime that has been slaked in water. It can be bought commercially in the form of a dry powder. The purpose of adding hydrated lime to concrete is to fill mechanically the voids in the latter. A small quantity added to the concrete when it is being mixed does not greatly reduce the strength of the latter. After the cement and lime are mixed until the color of the mixture is uniform, the sand, broken stone, and water are added as usual. The concrete should be made wet, and great care must be exercised in bonding old and new work. The amount of lime to be employed is usually given as a percentage, by weight, of the cement. Under ordinary conditions, for 1–2–4 concrete, 8% of lime will be found sufficient, and for 1–3–6 concrete, 17% of lime will be required.

In place of hydrated lime, concrete is often waterproofed by mixing with it finely ground *colloidal clay*. It is recommended that clay equal to about 10% of the weight of the cement be used in a mixture. The clay must be thoroughly dry and well mixed with the cement.

Sylvester Process.—The *Sylvester process* of waterproofing concrete consists in adding *powdered alum* and *soft soap* to the concrete. The sand and cement are mixed together dry, as usual. To this mixture is added alum equal to 1% of the weight of the mixture. To the water to be used is added 1%, by weight, of soft soap, and this soap is then thoroughly dissolved. The mortar is made wet, and the broken stone

is added in the usual manner. Ordinary precautions should be taken to make the concrete dense, and as a rule it is mixed rather wet.

Metallic Stearates and Other Compounds.—Besides the materials mentioned, various other chemicals are used to make concrete waterproof. One of the most successful of these is *calcium stearate*, which is a salt of a fatty acid. Usually, the quantity of finely ground metallic stearate added to the cement to make the concrete waterproof is about 2% of the weight of the cement.

Besides the metallic stearates, other substances are used more or less to waterproof concrete by the integral method. A mixture of oil and water has been used with some success, as has also chloride of lime. The purpose of the chemicals is in every case to fill the voids, and, in addition, some of them are water repellants.

SUPERFICIAL METHOD

Waxes.—To make concrete waterproof, it is sometimes coated with *wax*. The wax sinks into the pores of the concrete, filling the voids therein and thus preventing the percolation of water.

Paraffin is used for waterproofing with considerable success. A paraffin especially hardened to resist the sun's rays is used. The concrete surface on which the paraffin is to be applied should be thoroughly dry and not very cold. In fact, it is better to warm the surface with a torch if possible. The paraffin is heated and then applied with a brush. The hot paraffin is absorbed by the concrete, into which it penetrates for a short distance. Paraffin will resist the action of acids and alkalies.

Instead of applying paraffin while hot, it may be dissolved in some volatile carrier, such as benzine. The dry wall is painted with this solution, which is readily absorbed. The carrier then evaporates and leaves the paraffin to fill the pores in the surface of the wall. If the carrier is inflammable, care must be taken in using it.

Cements.—A common method of waterproofing concrete is to coat the surface of the concrete with an impervious cement

coating. Sometimes this coating is simply a rich, dense cement mortar; sometimes it contains a waterproofing compound similar to those mentioned in connection with the integral method; and sometimes a special waterproof cement is used.

The adhesion between the coating and the concrete work requires the most careful consideration. The concrete surface on which the waterproof coating is to be placed must first be chipped and scraped to remove all glaze and to obtain a rough texture. The surface must then be thoroughly washed with clean water to remove all dust, and all cracks must be filled with mortar. While the surface is still wet, it is painted with a mixture of cement and water of about the consistency of thick cream. This mixture is applied with a stiff brush and must be well rubbed into the surface. Before this coat dries, the first coat of waterproof mortar is put on, usually about ½ in. thick. This coat must be troweled into the surface with great care. Then, before this coat sets, another coat of waterproof mortar of about the same thickness or a little thicker is applied.

MEMBRANE METHOD

Bituminous Membranes.—The *bituminous-membrane method* consists in the use of a felt saturated with bituminous material and a bituminous binder. By a *binder* is meant a material that binds two surfaces together; that is, an adhesive material. By a *bituminous material* is meant any material containing a large proportion of solid or semi-solid bitumen, *bitumen* being that portion of pitch or asphalt that is soluble in carbon bisulphide, benzol, petroleum, ether, or other similar solvent.

The bituminous membrane is built up in place in successive layers, and should form, when finished, a practically homogeneous and continuous waterproof envelope. The felt serves to hold the bitumen in place, while the latter is the waterproofing material.

Specifications for Coal-Tar Pitch and Felt Roof Over Concrete.—The following specifications, known as *Barrett specifi-*

cations, will be found excellent for placing a coal-tar pitch and felt roof over concrete. These specifications also describe how the work should be carried out.

"There shall be used five thicknesses of approved felt weighing not less than 14 lb. per 100 sq. ft., single thickness, not less than 200 lb. of approved pitch, and not less than 400 lb. of gravel or 300 lb. of slag from $\frac{1}{4}$ to $\frac{5}{8}$ in. in size, free from dirt, per 100 sq. ft. of completed roof.

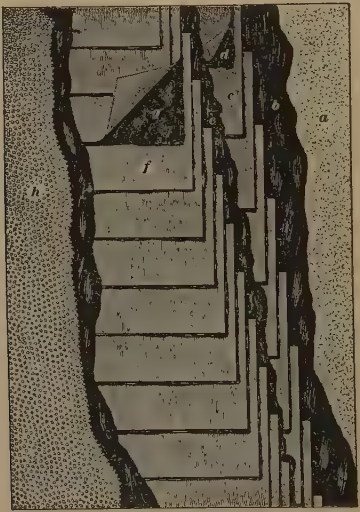


FIG. 17

"The material shall be applied as follows: (1) Coat the concrete *a* (see accompanying figure), with hot pitch *b* mopped on uniformly. (2) Lay two full thicknesses of tarred felt *c*, lapping each sheet 17 in. over the preceding one,

and mop with hot pitch *d* the full width of the 17-in. lap, so that in no case shall felt touch felt. (3) Coat the entire surface with hot pitch *e* mopped on uniformly. (4) Lay three full thicknesses of felt *f*, lapping each sheet 22 in. over the preceding one and mopping with hot pitch *g* the full width of the 22-in. lap between the plies, so that in no case shall felt touch felt. (5) Spread over the entire surface of the roof a uniform coat of pitch, into which, while hot, embed the gravel or slag *h*. The gravel or slag in all cases must be dry."

The preceding specifications are designed for roofs having an incline not exceeding 1 in. to the ft., and by adding the words "such nailing as is necessary shall be done so that all nails will be covered by at least two plies of felt," the specifications are suitable for inclines not exceeding 3 in. to the ft. For surfaces steeper than 3 in. to the foot, nailing strips of wood must be provided. These should be embedded in the concrete from 3 to 6 ft. apart, running at right angles to the pitch of the roof, and the felt nailed to these strips.

FIELD OPERATIONS

CONCRETE MIXERS

BATCH MIXERS

Cube Mixers.—One of the oldest and best-known forms of batch mixer is the *cube mixer*. In Fig. 1 is shown what is known as a *Carlin cube mixer*. The cube is driven by a belt *a*, and gears *b* serve to reduce the speed of rotation. The cube is shown in the position it occupies when being filled. The trap door for filling and discharging is shown open and swung back at *c*. The hopper above the cube is made in two parts. The lower part is hung from one of the posts of the platform, as shown. When the cube is revolving, this lower part can be swung out of the way. At *d* is shown the water tank, which feeds water into the cube through the pipe *e*. Below the cube is shown a car *f* into which the finished concrete is dumped.

The following conditions have been found to produce excellent results with a 4-ft. cube mixer: Each batch is mixed $1\frac{1}{2}$ min. at a speed of 10 rev. per min., making a total of 15 turns per batch. The time allowed for dumping, cleaning and charging the mixer and for temporary stoppage of the work is about $3\frac{1}{2}$ min., making an average time interval between successive batches of 5 min., which is at the rate of 12 batches per hour. Allowing only $\frac{7}{8}$ cu. yd. of rammed concrete per batch, this gives for a 10-hr. day an output of 105 cu. yd.

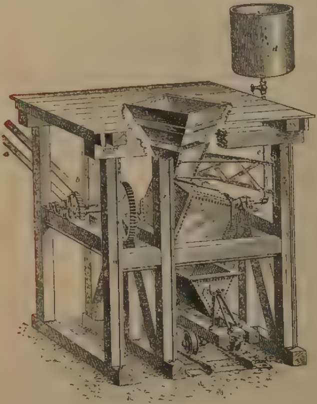


FIG. 1

Ransome Mixer. Another form of batch mixer, called the *Ransome mixer*, is illustrated in two views in Fig. 2. This machine consists essentially of a hollow cylindrical drum that is mounted on a horizontal axis and has a circular opening in each end. As shown in (*a*), the mixer is charged through one of these openings *a*. In the other opening, shown in (*b*), is a chute *b* that receives the mixed concrete from the drum and delivers it to a wheelbarrow or other receptacle. Inside the drum are several steel blades arranged in such a manner as to deflect the material from side to side as the drum revolves. The drum is mounted on four rollers and is supported on a truck, which is either made stationary or mounted on wheels, as required. The mixer is turned by power applied to the rim of the drum either from an engine mounted on the same frame or from a belt or a chain. The illustration shows one driven by a belt.

The chute b, Fig. 2 (b), through which the mixer discharges, is kept in the position shown in Fig. 3 (a) while the concrete is being mixed. When it is time to empty the mixer, the handle c, Fig. 2, is turned. This turning of the handle, which is connected to the chute by means of a chain, as shown, tips the chute b, as illustrated by dotted lines in the figure. Meanwhile, the drum is revolving and lifting masses of concrete up one side in the direction of its motion. These masses finally fall by the action of gravity to the bottom of the drum.

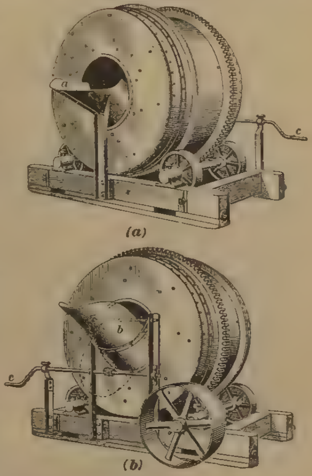


Fig. 2

The path followed by the concrete in the mixer is shown diagrammatically in Fig. 3. The blades are represented by short radial lines. When the chute is tipped down, as shown by dotted lines in Fig. 2 (b), the upper end intercepts the flow of concrete, as shown in Fig. 3 (b), and the mass slides down the chute and out of the drum.

The Ransome mixer is made in several sizes, with varying capacities.

Smith Mixer.—In Fig. 4 is illustrated the *Smith mixer*. The machine consists of a drum of double conical form that is supported and guided by a frame that can be tilted at will while the drum is revolving. The ingredients are fed in at one end of the drum, and after the required number of revolutions the mixed concrete is discharged at the other end by tilting the drum while it is running at full speed.

The interior of the drum is provided with blades so as to insure thorough mixing. The capacity of the mixer is from

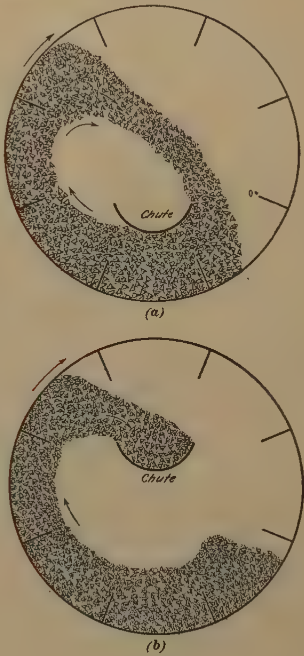


Fig. 3

10 to 35 cu. yd. of concrete per hour, according to the size of the machine.

CONTINUOUS MIXERS

Drake Mixer.—One of the best known forms of continuous-mixing machines is the *Drake mixer*. This mixer consists of an open trough fitted with a longitudinal shaft, to which are fastened blades, or paddles, some of which are set at an inclination, so that they will not only mix the ingredients, but also feed the mixture toward the discharge end. The mixer is so set that the shaft revolves on a horizontal axis. The ingredients are deposited by wheelbarrows or from a

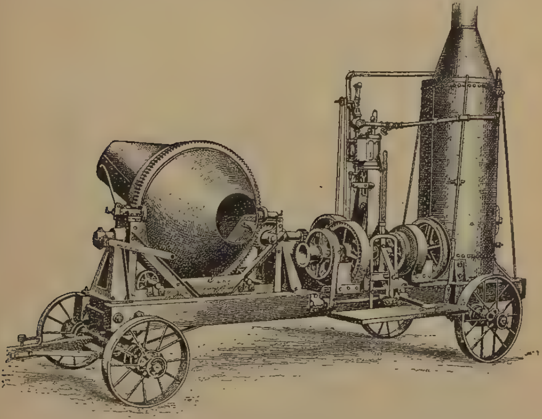


FIG. 4

measuring box at the upper, or feeding, end of the trough. The straight blade, or knife, at that end cuts or stirs the mass, which is then turned over by the adjacent curved blade, or scoop, and advanced to the next knife, where it is again cut and then turned over by the next scoop, and so passed on. This process is repeated until the end of the trough is reached, when the material is pushed out and falls into a receptacle as mixed concrete. The ingredients are mixed dry for about one-half the length of the trough and

then a spray from water pipes wets the material as it is cut and turned over.

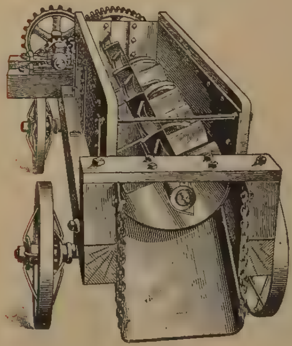


FIG. 5

Fig. 5 shows a Drake mixer of usual design. The ingredients are dumped in at the far end, and when they are entirely mixed are delivered down the chute. The Drake mixer is made in various styles.

Cockburn Mixer.—Another type of continuous mixer, called the *Cockburn mixer*, is illustrated in Fig. 6. This machine consists essentially of a long box of square cross-section, mounted on a substantial iron frame or truck and revolving on a longitudinal axis on friction rollers, as shown. The axis of the box is inclined slightly from the

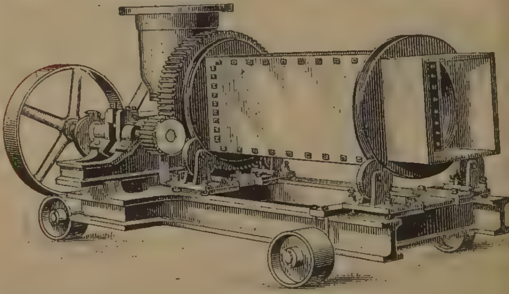


FIG. 6

horizontal. The ingredients are fed in through a hopper o chute, at the upper end, and, as the mixer revolves, the mate

rial moves by gravity toward the lower end. The volume of its output depends largely on the rapidity with which the ingredients are fed into the mixer.

QUANTITATIVE MIXERS

Quantitative mixers are mixers that measure as well as mix the ingredients used to make concrete. They usually consist of continuous mixers similar to those just described, to which is added a device for measuring automatically the

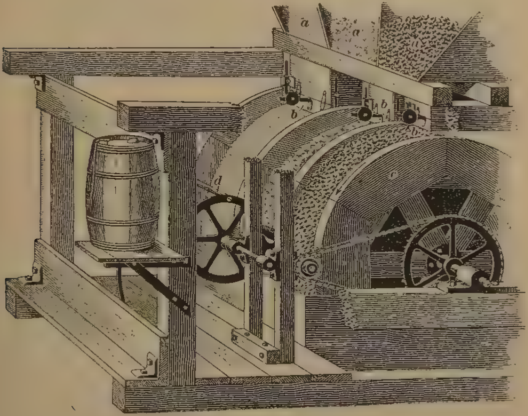


FIG. 7

cement, and broken stone as it enters the mixer. Sometimes the measuring apparatus and mixer proper are built as one machine, and sometimes the measuring apparatus is built separate, so that it may be used with any suitable mixer.

Gilbert Measurer and Feeder.—A machine that can be applied to any continuous mixer is the *Gilbert accurate measurer and feeder.* Fig. 7 shows a view of this measurer. At *a* are shown the bottoms of the three storage bins, which

are built hopper shaped. In one of these bins is stored cement, in another sand, and in the third, broken stone. These bins, although hopper shaped, have no bottoms. They are closed at the bottom by the cylinder *c*, which is of sufficient diameter to act as a practically flat bottom to the bin. In the front of each hopper there is a door *b* that may be adjusted by a weight, as shown in the figure. The cylinder *c* is turned by a man at the handle *d*. As the cylinder revolves, it carries with it a layer of cement, sand, and broken stone. The amount of each material carried forwards is controlled by adjusting the gate at *b*. As the wheel revolves, the materials finally slide off into a conveyer or directly into the mixer. The faster the operator revolves the cylinder, the more materials he will discharge, but they will always be discharged in the same proportion.

OPERATION OF MIXERS

A concrete mixer may be operated by a steam engine, a gas engine, or an electric motor; also, where the water-power is available, it can be operated by an impulse wheel or by a turbine; or, if the power is available from shafting in an adjacent building, the mixer may be operated by a chain or belt drive.

The superintendent or foreman of a reinforced-concrete job should take good care of the machinery in his charge. It is well for him to obtain from the manufacturers written directions as to the use and operation of their machines. Usually, these directions are explicit and are based on experience with the particular machine in question. As an example, the directions published by the Ransome Machinery Company are given in part as follows:

"**Rules for Operating a Mixer.**—If the machine is mounted on wheels, see that the weight is first taken off the wheels and carried on suitable sills, as shown in Fig. 8. The points of support should be beneath each of the roller shafts, beneath the bed of the engine, and beneath the boiler. The mixer frame should be carefully leveled in both directions. Remove the hook bolts that hold the drum to the frame

Fill all grease and oil cups and grease carefully the traction rings and roller faces. See that in all cases the lubricant is fed to the bearings. Use good graphite, hard oil, or grease in all compression cups, and screw the caps down so as to force the grease through the journal box. When the machine is in operation, a turn should be given on all compression cups at least once every 2 hr.

"**Instructions for Starting and Managing Boilers.**—See that all connections with the boiler are properly made and are tight. Fill the boiler up to or above the second gauge with water, and take particular notice while it is being filled that

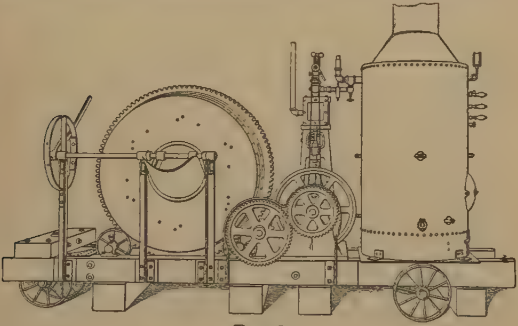


Fig. 8

all handhole plates and connections around it are tight; particularly note that the check-valve does not leak. Build a slow fire in the boiler until the water becomes hot; under no circumstances force the fire until after steam begins to generate; this can be determined by leaving the top gauge-cocks open until steam appears. After about 10 or 15 lb. of steam has been raised, note whether there are any slight leaks appearing in the boiler or its connections. After steam has been raised to the pressure to be carried, try the safety valve and be sure that it is in good working order. It is advisable to lift the safety valve from its seat at least twice

a day. Always carry the water in the boiler at a height that will best allow the engine to operate without carrying over water with the steam. It is always best to carry the water-line in the boiler as high as possible. Never allow the fire-door of the boiler to be opened except when firing the boiler. In checking steam, always close the ash-pit doors and damper in the stack; if this is not sufficient to check the steam, the fire should be banked. When shutting the boiler down at night, under no circumstances allow the fire-door to remain open.

"**Starting and Operating the Mixer.**—In starting the mixer, turn the machine over light a few times, meanwhile setting up such runways as may be required. See that the discharge chute is in position. Feed into the machine the amount of water required for the batch. Follow with stone, sand, and cement in the order named. Let the material remain in the machine about ½ min., which is long enough under average conditions, and then reverse the chute. Then discharge direct into wheelbarrows, buckets, or other vehicles, the whole batch or part, as desired. Reverse the chute and feed into the machine the next batch.

"In securing results as to output, watch the delivery side of the machine; get all the material out at once, so that the next batch can be mixing. If it is necessary to discharge only part at a time, use the largest cart or barrow that can be obtained.

"An occasional inspection of the journal boxes will guard against undue wear, which may result in bottoming the main gears, with disastrous results to both pinion and spur. Also watch that the rollers do not wear down so as to cause bottoming."

BUILDING LAWS, SPECIFICATIONS, AND COST

BUILDING LAWS

The construction of reinforced-concrete buildings is regulated by the laws of the municipality in which the work is carried on. These building laws differ for each city. The following laws are used in the City of Philadelphia. They are among the latest and may be considered as a good example.

"The term *reinforced concrete* shall be understood to mean an approved concrete mixture, reinforced by steel or iron of any shape, so that the steel or iron will take up all the tensional stresses and assist in the resistance to compression and shear.

"Before a permit to erect any reinforced-concrete structure is issued, complete specifications and drawings shall be filed with the Bureau of Building Inspection, showing all details of the construction, size and position of all reinforcing rods, stirrups, etc., and giving the composition and proportions of the concrete.

"The execution of the work shall be performed by workmen under the direct supervision of a competent foreman or superintendent.

"Reinforced-concrete construction will be accepted for fireproof buildings of the first class, if designed as hereinafter prescribed; provided, that the aggregate for such concrete shall be clean, broken, hard stone, or clean, graded gravel, together with clean siliceous sand or fine-grained gravel; should the concrete be used for flooring between rolled-steel beams, clean furnace clinkers entirely free of combustible matter, or suitable seasoned furnace slag may be used; when stone is used with sand or gravel it must be of a size to pass through a 1-in. ring, and 25% of the whole must not be more than one-half the maximum size; and provided further, that the minimum thickness of concrete

surrounding the reinforcing members of reinforced-concrete beams and girders shall be 2 in. on the bottom and 1½ in. on the sides of the said beams and girders. The minimum thickness of concrete under slab rods shall be 1 in. All reinforcement in columns to have a minimum protection of 2 in. of concrete.

"All the requirements herein specified for the protection of steel and for fire-resisting purposes shall apply to reinforced-concrete flooring between rolled-steel beams, as well as to reinforced-concrete beams and to entire structures in reinforced concrete. Any concrete structure or the floor filling in same, reinforced or otherwise, which may be erected on a permanent centering of sheet metal, of metal lathing and curved bars or a metal centering of any other form, must be strong enough to carry its load without assistance from the centering, unless the concrete is so applied as to protect the centering as herein specified for metal reinforcement.

"Exposed metal centering or exposed metal of any kind will not be considered a factor in the strength of any part of any concrete structure, and a plaster finish applied over the metal shall not be deemed sufficient protection unless applied of sufficient thickness and properly secured, as approved by the Chief of the Bureau of Building Inspection.

"All concrete shall be mixed in a mechanical batch mixe to be approved by the Bureau of Building Inspection, excep when limited quantities are required or when the conditio of the work makes hand mixing preferable; hand mixing t be done only when approved by the Bureau of Buildin Inspection. In all mixing the material shall be measure for each batch.

"When hand mixing is done under the aforesaid limita tions, the cement and fine gravel or coarse sand shall b first thoroughly mixed dry and then made into a mortar b gradually adding the proper amount of water. The crushe stone or gravel shall be spread out to a depth not to excee 6 in., in a tight box or upon a proper floor, and be sprinkle with water as directed; the mortar is then to be evenly sprea over the crushed stone, and the whole mass turned over

sufficient number of times to effect the thorough mixing of the ingredients.

"All forms and centering for concrete shall be built plumb and in a substantial manner, made tight so that no part of the concrete mixture will leak out through cracks or holes, or joints, and after completion shall be thoroughly cleaned, removing shavings, chips, pieces of wood and other material, and no debris of any kind shall be permitted to remain in the forms. All forms to be properly supported and braced in a manner to safely sustain the dead load that may be imposed upon them during construction.

"The reinforcing steel shall be accurately located in the forms and secured against displacement.

"Concrete shall be placed immediately after mixing.

"Whenever fresh concrete joins concrete that is set, the surface of the old concrete shall be roughened, cleaned, and spread with cement mortar, which mortar shall be mixed in proportions of 1 of cement to 2 of sand.

"Concrete shall not be mixed or deposited in freezing weather, unless precautions are taken to avoid the use of materials covered with ice or that are in any other way unfit for use, and that further precautions are taken to prevent the concrete from freezing after being put in place. All forms under concrete so placed to remain until all evidences of frost are absent from the concrete and the natural hardening of the concrete has proceeded to the point of safety.

"Concrete laid during hot weather shall be drenched with water twice daily, Sunday included, during the first week. The broken stone, if hot and dry, must be wet before going to the mixer.

"The time at which props, or shores, may safely be removed from under floors and roofs will vary with the condition of the weather, but in no case should they be removed in less than 2 wk.; provided, that column forms shall not be removed in less than 4 da.; provided further, that the centering from the bottom of slabs and sides of beams and girders may be removed after the concrete has set 1 wk.; provided, that the floor has obtained sufficient hardness to sustain the dead weight of the said floor and that no load or weight shall be

placed on any portion of the construction where the saic centers have been removed.

"The concrete for all girders, beams, slabs, and columns shall be mixed in the proportions of 1 of cement, 2 of sand o: fine gravel, and 4 of other aggregates, as before provided The concrete used in reinforced-concrete-steel constructioi must be what is usually known as a *wet* mixture. When the con crete is placed in water it must be placed in a semidry state

"Only Portland cement shall be permitted in reinforced concrete constructed buildings. All cement shall be testec in carload lots when so delivered or in quantities equal t same, and report filed with the Bureau of Building Inspectioi before using it in the work. Cement failing to meet th requirements of the accelerated test will be rejected.

"*Soundness, Accelerated Test.*—Pats of neat cement wil be allowed to harden 24 hr. in moist air, and then be sub mitted to the accelerated test as follows: A pat is expose in any convenient way in an atmosphere of steam, abov boiling water, in a loosely closed vessel, for 3 hr., afte which, before the pat cools, it is placed in the boiling wate for 5 additional hr.

"To pass the accelerated test satisfactorily, the pats sha remain firm and hard, and show no signs of cracking, di tortion, or disintegration.

"Such cements, when tested, shall have a minimum tensi strength as follows: Neat cement shall, after 1 da. in moi air, develop a tensile strength of at least 150 lb. per sq. in and after 1 da. in air and 6 da. in water shall develop a tensi strength of at least 500 lb. per sq. in.; and after 1 da. in a and 27 da. in water shall develop a tensile strength of least 600 lb. per sq. in. Cement and sand tests composed 1 part of cement and 3 parts of crushed quartz shall, aft 1 da. in air and 6 da. in water, develop a tensile strength at least 175 lb. per sq. in., and after 1 da. in air and 27 d in water shall develop a tensile strength of at least 240 l per sq. in. These and other tests as to fineness, set, et made in accordance with the standard method prescribed l the American Society of Civil Engineers, may, from time time, be required by the Bureau of Building Inspection.

"*Walls.*—Reinforced concrete may be used in place of brick and stone walls, in which cases the thickness may be two-thirds of that required for brick walls, as shown in the Schedule, Section 18 of the Act of Assembly, No. 123, of the Commonwealth of Pennsylvania, approved June 5, 1901, provided the unit stresses as set forth in these regulations are not exceeded.

"Concrete walls in such cases must be reinforced in both directions in a manner to meet the approval of the Chief of the Bureau of Building Inspection.

"*Steel.*—All reinforcements used in reinforced concrete shall be of standard grade of structural steel or iron of either grade to meet the Manufacturers' Standard Specifications, revised February 3, 1903.

"Reinforced-concrete slabs, beams, and girders shall be designed in accordance with the following assumptions and requirements:

"(*a*) The common theory of flexure to be applied to all beams and members resisting bending.

"(*b*) The adhesion between the concrete and the steel is sufficient to make the two materials act together.

"(*c*) The design shall be based on the assumption of a load four times as great as the total load (ordinary dead load plus ordinary live load).

"(*d*) The steel to take all the tensile stresses.

"(*e*) The stress-strain curve of concrete in compression is a straight line.

"(*f*) The ratio of the moduli of elasticity of concrete to steel:

Stone or gravel concrete	1 to 12
Slag concrete	1 to 15
Cinder concrete	1 to 30

"The allowable unit transverse stress upon concrete in compression:

	LB. PER SQ. IN.
Stone or gravel concrete	600
Slag concrete	400
Cinder concrete	250

"The allowable unit stress in tension:

	Lb. per Sq. In
Iron	12,000
Steel	16,000

"The allowable unit shearing strength upon concrete:

	Lb. per Sq. In
Stone or gravel concrete	75
Slag concrete	50
Cinder concrete	25

"The allowable unit adhesive strength of concrete:

	Lb. per Sq. In
Stone or gravel concrete	50
Slag concrete	40
Cinder concrete	15

"The allowable unit stresses upon concrete in direct com-pression in columns:

	Lb. per Sq. I
Stone or gravel concrete	500
Slag concrete	300
Cinder concrete	150

"The allowable unit stress upon hoop columns composed stone or gravel concrete shall not be over 1,000 lb. per sq. in figuring the net area of the circle within the hooping. Th percentage of longitudinal rods and the spacing of the hoo to be such as to permit the concrete to safely develop t above unit stress with a factor of safety of 4.

"When steel or iron is in the compression sides of bean the proportion of unit stress taken by the steel or iron sh be in the ratio of the modulus of elasticity of the steel or ir to the modulus of elasticity of the concrete; provided th the rods are well tied with stirrups connecting with the low rods of the beams; provided, further, that when rods are us in compression, the approval of the Chief of the Bureau Building Inspection must be obtained.

"In the design of structures involving reinforced-concre beams and girders, as well as slabs, the beams and gird shall be treated as T beams, with a portion of the slab acti as flange in each case. The portion of the slab that may used to take compression shall be dependent upon the ho zontal shearing stress that may exist in the beam, and in

case shall the slab portion exceed twenty times the thickness of the slab.

"All reinforced-concrete T beams must be reinforced against the shearing stress along the place of junction of the rib and the flange, using stirrups throughout the length of the beam. When reinforced-concrete girders carry reinforced-concrete beams, the portion of the floor slab acting as flange to the girder must be reinforced with bars near the top, at right angles to the girder, to enable it to transmit local loads directly to the girder and not through the beams, thus avoiding an integration of compressive stresses due to simultaneous action as floor slab and girder flange.

"In the execution of the work in the field, work must be so carried on that the ribs of all girders and beams shall be monolithic with the floor slabs.

"In all reinforced-concrete structures special care must be taken with the design of joints to provide against local stresses and secondary stresses due to the continuity of the structures.

"Shrinkage and thermal stresses shall be provided for by the introduction of steel.

"In the determination of bending moments due to the external forces, beams and girders shall be considered as simply supported at the ends, no allowance being made for continuous construction over supports. Floor slabs, when constructed continuously, and when provided with reinforcement at top of slab over the supports, may be treated as continuous beams, the bending moment for uniformly distributed loads being taken at not less than $\frac{WL}{10}$. In the case of square floor slabs, that are reinforced in both directions and supported as continuous beams on all four sides, the bending moment may be taken at $\frac{WL}{20}$. In floor slabs in juxtaposition to the walls of the building, which are acting as simple beams, the bending moment shall be considered as $\frac{WL}{8}$ when reinforced in one direction. If the floor slab is square and reinforced in both directions, but acting as a

simple beam, the bending moment shall be taken as $\frac{WL}{16}$.

"When the shearing stresses developed in any part of a reinforced-concrete building exceed the shearing strength as fixed in this section, a sufficient amount of steel shall be introduced in such a position that the deficiency in the resistance to shear is overcome.

"When the safe limit of adhesion between the concrete and steel is exceeded, provision must be made for transmitting the strength of the steel to the concrete.

"Reinforced concrete may be used with the unit stresses, previously given, for columns in which the ratio of the length to least side or diameter does not exceed fifteen. If more than fifteen diameters, the allowable stress shall be decreased proportionally. Reinforcing rods that are introduced for lateral stresses must be tied together at intervals of not more than the least side or diameter of the columns.

"Longitudinal reinforcing rods will not be considered as taking any direct compression.

"The contractor must be prepared to make load tests in any portion of a reinforced-concrete building within a reasonable time after erection, and as often as may be required by the Chief of the Bureau of Building Inspection. The tests must show that the construction will sustain a load equal to twice the calculated live load without signs of cracks."

COST DATA

The cost of reinforced-concrete construction depends on the local cost of labor and materials. In making an approximate estimate of the cost of a building, unit prices per cubic yard or per square foot of floor surface may be used. These prices are obtained from the cost of similar buildings already erected in the same locality. In making an exact estimate of the cost of a building, the materials can usually be accurately determined and their cost found from the current market prices. The cost of labor, including the placing and mixing of concrete, and the cost of form construction are usually more difficult to determine. The latter cost is greatly

influenced by the amount of experience local carpenters have had in this class of work.

In assuming approximate unit prices for reinforced concrete, it is customary to include in the estimate the cost of the forms; thus, the cost of reinforced-concrete floor and wall construction per cubic yard will usually be between \$16 and \$20. This price includes the steel reinforcement, the cost of the labor on the forms, and the cost of the concrete. This cost per cubic yard increases as the thickness of the wall decreases.

The cost of reinforced-concrete floor systems can be estimated at a square foot. The total cost per square foot of floor, including the steel reinforcement, form work, etc., will usually be found to vary between 60 and 65 cents.

It must be understood that the preceding figures are subject to considerable changes in different sections of the country. They indicate only that such costs can be obtained fairly accurately for certain localities, for a period of 1 or 2 yr.

Sometimes the cost of a building is estimated per square foot of floor surface, this cost including also the cost of the walls, etc. Thus, if it costs in a certain locality \$1.50 per square foot of floor to build a concrete mill building, the approximate cost on a proposed building two stories high, 50 ft. wide, and 200 ft. long, can be found as follows: The floor space on each floor is $200 \times 50 = 10{,}000$ sq. ft. For the two floors, the surface is 20,000 sq. ft., and at \$1.50 per square foot, the price is $20{,}000 \times \$1.50 = \$30{,}000$. This cost is for the building complete. It includes ordinary plumbing, painting, etc., such as would be necessary for a factory building. If the building were more ornamental, the unit cost would be taken higher—at perhaps \$2 per square foot, or whatever the current figures in the same locality happened to be.

In making an exact estimate of a reinforced-concrete building, the following suggestions will be of interest:

The principal steel reinforcement is carefully estimated. For most styles of patented bars or round and square bars, the manufacturers give weights that correspond to various

sections, as steel reinforcement is bought by the pound Tables of these weights will be found on pages 253 to 265 If such weights cannot be found in manufacturers' catalog or elsewhere, the following rule may be used:

Multiply the sectional area of the steel in square inches b its length in feet and by 3.4. The product will be the weigh of the steel in pounds. Usually it is not necessary to estimat the quantity of steel used for stirrups, as 10% of the mai reinforcement is a close approximation. This 10% include such additional steel as would be used for column ties wiring, etc. It should be noted that the different sizes o steel should be kept separate, as in most localities the pric of steel per pound increases as the size of the bar decreases

It is not necessary in estimating the volume of concret to deduct the volume taken up by the steel reinforcements After the volume of concrete has been obtained, the amoun of sand, cement, and broken stone can be estimated fron the table on page 244. The cost of mixing concrete is in th neighborhood of 30 cents per cu. yd., using labor at $1.5 per da. In localities where the price is higher, this figur must be increased. Machine-mixed concrete costs fron 50 to 60% as much as hand-mixed concrete.

The entire cost of mixing, wiring, and placing, includin foreman's wages, in a large eastern city amounted to $1.3 per cu. yd. However, this value must be determined fo every different locality having different scales of wages.

Sand is usually bought by the ton, but measured by th cubic yard. One long ton is equal to about 1 cu. yd. of dr sand. The cost of steel reinforcement varies in differer localities, and no exact figures can be given. Usually, it is of the girder-frame type, a man can place in the forn about 1,600 lb. of reinforcement in a day. If it consists loose rods, one man can place from 800 to 1,200 lb. per d

The cost of form work cannot be given in general term The estimator must know the quality of forms to be mad and measure up the approximate amount of wood require If possible, the forms should be used two or three times the same job, and they should have a certain selling pri after the work is completed.

The amount of carpenter work on the forms is variable, depending on the experience of the carpenters and the intricacy of the work. Usually, a carpenter can erect 20 lin. ft. of column form in a day, or 40 sq. ft. of floor and girder form. These figures are for smooth, finished work, and not for rough work.

SPECIFICATIONS

CONDENSED SPECIFICATIONS FOR REINFORCED-CONCRETE WORK

The following specification is the briefest form that can be used in specifying reinforced-concrete construction. It briefly states the requirements of the architect regarding the submission of the construction for approval, the character of the concrete and the steel reinforcement, and the floor load upon which the calculation for the design of the floor system is based.

REINFORCED-CONCRETE CONSTRUCTION

General.—The reinforced-concrete construction shall be of an approved and successful type, and the contractor shall submit for the architect's approval framing plans, and details of construction, together with schedules showing the amount of steel reinforcement in all the beams, girders, and columns.

The contractor is to assume all responsibility for the safety and protection of the work during construction, and shall deliver the same in a complete and finished condition in compliance with the specifications and the accompanying plans.

Concrete.—The concrete for beams, girders, and slabs shall be proportioned of 1 part Portland cement, $2\frac{1}{2}$ parts sand, 5 parts broken stone or gravel. The concrete for columns shall be a 1–2–4 mixture.

Cement.—The cement shall conform to the requirements of the standard specifications for cement adopted by the American Society for Testing Materials, on November 14, 1904.

Aggregates.—The sand shall be clean, and not contain over 3% of loam. Broken stone and gravel shall be hard and close-grained, and free from dust and dirt; they shall be of such size as to pass through a ring 1 in. in diameter.

Reinforcing Steel.—The steel for the reinforcement shall be manufactured by the open-hearth process. This steel should have an ultimate tensile strength of from 60,000 to 70,000 lb. per sq. in. and an elastic limit of at least half that amount, with an elongation of at least 20%. A bar shall bend cold through an angle of 180° and close down on itself without showing signs of fracture. High-carbon steel or steel with an elastic limit greater than 45,000 lb. per sq. in. shall not be used.

The beam and girder reinforcement shall consist of square-twisted or approved type of deformed bar, while the slab reinforcement may be of square-twisted bars, deformed bars, or of an approved form of expanded metal or woven or electrically welded wire fabric. The column rods may be of plain round or square-twisted bars. The girders and beams must have the rods so bent and arranged as to provide amply for the bending moments throughout their length and also to take care of the negative bending moments near the bearings and supports. Sufficient stirrups must be provided near the bearings or throughout the length of the beams and girders to resist the horizontal, oblique shearing, and tensile stresses.

Floor and Test Loads.—The floors and columns throughout shall be designed to sustain a live floor load of 150 lb. per sq. ft. The finished floor systems shall be tested in two places, to be designated. The test load shall be equal to one and one-half times the amount of the live floor load. The test loads shall be so placed as to fully load the girders and beams of one bay.

MEMORANDA

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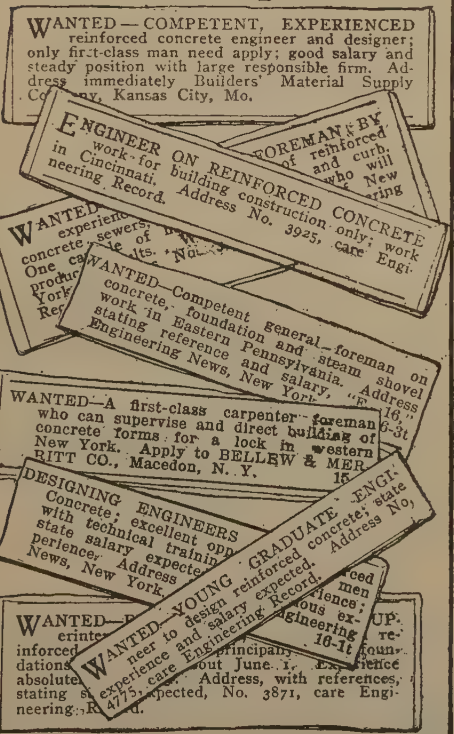
Superintendent of Construction of Million-Dollar Concrete Building

There is now being built in San Bernardino, Cal., a great precooling plant for citrus fruits that will cost one million dollars. The superintendent of construction in charge of this plant, which is built of reinforced concrete, is Wm. L. Snook, 1643 W. Thirty-Fifth Street, Los Angeles, Cal., an I. C. S. trained man. Mr. Snook states: "For the last 3 years I have been superintendent of construction of reinforced-concrete buildings. At the present time I am superintending the construction of the largest ice and precooling plant of its kind in the country."

Many Opportunities for Concrete Engineers



Field Engineer in United States Service

When I enrolled I was doing odd jobs wherever I could get work to do. Through study of my Course I have filled important positions in the United States Reclamation Service, Inspector in the General Land Office, and other United States Government positions, ranging in salary from $75 to $200 a month. I am now Field Engineer with the United States Reclamation Service in charge of the location and construction for the irrigation of about 25,000 acres of land on the Payette-Boise Project.

I am enrolled for the Concrete Engineering Course. The Schools have done wonders for me and I highly recommend them to any one who wishes to learn and advance and be worth more to their employers.

W. W. Pefley,
215 N. 14th St., Boise, Idaho

CREDITS HIS SUCCESS TO HIS COURSE

Heber Scott, Box 408, Denison, Tex., has been a student of the Schools for about 2 years. At the time of his enrolment he was attending high school. Last summer he served on a preliminary survey of the county roads as chainman, rodman, etc. When the ancient wooden bridges were to be replaced with concrete, he was appointed concrete inspector on all bridge and concrete work, with an advance in his salary of 20 per cent. He says that he will take pleasure in recommending the schools to any one, since he believes his Course with the I. C. S. was the sole cause of his advancement.

NOW PROPRIETOR

M. F. Landreth, Box 92, Wenatchee, Wash., was working for the State University, taking care of the buildings, shrubbery, lawns, etc., at the time of his enrolment with us. At that time he had only a grammar-school education. Today he holds a one-third interest in the Wenatchee Concrete Block Company and is also a cement and concrete building contractor.

NOW A LICENSED ARCHITECT

Miles E. Miller, Sharon Block, Salt Lake City, Utah, was a carpenter when he enrolled for his first Course with the I. C. S.—the Building Contractors'—enrolling afterward for Structural Engineering and Concrete Engineering Courses. He has made such good use of the knowledge gained from these that he is now a licensed architect in the state of Utah.

CEMENT WORKS SUPERINTENDENT EARNS $5,000 A YEAR

Norman L. Warford, Concrete, Wash., was working at carpentry, earning $2.50 a day, when he woke up to the fact that he needed more knowledge of engineering to get ahead, and enrolled for an I. C. S. Course. He says this was the best thing he ever did, since it has enabled him to climb the ladder in the business of cement manufacturing, until he was able to build and equip the Washington Portland Cement Company's plant, having a capacity of 1,800 barrels a day. He is now superintendent of this plant at a salary of $5,000 a year.

BUILDING A MILLION-DOLLAR PIER

Although he had considerable practical experience in concrete work, Mr. Wm. E. Chesney, Bower Lodge, Halifax, N. S., Canada, enrolled for the Concrete Engineering Course and, by hard study, completed it in about 1 year. As a result of this study he was appointed to the position of inspector of concrete construction on the new concrete pier for the Intercolonial Railway of Canada, with a substantial increase in salary. The new pier is the first of its kind to be constructed in Halifax harbor and will, when finished, cost $1,000,000.

Now Earns $300 a Month

M. P. Kellogg, of San Diego, Cal., enrolled for and studied in the I. C. S. Complete Architectural Course. For 16 years he had been a carpenter and contractor. After studying his Course he began the practice of architecture. Mr. Kellogg has won several prizes, over many competitors, for the excellence of his designs.

In a letter to us regarding his progress Mr. Kellogg says: "My success is due to the instruction received from the International Correspondence Schools. I am well satisfied with your system of teaching; in fact, I think it as good as personal instruction. I consider my Scholarship the best investment I ever made—it has been worth several times the cost to me. My Bound Volumes have been especially helpful. I shall be glad to answer any letters regarding the Schools and my Course with them. My business now nets me about $300 a month."

MACHINIST TO STRUCTURAL DESIGNER

Edmund B. La Salle, Batavia, Ill., was employed as a machinist when he enrolled in the Structural Engineering Course of the I. C. S. He was ambitious, of course, to become a structural draftsman and designer, but the numerous books he bought were found to be a waste of money. After studying the Course, Mr. La Salle obtained work as a draftsman. Recently, he has accepted a position in Chicago as structural designer, at a salary of $130 a month. He designs standpipes and steel tanks; also towers for water-supply stations.

HIS COURSE BROUGHT RAPID INCREASE

Ernest N. Woodbeck, 133 Crawford Ave., Detroit, Mich., was learning the business of carpenter for reinforced concrete work when he enrolled with the I. C. S. for a Concrete Engineering Course. His studies made it possible for him to become layerout of work from building plans. He had only a seventh grade education when he enrolled, but in 18 months he increased his salary nearly 40 per cent.

NOW OWNS A PROSPEROUS BUSINESS

Wm. F. Gleason, 709 Crawford Ave., Augusta, Ga., was earning $2.50 a day as a carpenter when he enrolled for the Complete Architectural Course with the I. C. S. This enabled him to become superintendent of construction for a concrete building block and tile company. He has since established a prosperous business of his own as a member of the Skinner & Gleason Contracting Company, builders, making a specialty of concrete construction and brick work. His income is at least $2,000 a year.

TEMPORARY EMPLOYMENT MADE PERMANENT

Harry L. Ackerley, 3 Richard St., West Lynn, Mass., says: "I first enrolled for a Course in Architecture. After taking the preliminary studies I applied for a position as draftsman and was given 'temporary' employment. I remained with the concern 4 years. I then enrolled for an Electrical Engineering Course, which has been invaluable to me. I am now employed by the General Electric Company and my salary has been increased 350 per cent."

INCOME INCREASED $500 A YEAR

Charles E. Werking, Hagerstown, Ind., a foreman carpenter, enrolled for the I. C. S. Complete Architectural Course. He says, "I never entered into anything I enjoyed more and gained so much." Mr. Werking is now an architect, with income increased from $750 a year to $1,250 a year.

Won Prizes in Five Contests

The fact that my house plans have won prizes in five different contests argues well for the Course I took with the International Correspondence Schools.

My early education was derived from the public schools. At the age of 15 I built a rowboat so well that I was advised to take up carpentering. I did so, serving my term with Henry Lorenz, of Matamoras, Pa., and working with the builders around Port Jervis. Then I decided to go into the building contracting business for myself, and knowing that a knowledge of architecture and drafting was necessary, I enrolled with the I. C. S., studying evenings, rainy days, and odd times until I earned my Diploma. I now do considerable planning and drafting, and I am satisfied that I will soon be able to give up all other work. The Course was a great help to me in many ways. I say that no mechanic has any chance to advance these days unless he takes up a good course of study along the line of his trade, such as the International Correspondence Schools give.

C. A. Wagner,
Port Jervis, N. Y.

NOW OWNS A CONCRETE FACTORY

Not being able to afford a college education, CHAS. F. CULP, New Rockford, N. Dak., enrolled with the I. C. S. for the Complete Architectural Course, paying for it with the first money he earned after becoming of age. At the time, his wages as a carpenter were only $1.25 a day. He is now a prosperous contractor and builder, being also the owner of a factory for the making of cement products. He attributes his success very largely to his work with the I. C. S.

CARPENTER TO $10-A-DAY DRAFTSMAN

ELMER B. CUTTS, Hinckley, Ill., was working for his father as carpenter, earning 17½ cents an hour at hard labor, when he took up a course of study in the I. C. S. The knowledge gained from the Course as placed him in the position of chief draftsman for the Robert Cutts Construction Company. He prepares plans for various dwellings, hotels, churches, and public buildings, earning $10 a day.

NOW SUPERINTENDENT

FRANK FREE, Greenfield, Ohio, was a common carpenter employed some eight months a year at $2.70 a day, when he enrolled for the Complete Architectural Course, and afterward for the Concrete Engineering Course. He is now superintendent of construction for Samuel Hannaford & Sons, Cincinnati, Ohio, engaged on reinforced concrete and other construction. His aslary is $1,500 a year.

NOW AN ARCHITECT

EDWARD THAL, 702 Ohio Building, Toledo, Ohio, was earning $18 a week as a paper hanger and painter when he enrolled with the Schools for the Complete Architectural Course. He is now an architect and engineer, making a specialty of reinforced concrete, with an income several hundred per cent. larger than when he enrolled.

NOW IS STRUCTURAL DRAFTSMAN

C. B. GILBERT, care Philadelphia Engineering Company, Philadelphia, Pa., prior to becoming a student in the I. C. S. Structural Engineering Course was general utility man in a shoe factory, with little or no chance to rise. Before he had completed his I. C. S. Course he secured a position as structural draftsman with the Niles-Bement-Pond Company. His prospects are bright and his income is greater.

CARPENTER TO CONTRACTOR

FRANK J. BERDEL, 333 Columbus Ave., Canton, Ohio, writes: "I enrolled for the Building Contractors' Course three years ago while employed as a carpenter, receiving $2.50 a day for 10 hours' work. After studying evenings for one year I started contracting. Though but 25 years old my income is $12 a day."

Now Reinforcement Contractor

I am a pretty busy man these days, but I must send you a little message of appreciation concerning your Concrete Engineering Course. Two years ago I was a machinist's helper in a railroad shop, earning 23 cents an hour. But I was ambitious and when your representative approached me I soon saw that I would make the mistake of my life if I did not sign up for a Course. I found your Concrete Engineering Course a veritable gold mine. I immediately secured work on a reinforced concrete building. Before this was completed I was tendered the position of steel foreman on a large reinforced concrete building in Portland, at just double my former salary. I am now a reinforcement contractor, bending, fabricating and placing steel reinforcement in all classes of reinforced concrete construction. I have now four large contracts under way. I am thankful for the fact that a representative of the I. C. S. yanked me out of my hole and pointed out a way for me to stay out.

C. G. Wilford, Jr.,
147 Front St.,
Portland, Ore.

BETTERED HIS POSITION AND SALARY

CHAS. A. CARPENTER, 16 West 3d St., Fulton, N. Y., was a steam fitter, earning $65 a month, when he enrolled with the Schools, at the age of 35, for a Mechanical Drawing Course and afterward for the Mechanical Engineering Course. He is master mechanic of reinforced concrete work for the Walter Bradley Concrete Company. His Courses have enabled him to design several new machines, one of which saves the company 25 per cent. of the cost of production. His salary has increased about 50 per cent. since he began to study his first Course.

FROM BRICKLAYER TO GENERAL CONTRACTOR

W. C. DAVIS, Michigan Building, Wichita, Kans., was working as a bricklayer, with no prospect of advancement, when he enrolled with the Schools for the Complete Architectural Course. After successfully superintending the construction of a large warehouse he became a member of the H. I. Ellis Construction Company, employing from 50 to 100 men. He says that his Course enables him to superintend the field work of the company, especially in the line of masonry and concrete. His income has increased at least 300 per cent.

PASSED A SUCCESSFUL EXAMINATION

OTIS M. TOWNSEND Eighth St., Ocean City, N. J., was an apprentice boy when he enrolled with the Schools for the Complete Architectural Course. He is now an architect and builder employing 40 carpenters and keeping a draftsman in his office. His Course enabled him to pass the state examination successfully and receive a certificate as an architect.

SALARY INCREASED 400 PER CENT.

When M. F. MAYNARD, 810 Belleville Ave., New Bedford, Mass., enrolled for the Building Contractors' Course he was a carpenter, earning $1.75 a day. He gives the I. C. S. the credit for establishing him as a contractor and builder and increasing his income about 400 per cent.

FROM POVERTY TO PROSPERITY

H. L. THOMAS, Englewood, Colo., enrolled in the I. C. S. while out of work, without money, and with a family of six to support. Just one year after starting the I. C. S. Course he secured a position as draftsman, at $55 a month. Soon he got a better position at $75 a month, laying out work, and making plans and designs, all of which he could do without instruction other than that received from the Course. At present Mr. Thomas is with the Denver Union Water Company, as structural engineer.

Earns $10,000 a Year

I enrolled with the I. C. S. for a Complete Architectural Course in the spring of 1901, at that time being a stone cutter drawing an average of $15 a week. As you will note, my advancement has been quite rapid. In the fall of 1901 I took the foremanship over 35 stone cutters; during the winter was promoted to superintendent of the general mason force. During 1902 was appointed superintendent of general construction for the State of Ohio, which position I held for 2½ years. Then I accepted the position of general superintendent of construction with a general contracting firm in Cleveland, afterward obtaining an interest in the business. After the lapse of 4 years I disposed of my interest and started an architectural and engineering office. I am handling some of the largest work in this section—mercantile, amusement, and hotel work. I employ from three to four men in the field and six to eight in the office. My present income is from $10,000 to $12,000 a year. I have every reason to be grateful to the I. C. S.

GEO. A. GRIEBLE,
510-12 Columbia Building,
Cleveland, Ohio

AN ENTHUSIASTIC STUDENT

J. W. Wylie, 115 Church St., Chester, S. C., was working as a farmer when he enrolled with the I. C. S. Since his early education had been greatly neglected, he found the mathematical part of his Course to be very useful in making estimates. He now takes contracts for cement work and declares that he is grateful for what the Schools have done for him.

HIS COURSE HELPED TO GET CONTRACTS

F. H. Schwartz, Galesburg, Ill., purchased our Concrete Engineering Course for the purpose of increasing his knowledge in that particular line. He found, when it became known that he was studying an I. C. S. Course, that he could obtain contracts which he would not have received if his knowledge stopped at the mixture of one-two-three.

THE I. C. S. DOES WHAT THE UNIVERSITY DOESN'T

After Ira S. Dole, Lewistown, Idaho, had spent 1 year in study on his I. C. S. Course, in 1910, he completed the freshman year in the State University. Beginning work as a contractor during the next December, he did some $8,000 worth of work. He says that the I. C. S. does some things for its students which are not as a rule accomplished at a university. It teaches him to dig out things for himself.

CARPENTER BECOMES FOREMAN

J. F. Harrington, Missoula, Mont., before enrolling in the I. C. S., was a journeyman carpenter, earning $4.50 a day. The Course of study taught him to understand building plans so that he has secured the position of foreman and earns $7 a day.

TEN TIMES HIS FORMER SALARY

When the National Convention of Carpenters met in Scranton some years ago, John G. Garbart, 16 Verner Avenue, Ingram, Pa., visited the I. C. S. Instruction Department. What he saw convinced him that he needed a Course and he enrolled in our School of Architecture. He was then earning $1.50 a day. His Course has enabled him to purchase the business of his former employer and he now earns ten times what he did when he enrolled.

NOW EARNS $5,000 A YEAR

Ed. J. Weaver, Youngstown, Ohio, considers that the work he did with the I. C. S. on his two Courses, Architectural Drawing and Complete Architectural, is the foundation of his success in his chosen profession. When he enrolled he was a machine hand in a planing mill, earning $2 a day. He is now a practicing architect, and has earned $5,000 a year for several years past.

Now a Successful Architect

I got my first lessons from the International Correspondence School of Civil Engineering several years ago. At that time I was working as a clerk in an express office at $40 a month. I had also wasted about $200 and several months of time in a business college. Through your assistance and instruction I was enabled to advance to the position of chief estimator for a large concern. Then I enrolled for the Structural Engineering Course and soon was offered a good position as a building designer, where I advanced very rapidly. The general public began to take notice of my achievements in modern building construction and I obtained many good jobs to handle on the side. Up to the first of this year I kept my position, but the demand for my grade of work was so great and I had so much work on hand that it was utterly impossible for me to retain my position. So I opened an office and started in business for myself. I have on the books work to the amount of over two hundred and twenty-five thousand dollars for this summer, on which my commission will be about twelve thousand dollars. I think this is not so bad for the first year.

RALPH M. SNYDER,
Fort Wayne, Ind.

Erected the Largest Reinforced Concrete Building in the World

Erik Holman, Room 657, Pacific Building, San Francisco, Cal., had worked as a carpenter and contractor on a small scale for some years before he enrolled with the I. C. S. for the Complete Architectural Course. Having successfully superintended the erection of the first large reinforced concrete building in Los Angeles, he was called upon to erect the Pacific Building, corner of Market and Fourth Sts., San Francisco, Cal., costing $1,500,000, said to be the largest reinforced concrete office building in the world. He is now a consulting engineer, with an average income of $300 a month.

From the New Jersey State Board of Architects

The New Jersey State Board of Architects in its second annual report, 1904, in recommendations under the heading of "Books for Study," makes the following statement indorsing the International Correspondence Schools' Courses of study:

"The Board having received numerous inquiries concerning the extent of the preparation necessary to fit one to engage in an examination for a certificate to practice architecture in the State, it has been thought best that the following information be issued.

"The Board recommends, where it is possible, that the student complete a full course in architecture at some well-known university or other institution of learning. A diploma of graduation supplemented by a proper amount of experience is sufficient evidence to the Board that the applicant for a certificate is worthy of one and that the formal examination may be waived.

"The Board realizes that to many this is impossible and to such it can recommend the Courses of study conducted through the mails by the International Correspondence Schools, Scranton, Pa. Any one who has taken one of these Courses will be well prepared to pass the Board's examination."